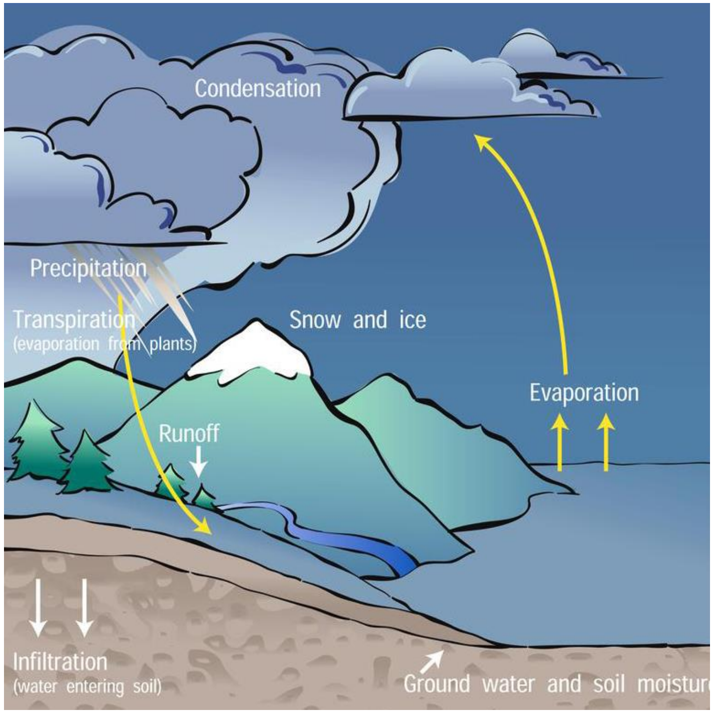
เอกสารประกอบการสอน

วศยธ 323 อุทกวิทยา (Hydrology)

ภาควิชาวิศวกรรมโยธาและสิ่งแวดล้อม คณะวิศวกรรมศาสตร์ มหาวิทยาลัยมหิดล

ผศ. ดร. อารียา ฤทธิมา



LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

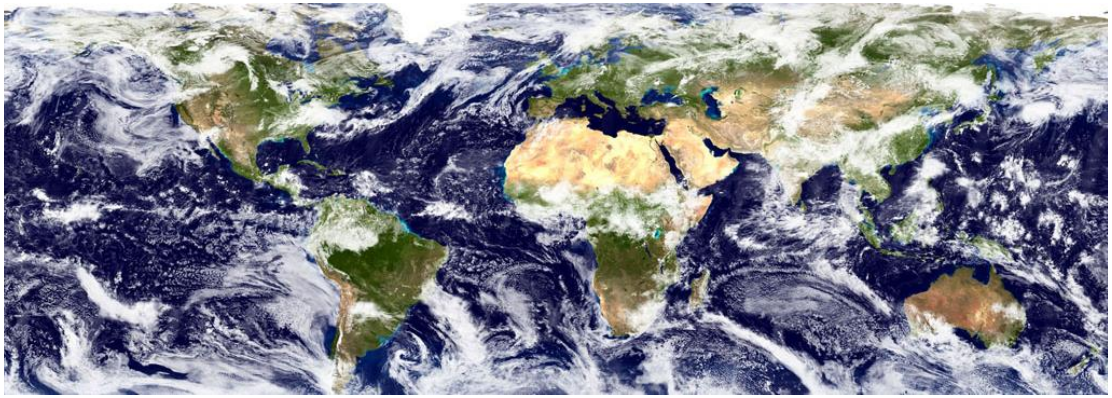
LECTURE OUTLINE

Introduction to Hydrology

- Hydrologic Cycle
- Systems Concept
- Hydrologic System Model

HYDROLOGY

- **Hydrology** is the scientific study of the movement, distribution, and quality of water on earth and other planets, including the water cycle, water resources and environmental watershed sustainability.
- A practitioner of hydrology is a "**Hydrologist**", working within the fields of earth or environmental science, physical geography, geology or civil and environmental engineering.
- The practical application of hydrology is called "**Applied Hydrology**".



APPLIED HYDROLOGY

Applied hydrology are found in such tasks as;

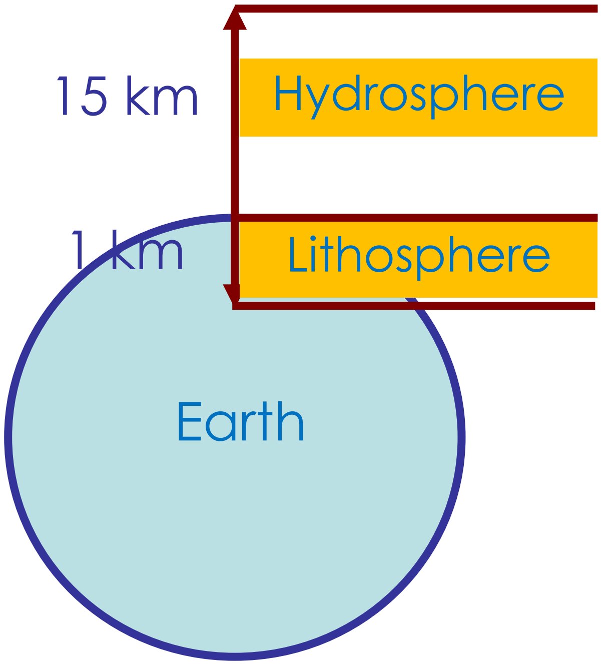
- Design and operation of hydraulic structures
- Water supply
- Wastewater treatment and disposal
- Irrigation
- Drainage
- Hydropower generation
- Flood control
- Navigation
- Erosion and sediment control
- Salinity control
- Pollution abatement
- Recreation use of water
- Fish and wildlife protection

APPLIED HYDROLOGY

Branches of Hydrology;

- **Chemical Hydrology**
  is the study of the chemical characteristics of water.
- **Ecohydrology**
  is the study of interactions between organisms and the hydrologic cycle.
- **Hydrogeology**
  is the study of the presence and movement of water in aquifers.
- **Hydroinformatics**
  is the adaptation of information technology to hydrology and water resources applications.
- **Hydrometeorology**
  is the study of the transfer of water and energy between land and water body surfaces and the lower atmosphere.
- **Surface Hydrology**
  is the study of hydrologic processes that operate at or near the Earth's surface.

HYDROLOGIC CYCLE



Water on earth exists :

- in a space called **Hydrosphere** (15 km up into the atmosphere)
- in the crust of the earth (1 km down into the **Lithosphere**

Water circulates in the hydrosphere through the maze of paths constituting the "**Hydrologic Cycle**".

HYDROLOGIC CYCLE

The area near the surface of the earth can be divided up into 4 parts :

Lithosphere : is the solid rocky crust covering entire planet. It covers the entire surface of the earth from the top to the bottom.
Hydrosphere : is composed of all of the water on or near the earth.
Biosphere : is composed of all living organisms.
Atmosphere : is the body of air which surrounds our planet.

- 79% - Nitrogen
- 21% - Oxigen
- The small amount remaining is CO_2 and other gases.

HYDROLOGIC CYCLE

Hydrologic Cycle

Describes the continuous movement of water on, above, and below the surface of the earth.

- Hydrologic cycle is also known as water cycle or hydrological cycle.
- Water can change states among liquid, vapor, and ice at various places in the water cycle.
- The cycle has no beginning or end.
- Its processes occur continuously.
- The mass balance of water on earth remains fairly constant over time but the partitioning of the water into the major reservoirs of ice, fresh water, saline water and atmospheric water is variable depending on a wide range of climatic variables.
- The hydrologic cycle is the central focus of hydrology.

HYDROLOGIC CYCLE

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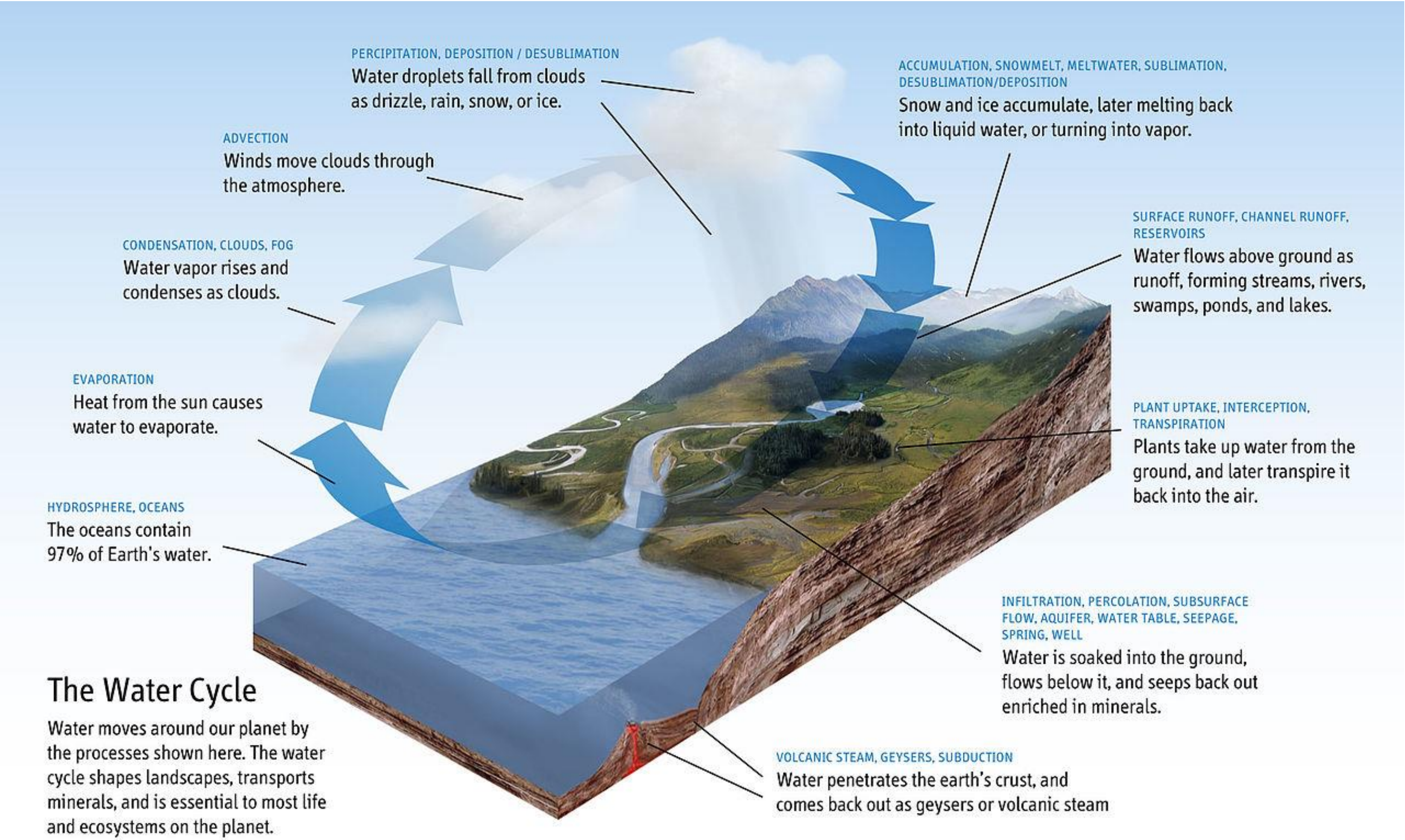
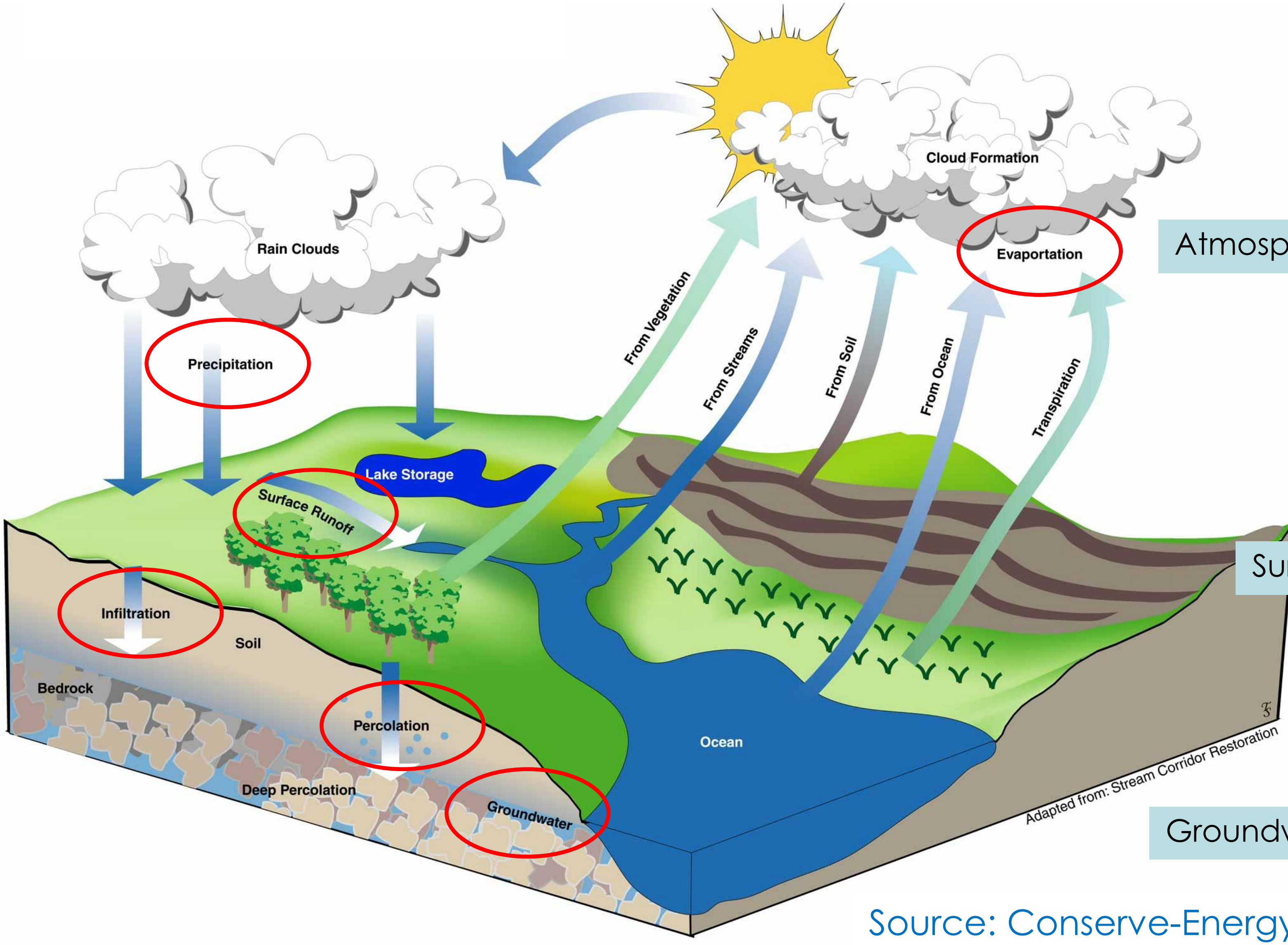


Source: Wikipedia (2018)

HYDROLOGIC CYCLE



Atmospheric Subsystem

Surface Subsystem

Groundwater Subsystem

Source: Conserve-Energy-Future (2018)

HYDROLOGIC CYCLE

Hydrologic Cycle
The water moves from one reservoir to another such as from a river to ocean, or from ocean to the atmosphere by the physical processes of evaporation, condensation, precipitation, infiltration, runoff, and sub-Surface flow.

Surface Runoff
Surface runoff is the excess water flows over the land. It occurs when soil is infiltrated to full capacity.

Transpiration
The evaporation of water from plants through their leave.

Interception
If the surface is covered by dense vegetation, much of precipitation may be held on leaves and plant limbs and stems.

WORLD WATER QUANTITY

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Estimated World Water Quantities

Item	Area (10^6 km^2)	Volume (km^3)	Percent of total water	Percent of fresh water
Oceans	361.3	1,338,000,000	96.5	
Groundwater				
Fresh	134.8	10,530,000	0.76	30.1
Saline	134.8	12,870,000	0.93	
Soil Moisture	82.0	16,500	0.0012	0.05
Polar ice	16.0	24,023,500	1.7	68.6
Other ice and snow	0.3	340,600	0.025	1.0
Lakes				
Fresh	1.2	91,000	0.007	0.26
Saline				
Marshes				
Rivers	148.8	2,120	0.0002	0.006
Biological water	510.0	1,120	0.0001	0.003
Atmospheric water	510.0	12,900	0.001	0.04
Total water	510.0	1,385,984,610	100	
Fresh water	148.8	35,029,210	2.5	100

96.5% of all the earth's water is in the oceans.

1.7% of all the earth's water is in the groundwater.

1.7% of all the earth's water is in the polar ices.

0.1% of all the earth's water is in the surface and atmospheric water system.

Source: Chow et al. (1988)

AVERAGE WATER BALANCE

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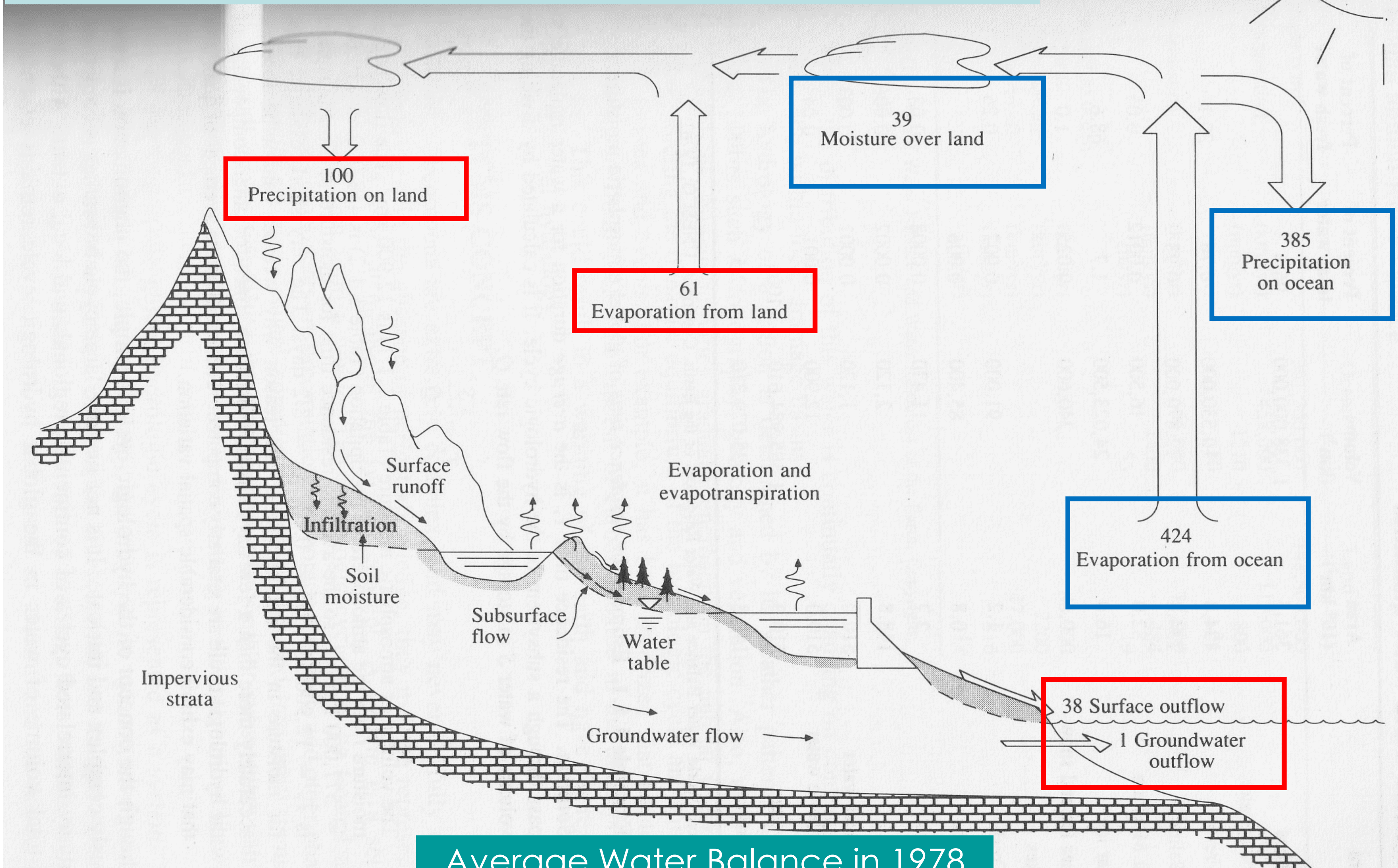
Hydrologic cycle with global annual average water balance



Average Water Balance in 1978

AVERAGE WATER BALANCE

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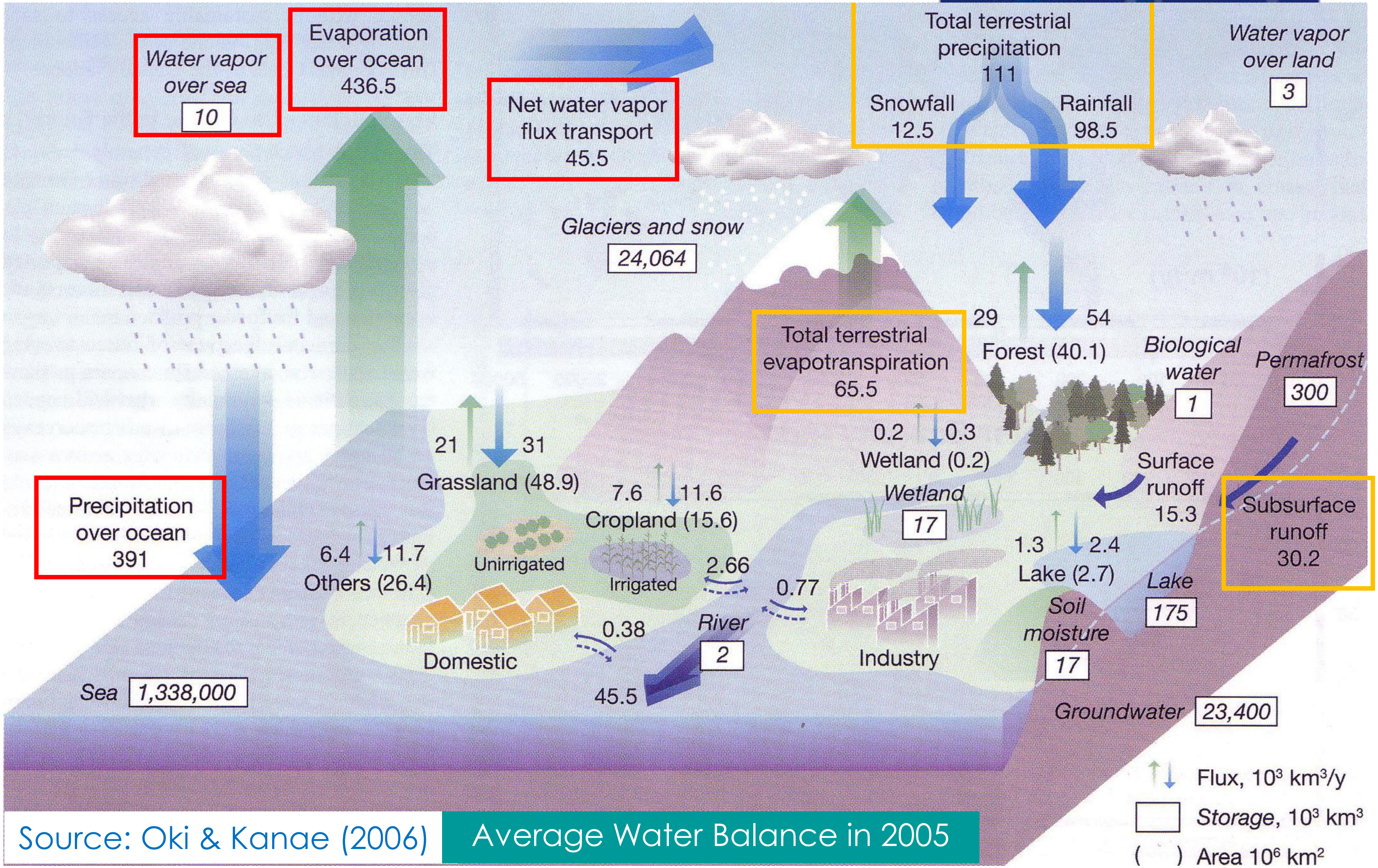

Source: Oki & Kanae (2006)

Average Water Balance in 2005

GLOBAL ANNUAL WATER BALANCE

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Global Annual Water Balance

			Ocean	Land
Area (km^2)			361,300,000	148,800,000
Precipitation	100%	(km^3/yr)	458,000	119,000
		(mm/yr)	1270	800
		(in/yr)	50	31
Evaporation	61%	(km^3/yr)	505,000	72,000
		(mm/yr)	1400	484
		(in/yr)	55	19
Runoff to ocean	39%			
Rivers		(km^3/yr)	–	44,700
Groundwater		(km^3/yr)	–	2200
Total runoff		(km^3/yr)	–	47,000
		(mm/yr)	–	316
		(in/yr)	–	12

Source: Chow et al. (1988)

RESIDENCE TIME: EXAMPLE 1

Estimate the residence time of global atmospheric moisture.

$$T_r = \frac{S}{Q}$$

Tr = Residence time (the average duration for a water molecule to pass through a subsystem of the hydrologic cycle).

S = Volume of water

Q = Flow rate

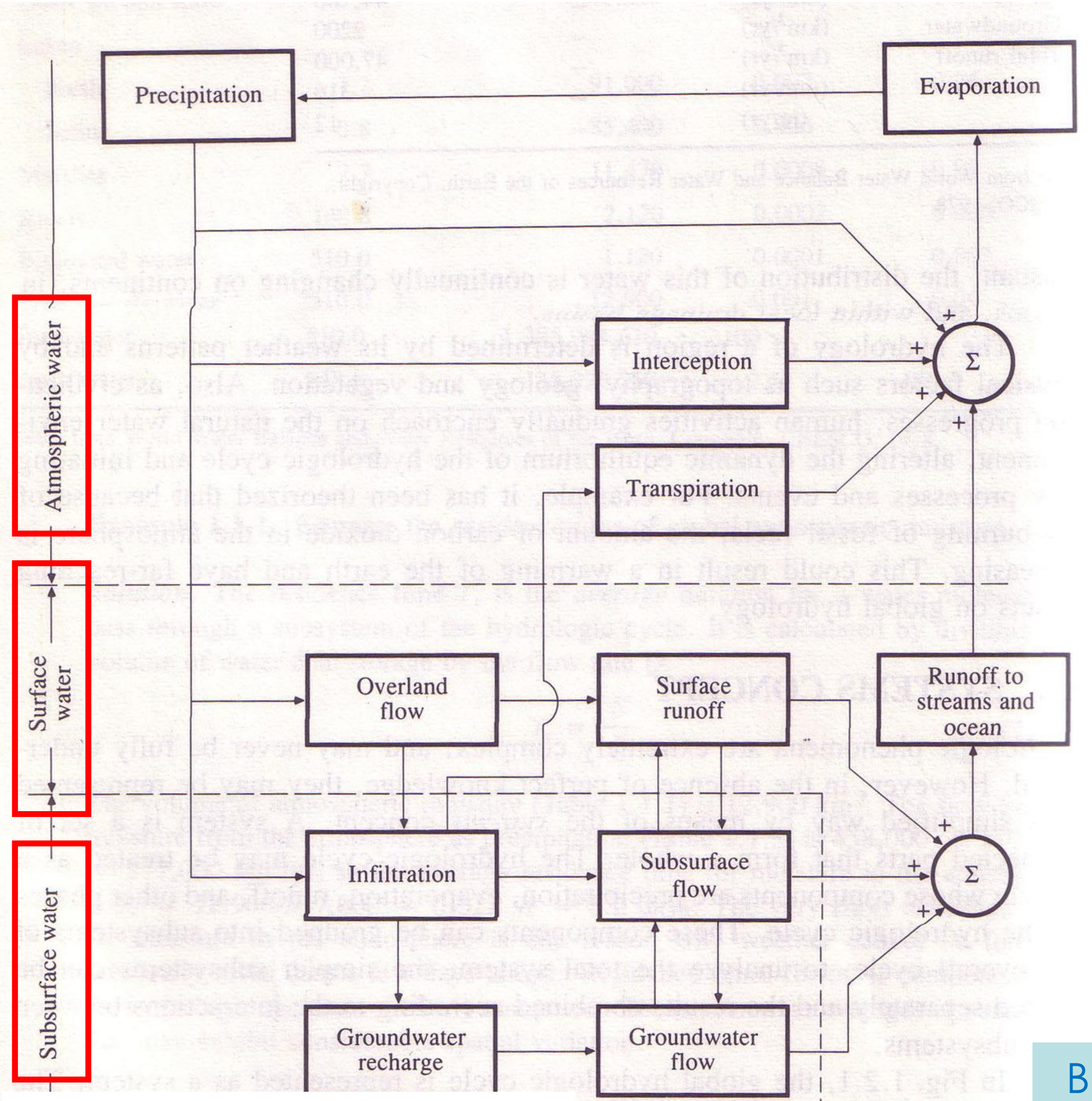
$S = 12{,}900\ km^3$ (Table)

$$Q = 458{,}000 + 119{,}000\ \frac{km^3}{yr}\ \text{(Table)}$$

$$= 577{,}000\ \frac{km^3}{yr}$$

Tr = S/Q = 12,900/577,000 = 0.033 years = 8.2 days

SYSTEM CONCEPT



Source: Chow et al. (1988)

Global hydrologic cycle is represented in a simplified way by means of "The System Concept"

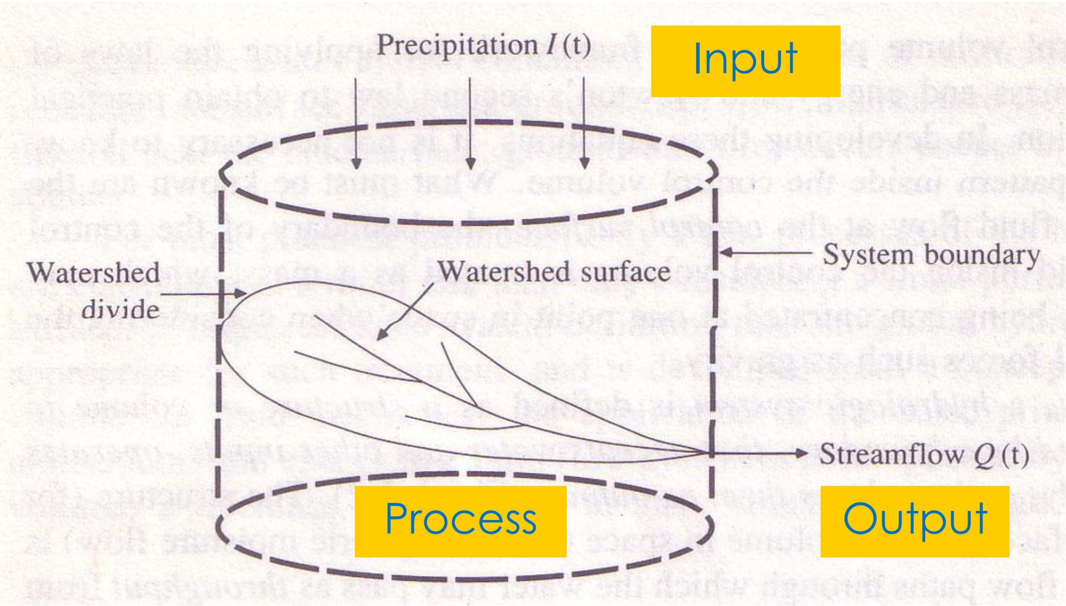
Most hydrologic system is inherently random, because their major input is precipitation, a highly variable and unpredictable phenomena.

The statistical analysis plays a large role in hydrologic analysis.

Block diagram representation of the global hydrologic system

SYSTEM CONCEPT: EXAMPLE 2

Represent the storm rainfall-runoff process on a watershed as a hydrologic system.



A Watershed as a Hydrologic System

A watershed is the area of land draining into a stream at a given location.

The watershed divide is a line dividing land whose drainage flows toward the given stream from land whose drainage flows away from that stream.

For practical problems, only a few processes of hydrologic cycle are considered at a time, and only considering a small portion of the earth's surface.

BASIC EQUATION OF HYDROLOGIC CYCLE

Simple Hydrologic System Model

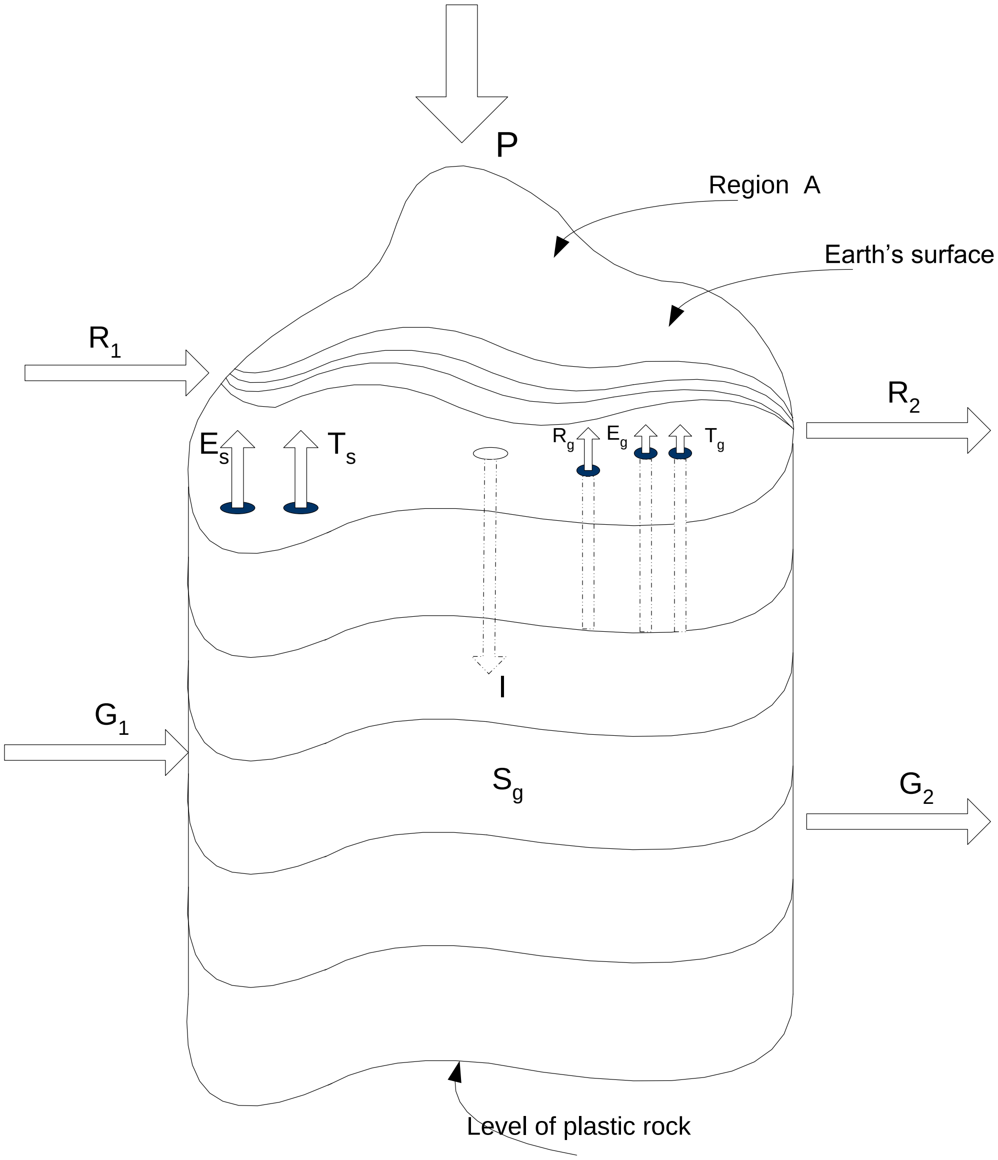
Mass Balance Equation ; I-Q=dS/dt

I = Input (volume/time)

O = Output (volume/time)

dS/dt = Time rate of change of storage

BASIC EQUATION OF HYDROLOGIC CYCLE



P=Precipitation
E=Evaporation
T=Transpiration
R=Surface Runoff
G=Groundwater Flow
R_g=Subsurface Flow
I=Infiltration
S=Storage
s=Land Surface
g=Groundwater

BASIC EQUATION OF HYDROLOGIC CYCLE

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- **Water Budget in Land Surface**

$$(P+R_1+R_g)-(R_2+E_s+T_s+I)=\Delta S_s \qquad (1)$$

- **Water Budget in Groundwater**

$$(I+G_1)-(G_2+R_g+E_g+T_g)=\Delta S_g \qquad (2)$$

$$P-(R_2-R_1)-(E_s+E_g)-(T_s+T_g)-(G_2-G_1)= \Delta S_s+\Delta S_g \qquad **(1)+(2)$$

R (Net Surface Flow)=R_2-R_1
E (Net Evaporation)=E_2+E_1
T (Net Transpiration)=T_s+T_g
G (Net Groundwater Flow)=G_2-G_1
$\Delta S= \Delta S_s+\Delta S_g$

$$P-R-E-T-G=\Delta S$$

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Chow, V.T., Maidment, D.R., & Mays, L.W. (1988). *Applied hydrology*. New York: McGraw-Hill Book Company.

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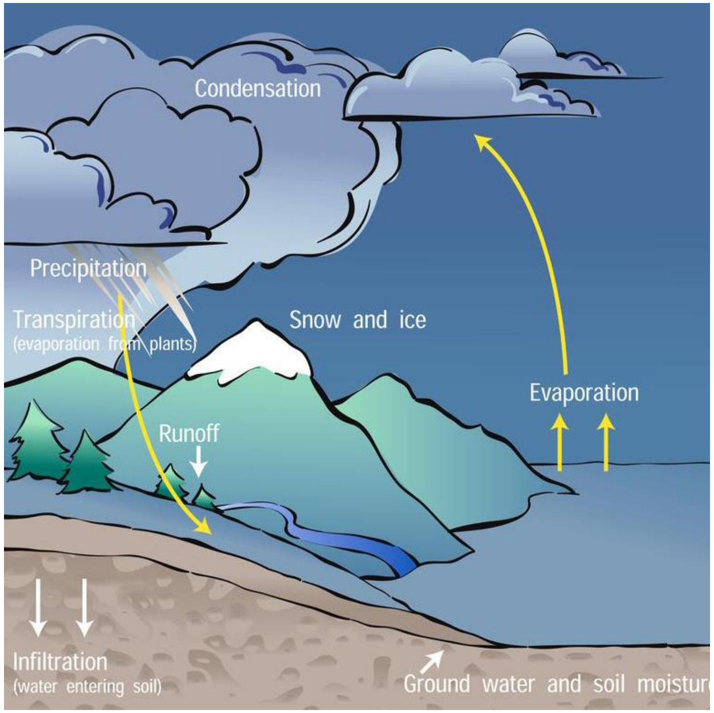

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Hydrologic Phenomenon

- Global Warming and Climate Change
- El Nino

HYDROLOGIC PHENOMENON

Hydrologic Phenomenon

Hydrologic phenomenon is an observable movement of water within the water cycle.

It includes;

- Precipitation
- Extreme Flood
- Severe Drought
- Global warming
- El Nino
- Cyclone
- etc.

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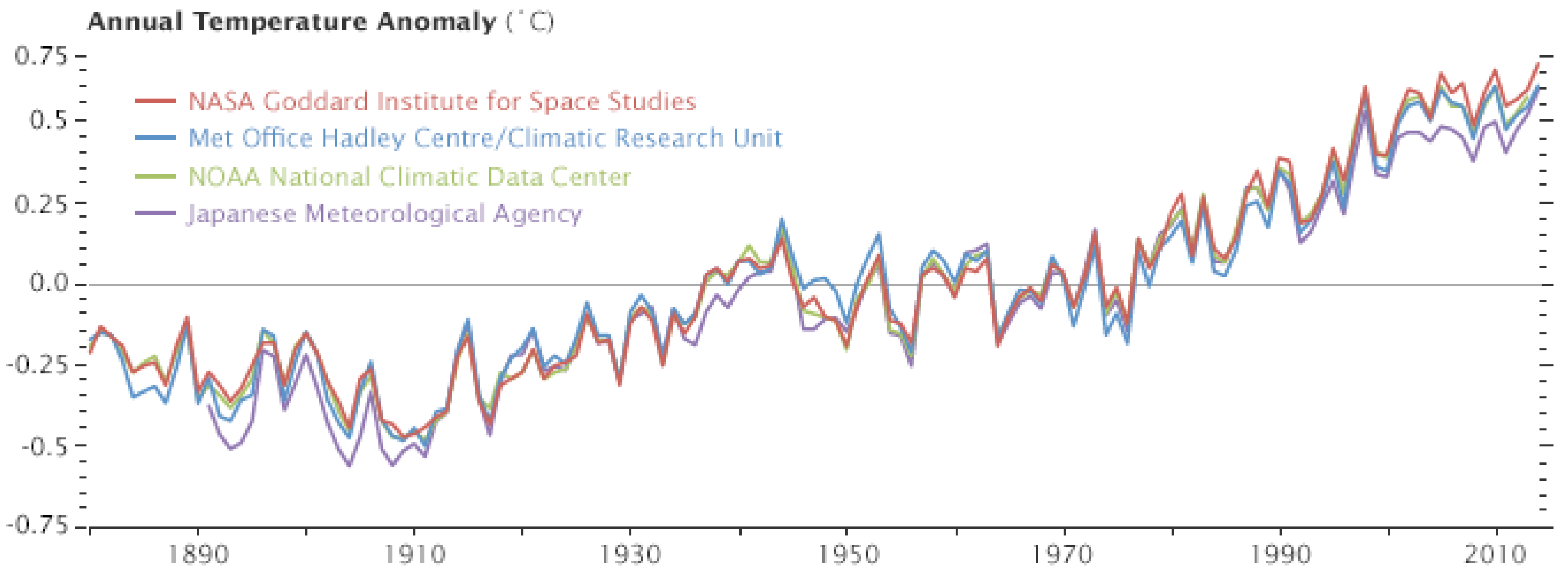
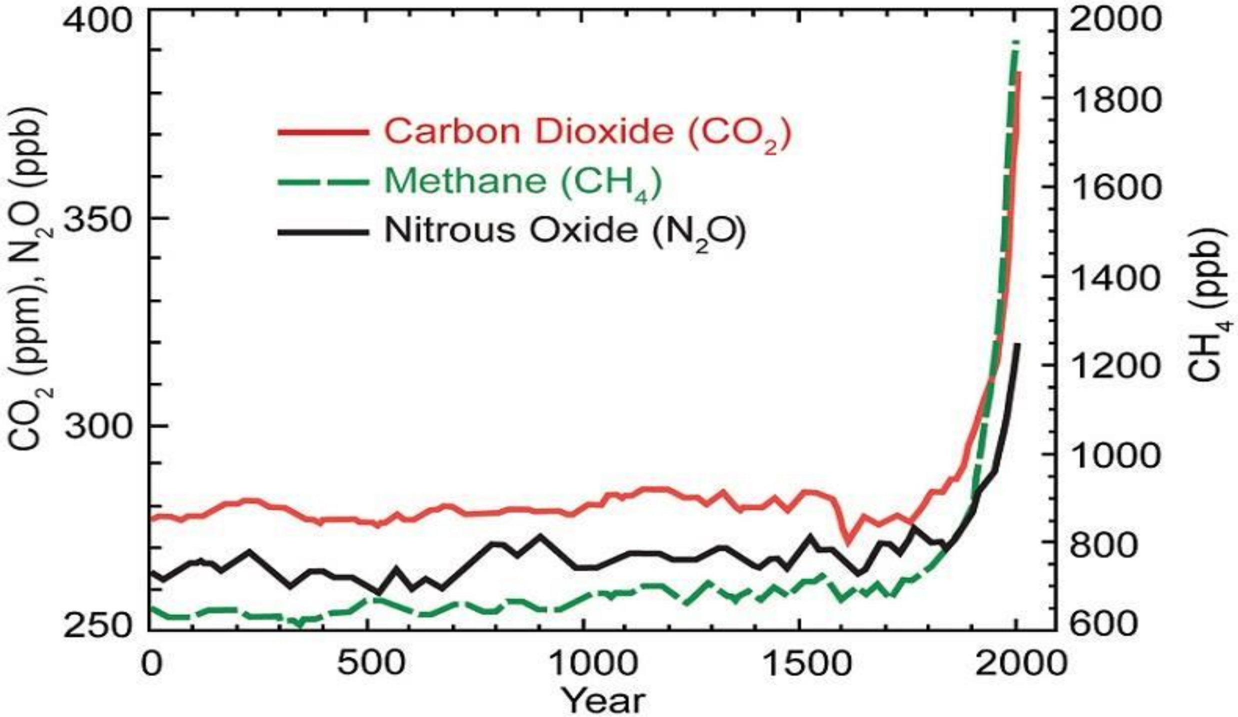
Global Warming and Climate Change

Global warming and climate change refer to an increase in average global temperatures. Natural events and human activities are believed to be contributing to an increase in average global temperatures. This is caused primarily by increases in greenhouse gases such as Carbon Dioxide (CO_2).



Source: NASA (2018)

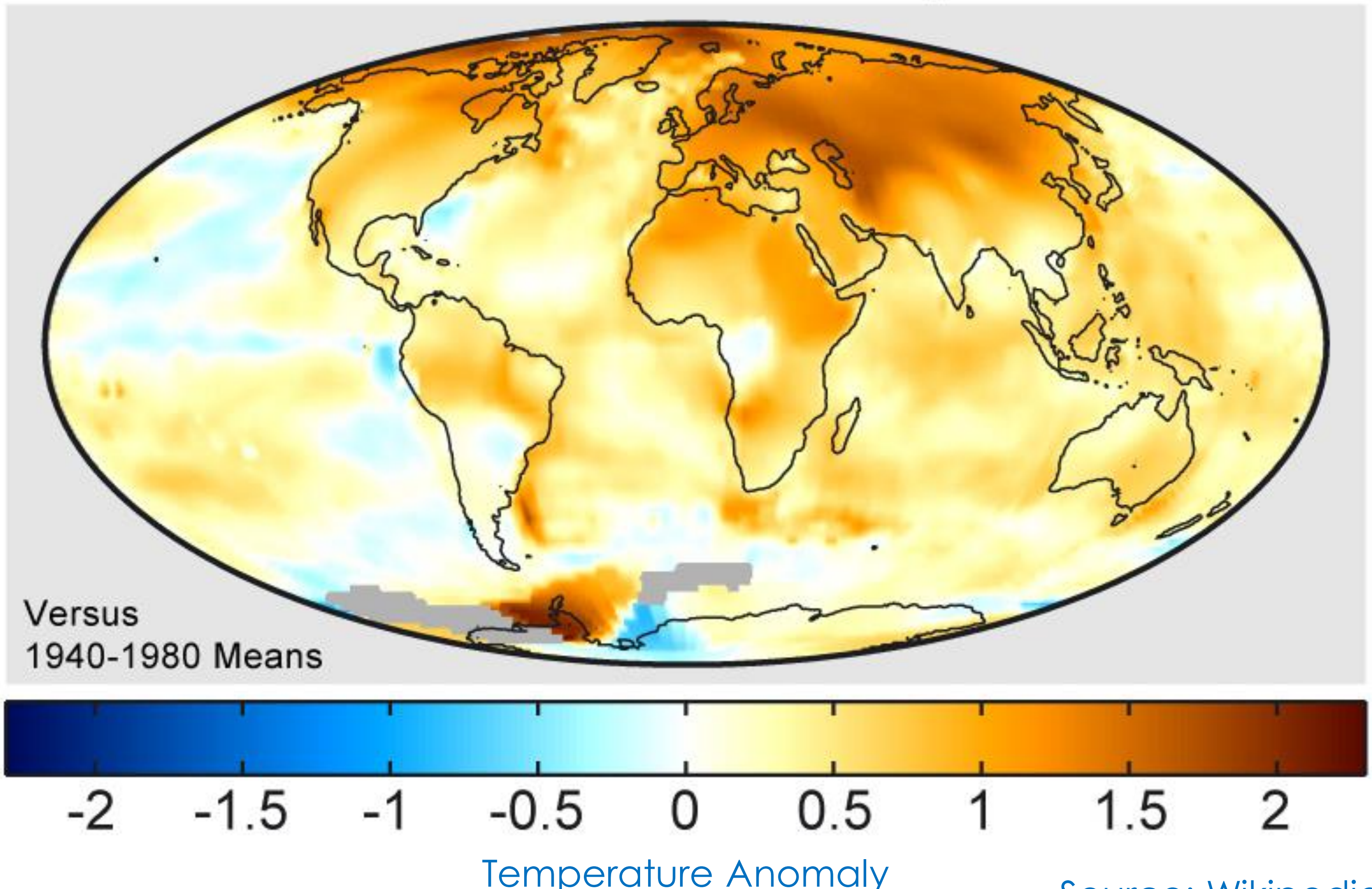
Changes in Carbon Dioxide-Methane-Nitrous Oxide



Source: Mcpherson (2018)

HYDROLOGIC PHENOMENON

1999-2008 Mean Temperatures

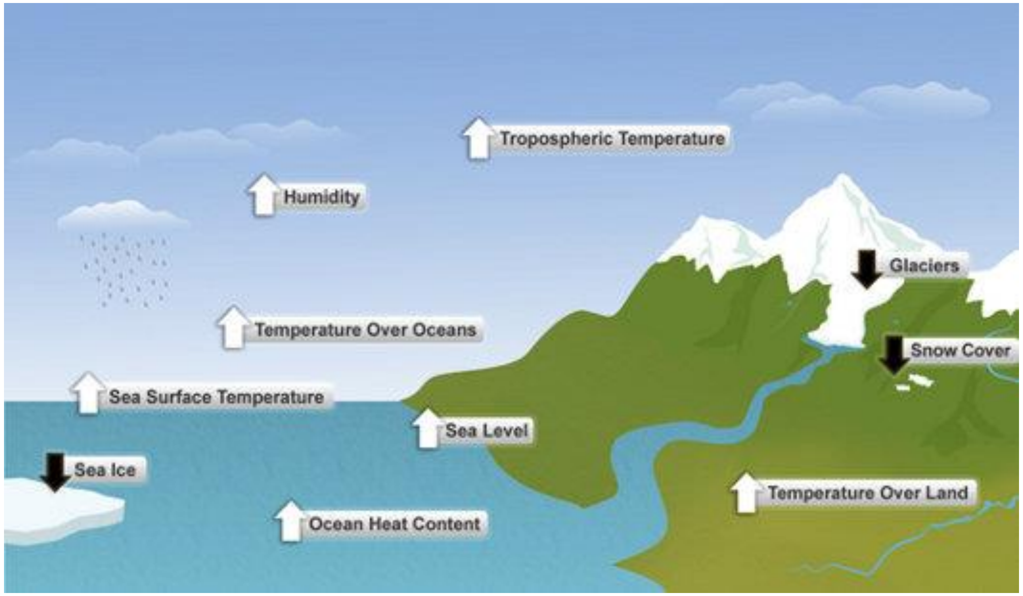


Source: Wikipedia (2018)

CLIMATE CHANGE

What are the main indicators of Climate Change?

As explained by the US agency, the National Oceanic and Atmospheric Administration (NOAA), there are 7 indicators that would be expected to increase in a warming world and 3 indicators would be expected to decrease.



7 indicators increased;

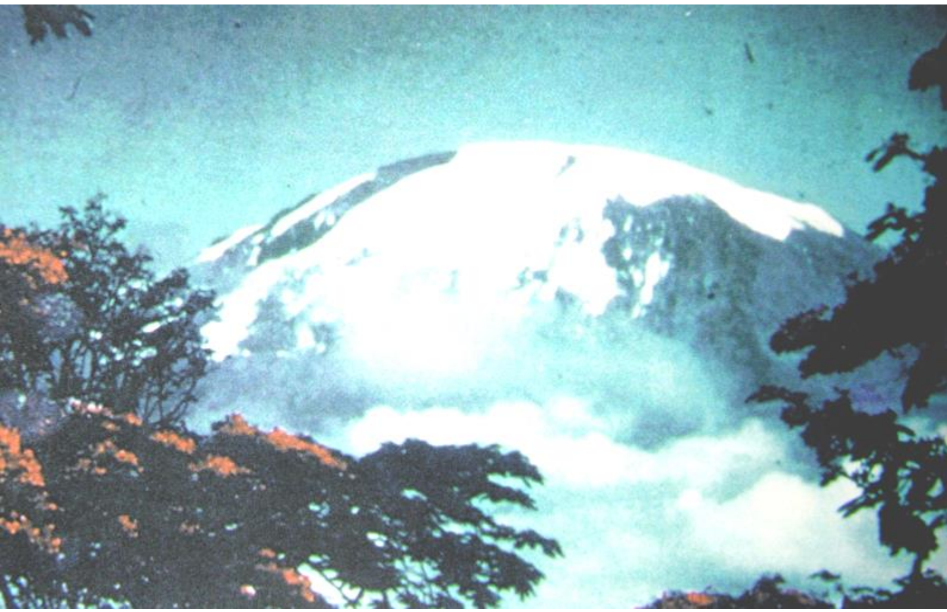
- Temperature over land
- Temperature over oceans
- Sea level
- Ocean heat content
- Sea surface temperature
- Humidity
- Tropospheric temperature

3 indicators decreased;

- Sea ice
- Glaciers
- Snow covers

Source: NOAA (2015)

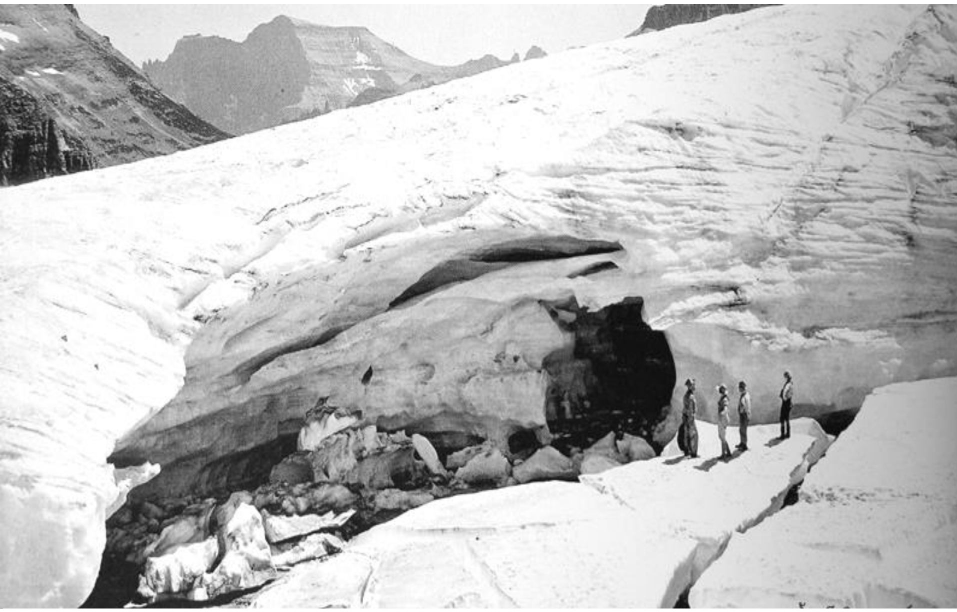
EFFECTS OF CLIMATE CHANGE



In 1970

In 2000

EFFECTS OF CLIMATE CHANGE



In 1932

In 1938

EFFECTS OF CLIMATE CHANGE

In 1978

In 2006

EFFECTS OF CLIMATE CHANGE

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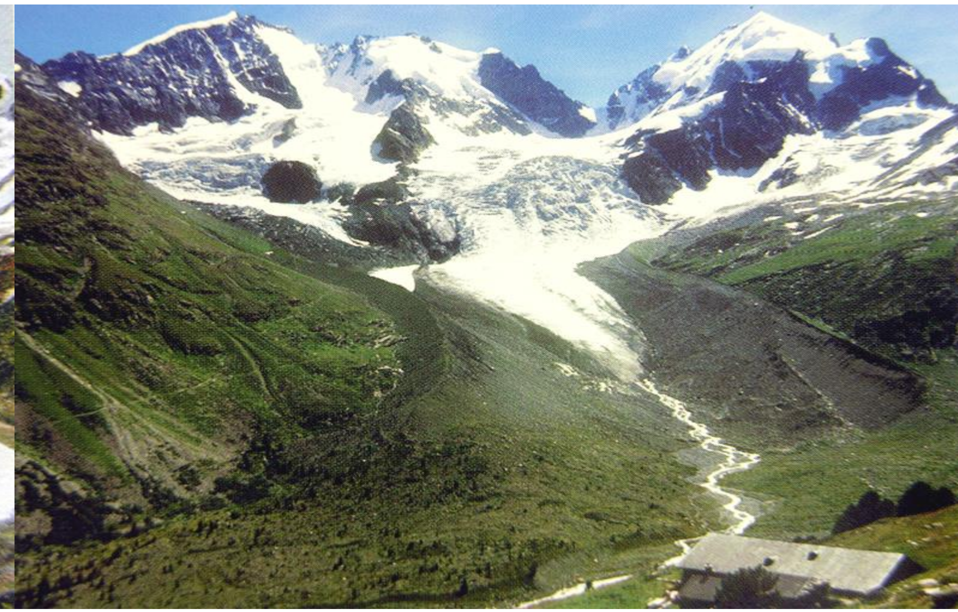
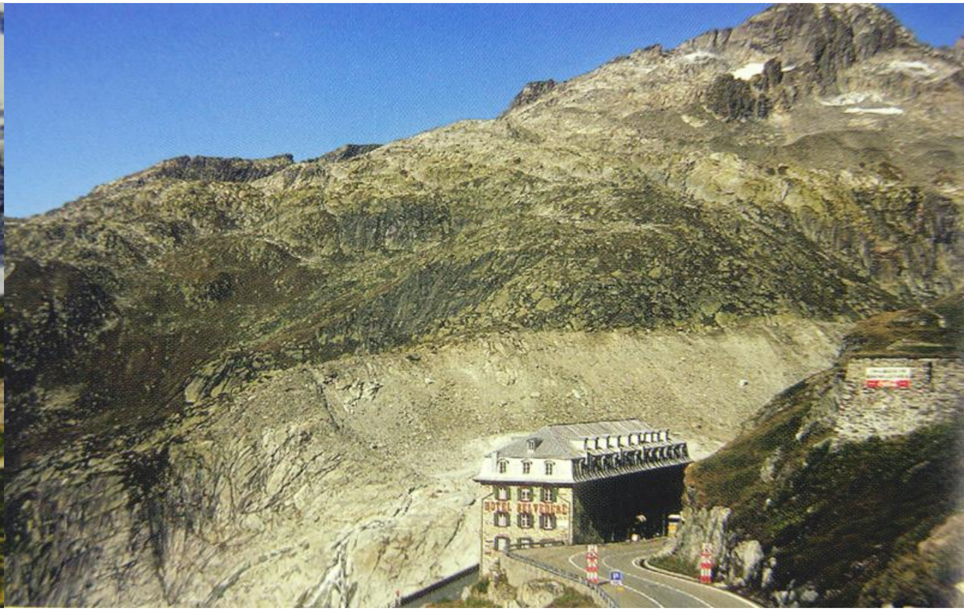
In 1910



In 2001

EFFECTS OF CLIMATE CHANGE

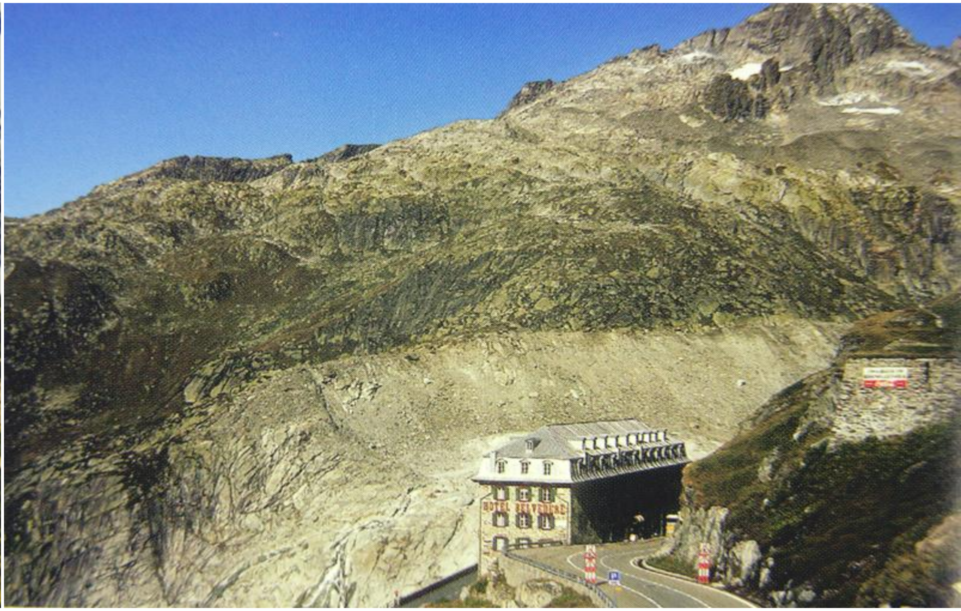
In 1906



In 2003

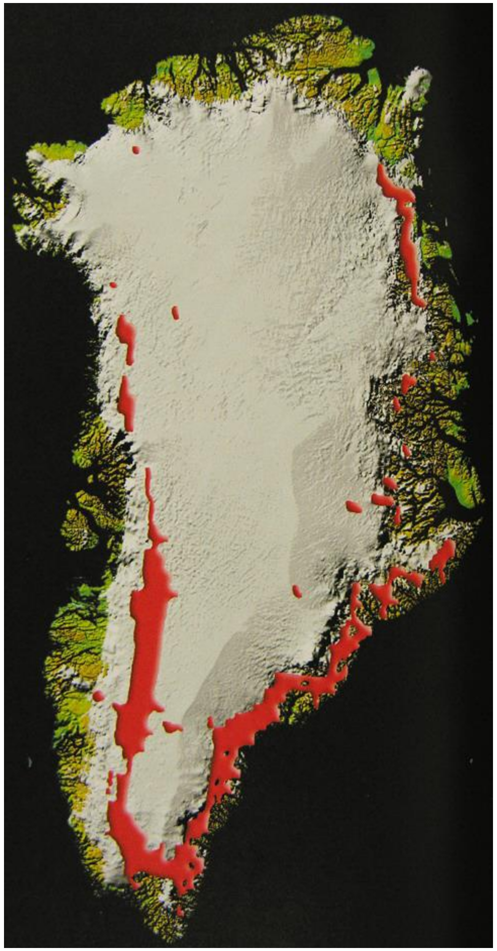
EFFECTS OF CLIMATE CHANGE

In 1906



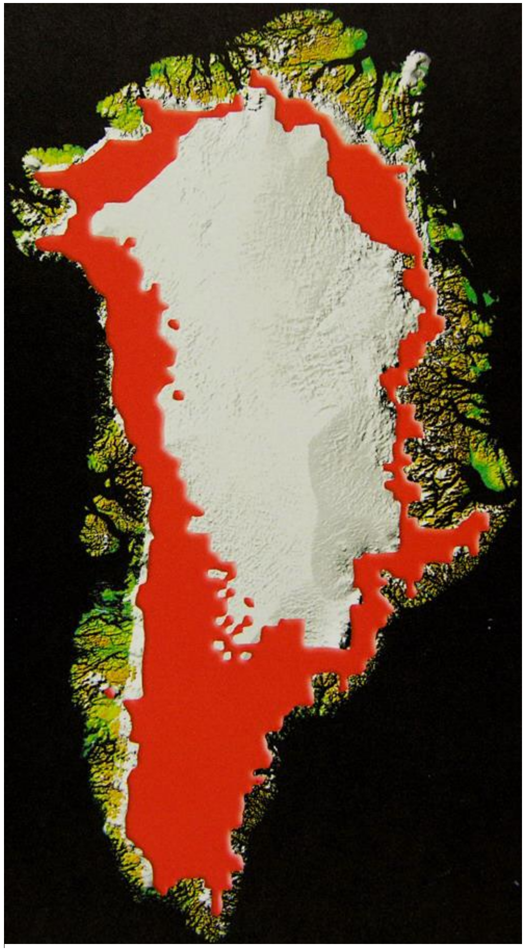
In 2003

EFFECTS OF CLIMATE CHANGE



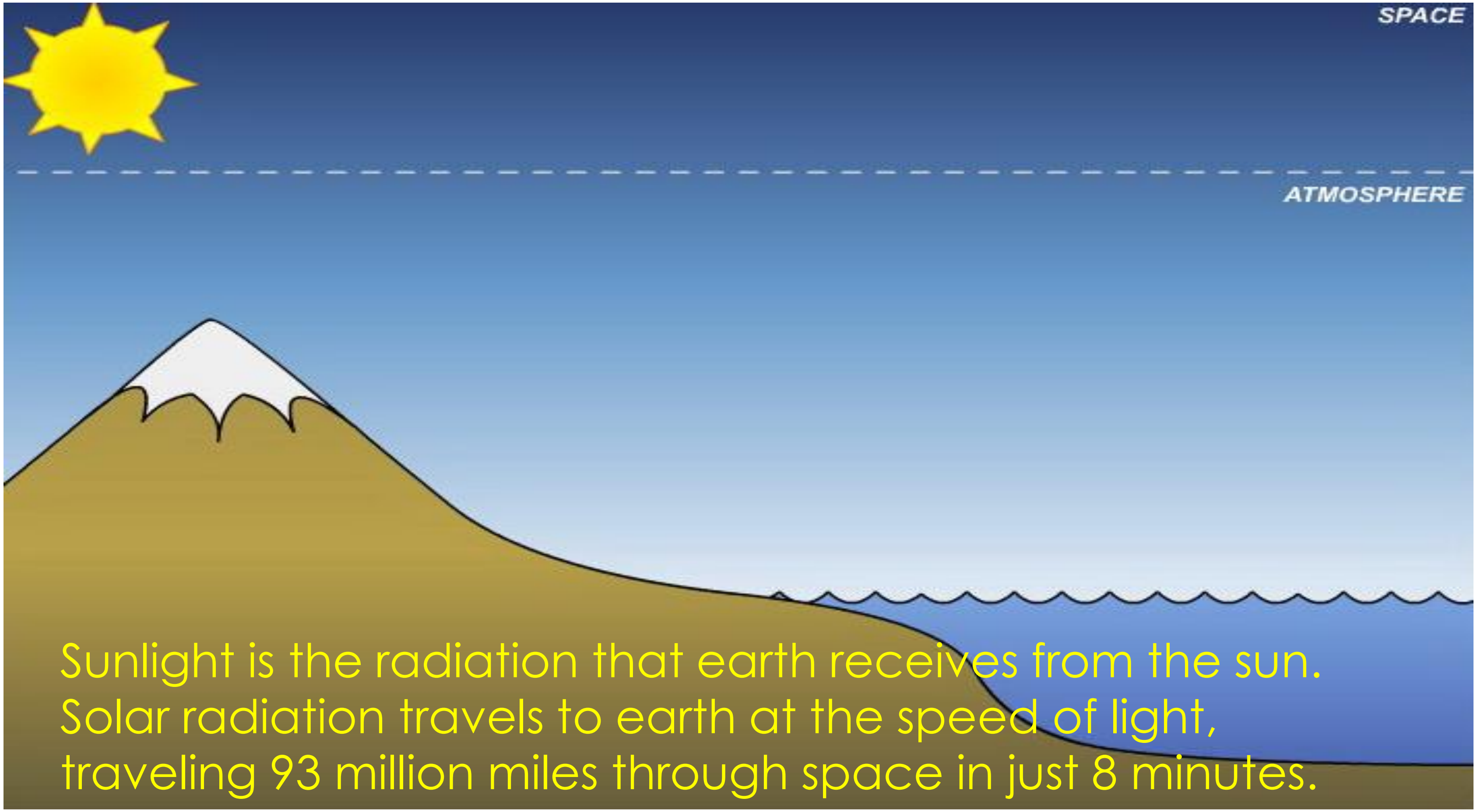
In 1992

In 2002



In 2005

GLOBAL ENERGY BALANCE



GLOBAL ENERGY BALANCE

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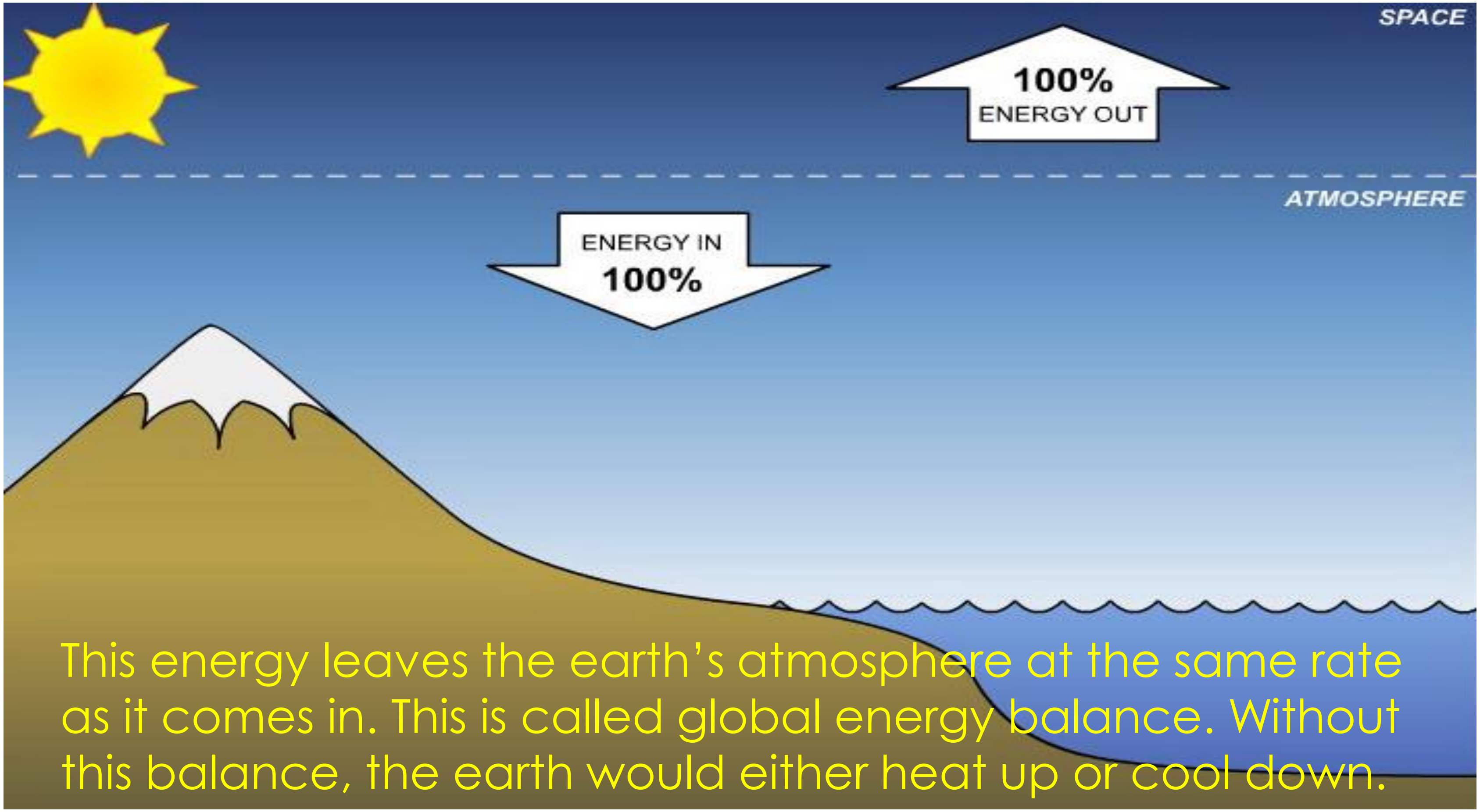
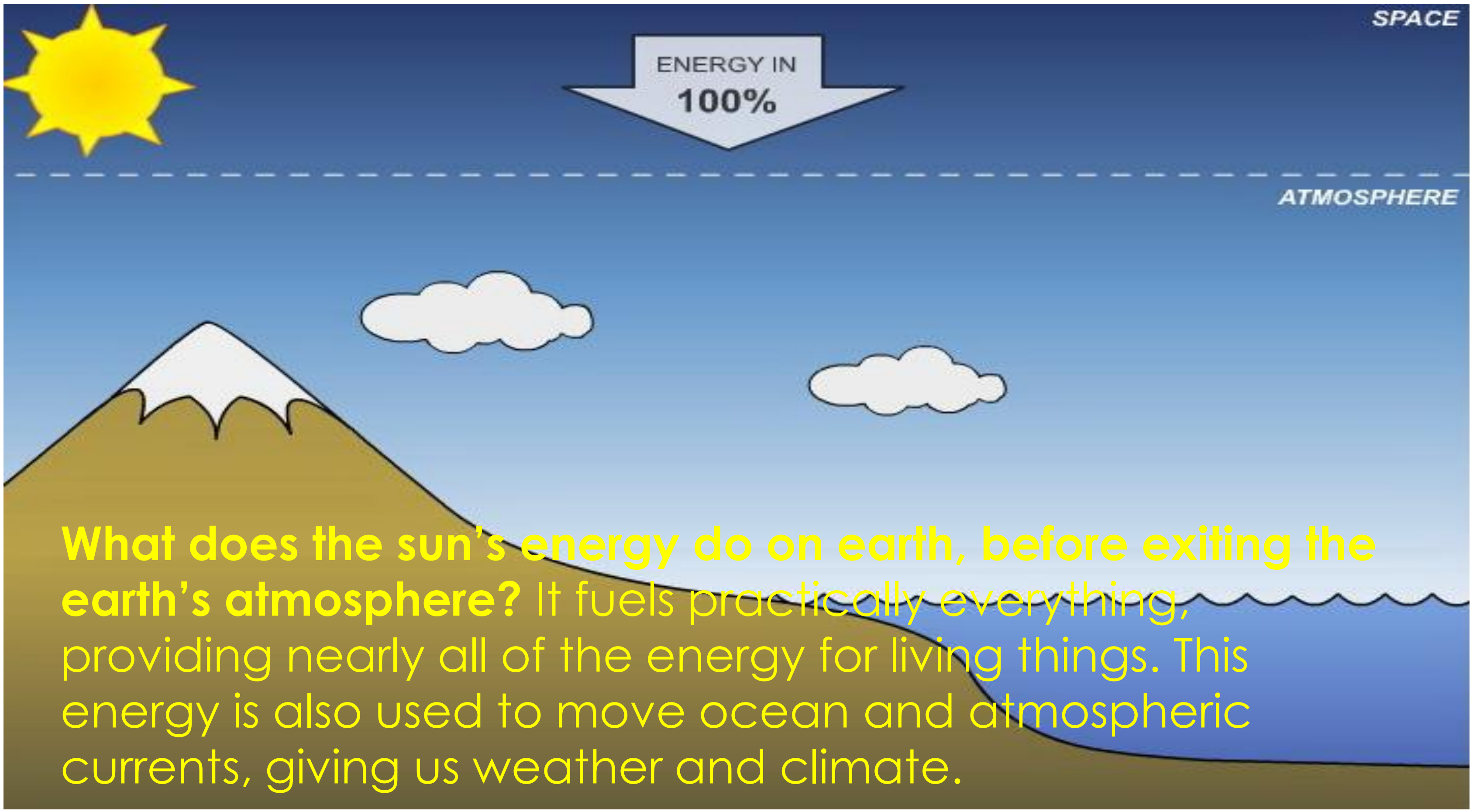
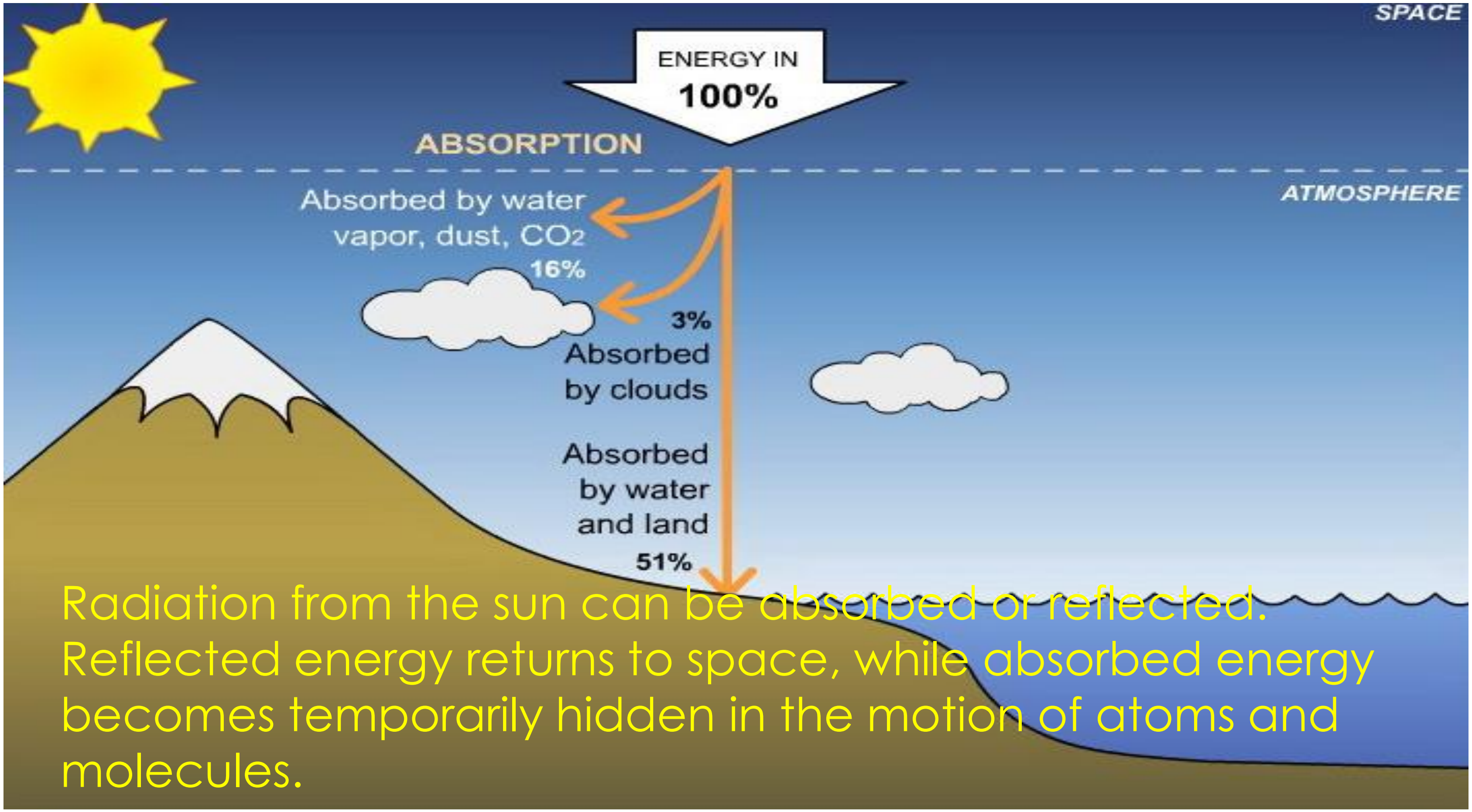


GLOBAL ENERGY BALANCE



What does the sun's energy do on earth, before exiting the earth's atmosphere? It fuels practically everything, providing nearly all of the energy for living things. This energy is also used to move ocean and atmospheric currents, giving us weather and climate.

GLOBAL ENERGY BALANCE



Radiation from the sun can be absorbed or reflected. Reflected energy returns to space, while absorbed energy becomes temporarily hidden in the motion of atoms and molecules.

GLOBAL ENERGY BALANCE

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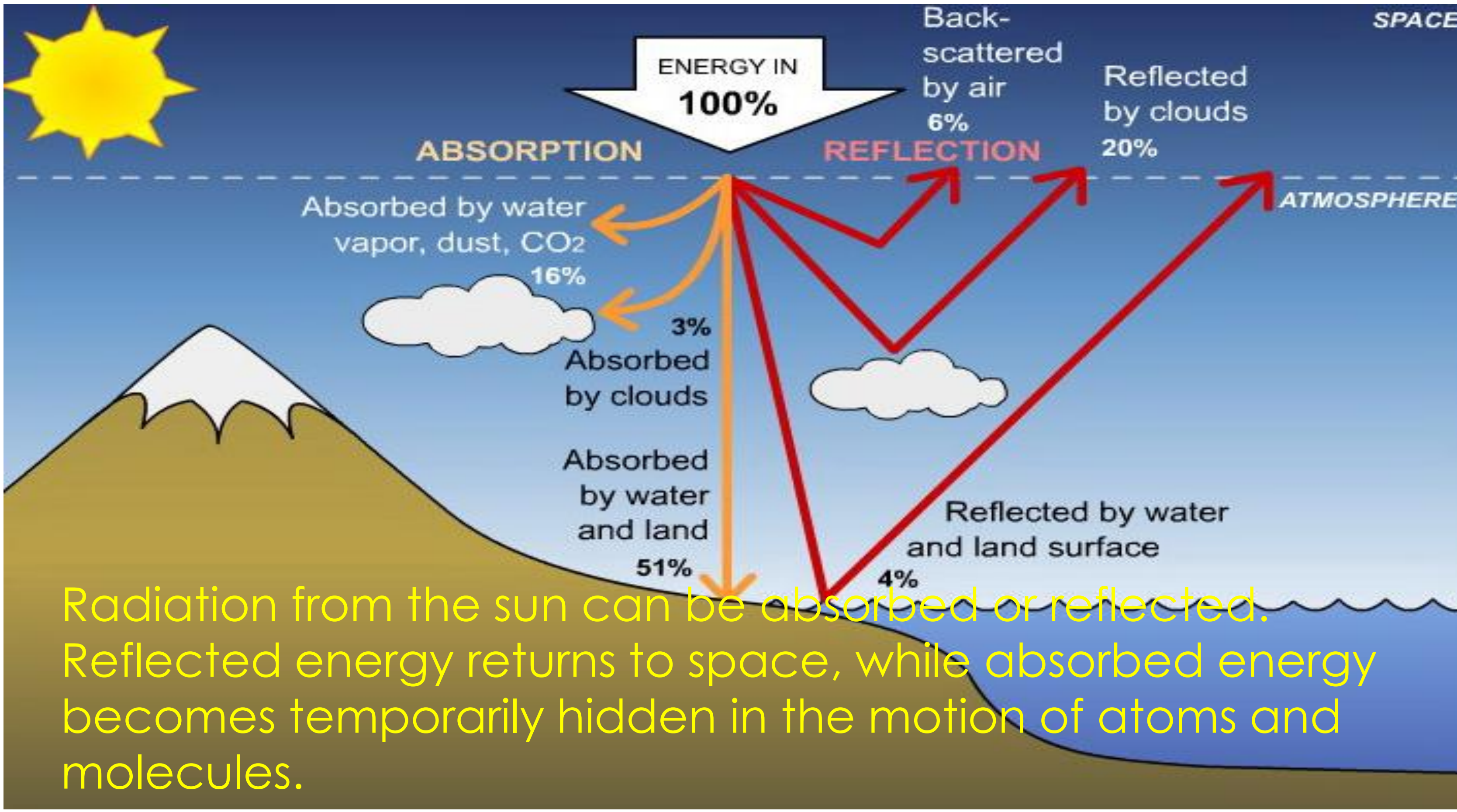

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GLOBAL ENERGY BALANCE

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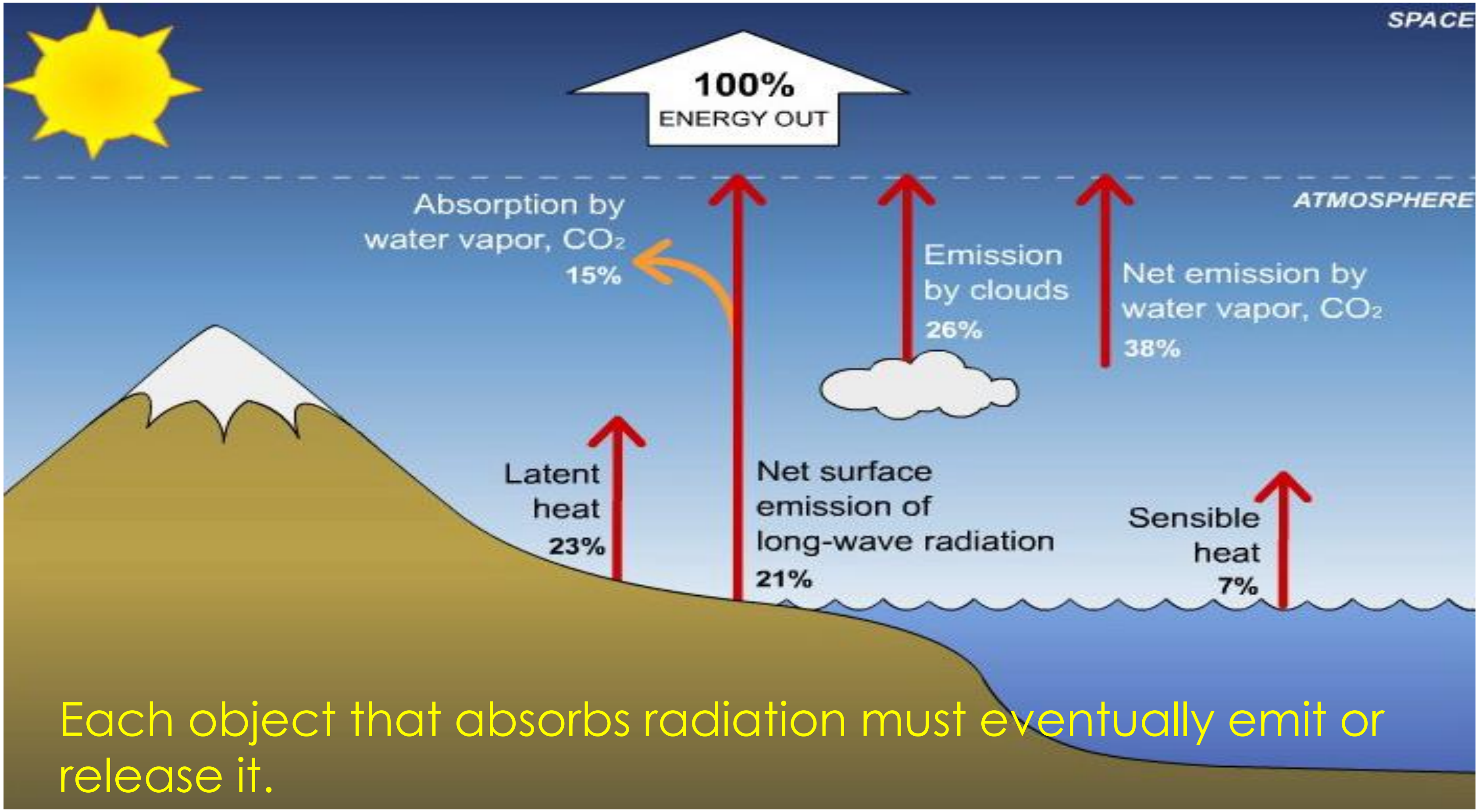

Each object that absorbs radiation must eventually emit or release it.

GREEN HOUSE GASES

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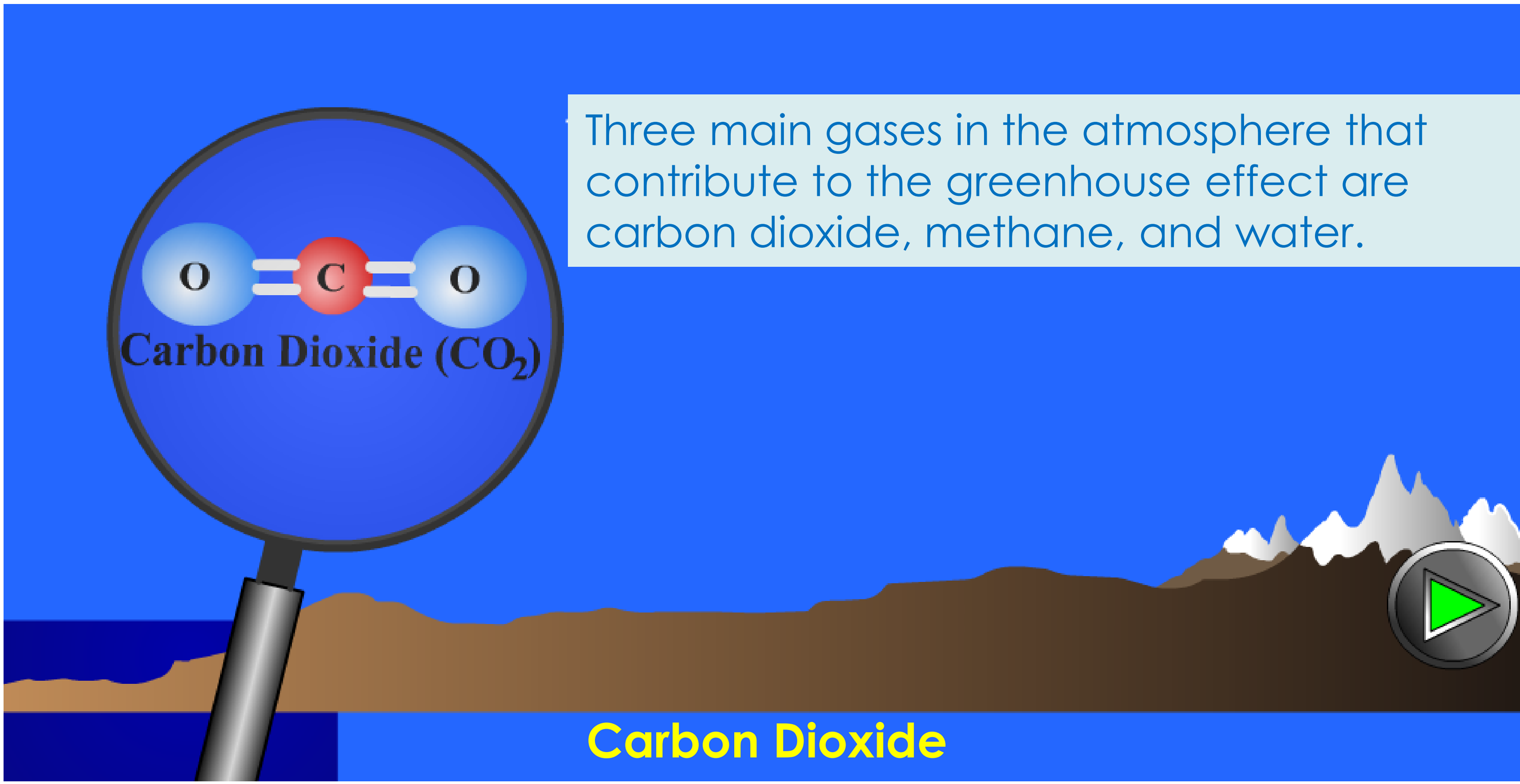
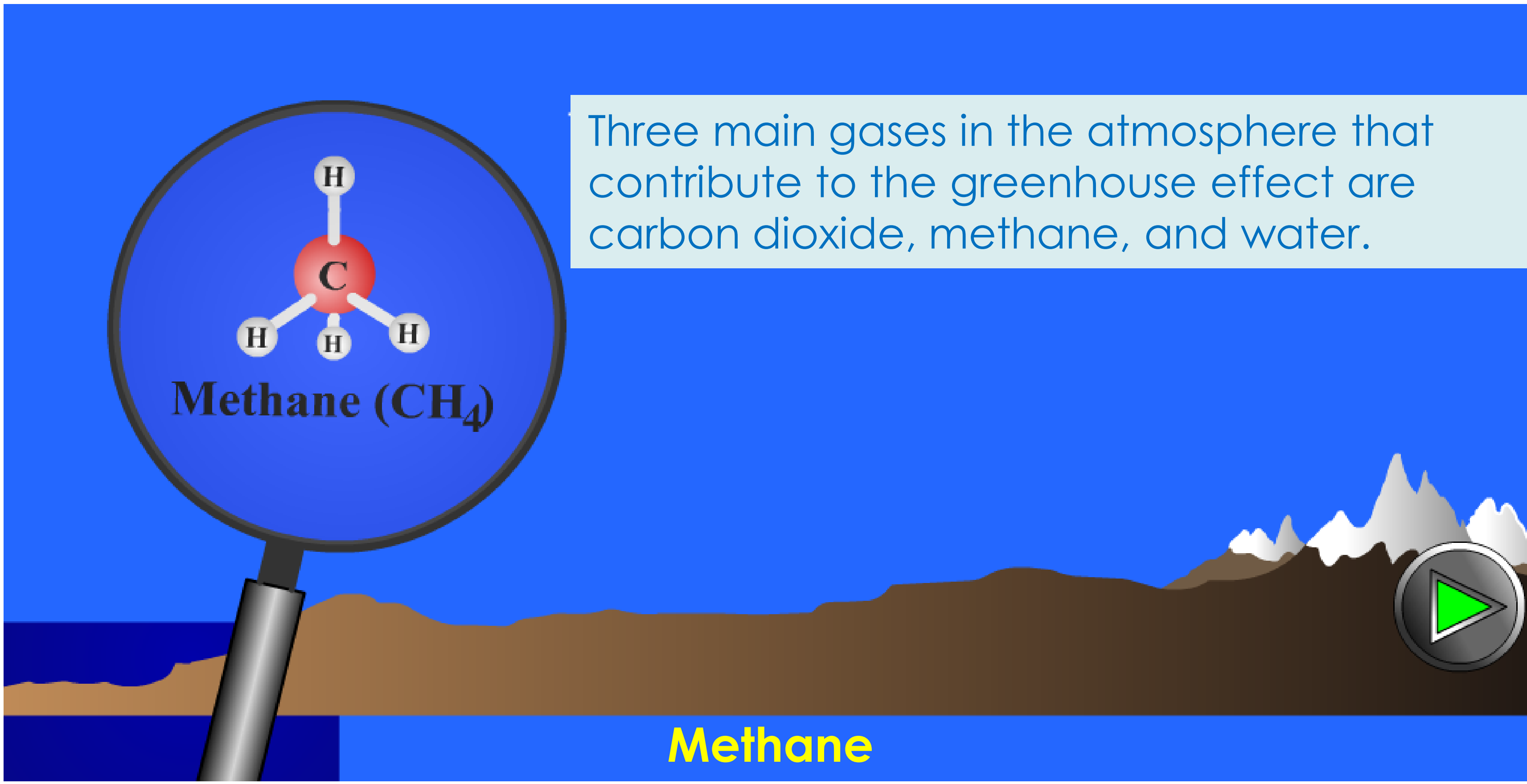
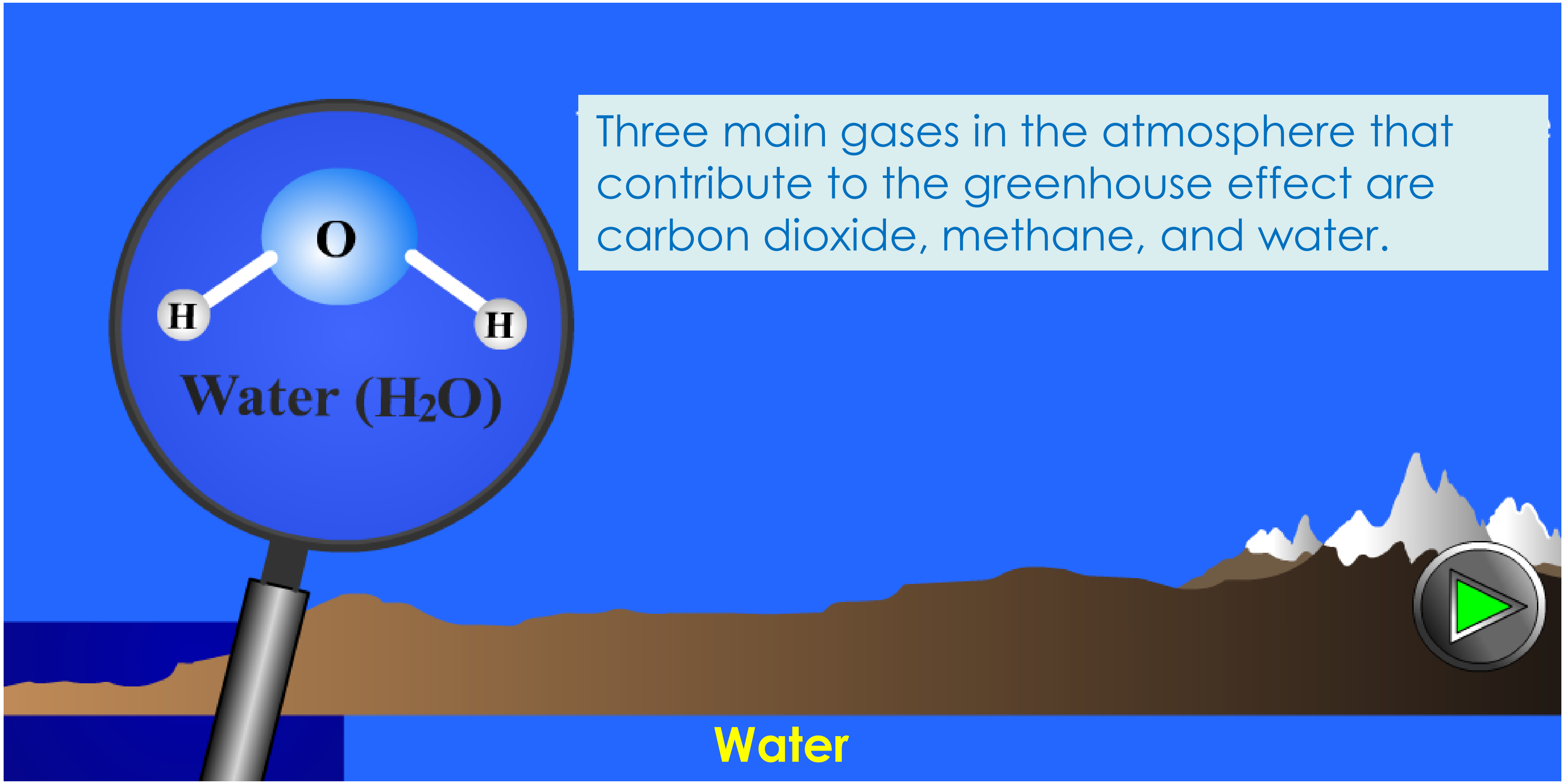


GREEN HOUSE GASES



GREEN HOUSE GASES



GREEN HOUSE GASES

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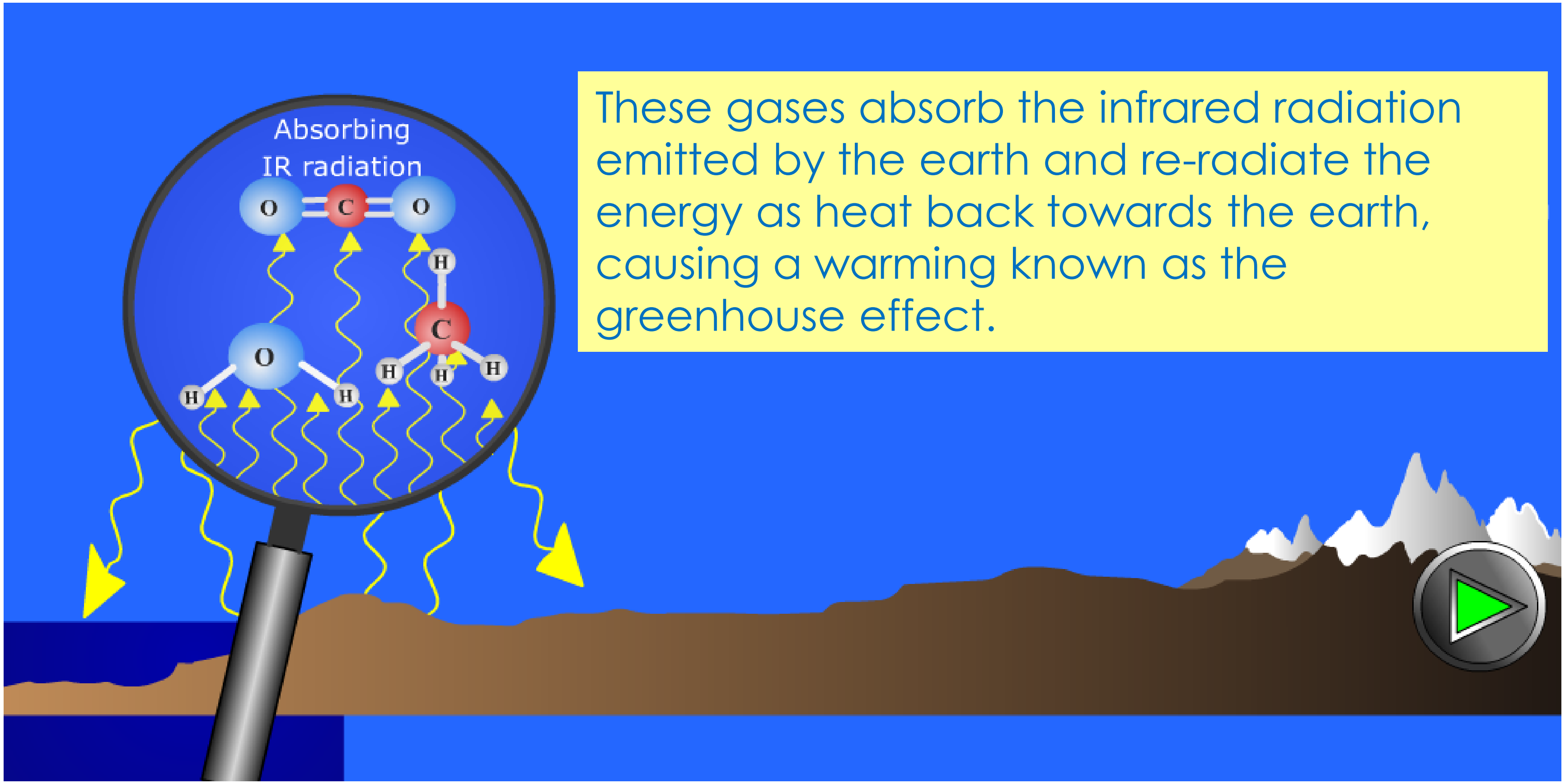


These gases absorb the infrared radiation emitted by the earth and re-radiate the energy as heat back towards the earth, causing a warming known as the greenhouse effect.

EL NINO

El Niño (Spanish name for the male child)

El Nino initially referred to a weak, warm current appearing annually around Christmas time along the coast of Ecuador and Peru and lasting only a few weeks to a month or more.

Every three to seven years, an El Niño event may last for many months, having significant economic and atmospheric consequences worldwide.

EL NINO

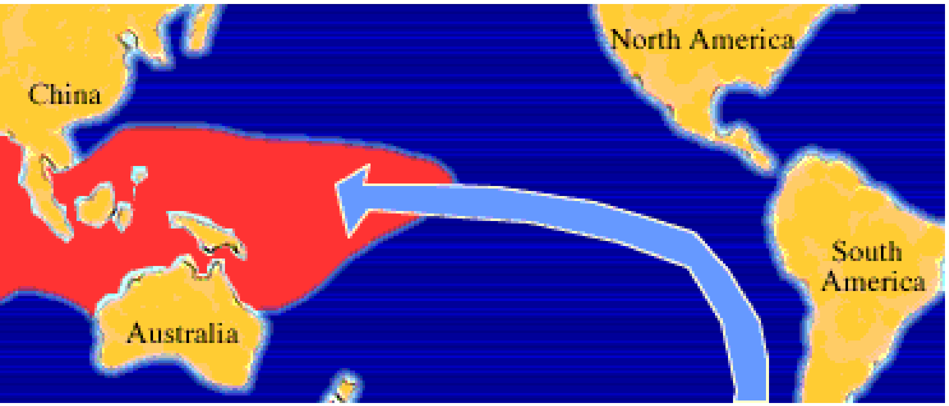
During the past forty years, ten of these major El Niño events have been recorded, the worst of which occurred in 1997-1998.

El Niño Years			
1902-1903	1905-1906	1911-1912	1914-1915
1918-1919	1923-1924	1925-1926	1930-1931
1932-1933	1939-1940	1941-1942	1951-1952
1953-1954	1957-1958	1965-1966	1969-1970
1972-1973	1976-1977	1982-1983	1986-1987
1991-1992	1994-1995	1997-1998	

EL NINO

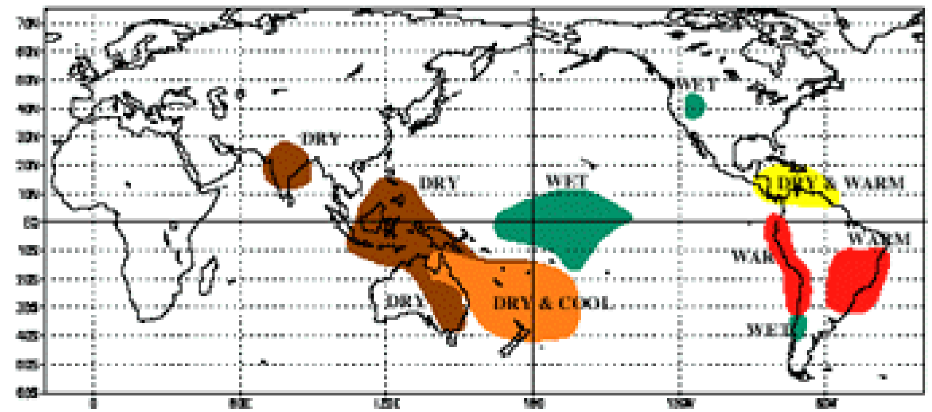
- In the tropical Pacific, trade winds generally drive the surface waters westward.
- The surface water becomes progressively warmer going westward because of its longer exposure to solar heating.
- El Niño is observed when the easterly trade winds weaken, allowing warmer waters of the western Pacific to migrate eastward and eventually reach the South American Coast (shown in orange).
- The cool nutrient-rich sea water normally found along the coast of Peru is replaced by warmer water depleted of nutrients, resulting in a dramatic reduction in marine fish and plant life.

EL NINO

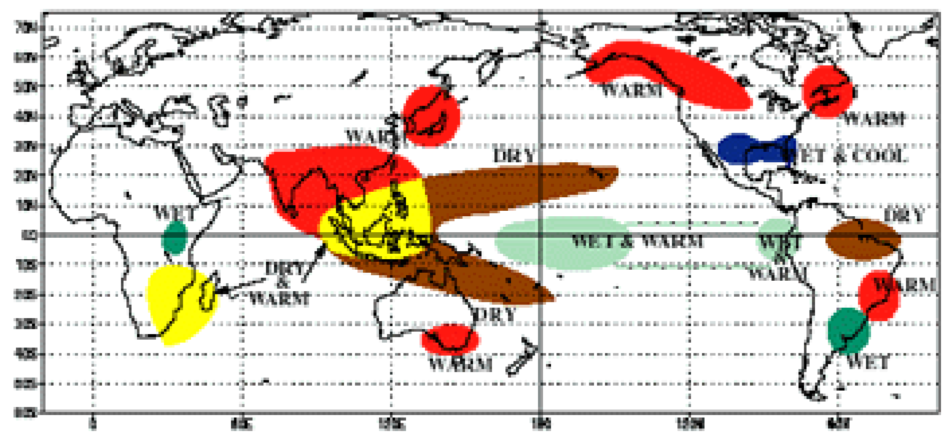


Droughts in the Western Pacific Islands and Indonesia as well as in Mexico and Central America were the early (and sometimes constant) victims of this El Niño.

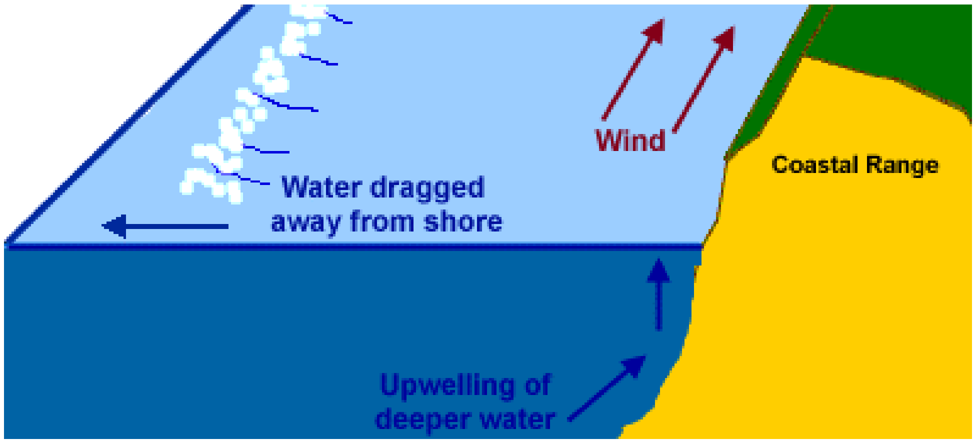
Warm Episode Relationships: Jun-Aug



Warm Episode Relationships: Dec-Feb



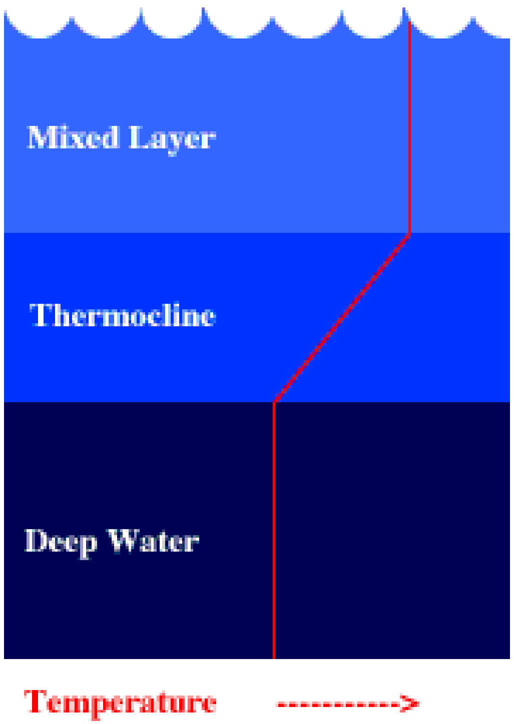
EL NINO



Upwelling
The transport of deeper water to shallow levels

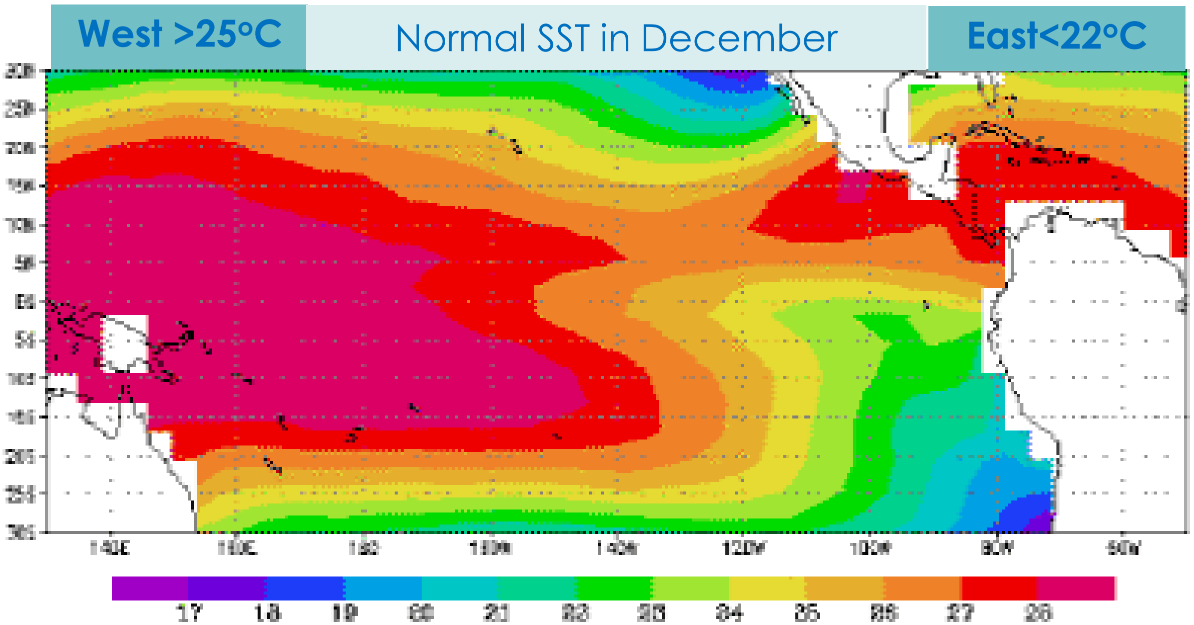
- One oceanic process altered during an El Niño year is upwelling, which is the rising of deeper colder water to shallower depths.
- The diagram above shows how upwelling occurs along the coast of Peru.
- Nutrient-rich water rises from deeper levels to replace the surface water that has drifted away and these nutrients are responsible for supporting the large fish population commonly found in these areas.

EL NINO



- The thermocline is the transition layer between the mixed layer at the surface and the deep water layer.
- The definitions of these layers are based on temperature.
- The mixed layer is near the surface where the temperature is roughly that of surface water.
- In the thermocline, the temperature decreases rapidly from the mixed layer temperature to the much colder deep water temperature.
- The mixed layer and the deep water layer are relatively uniform in temperature, while the thermocline represents the transition zone between the two.

EL NINO

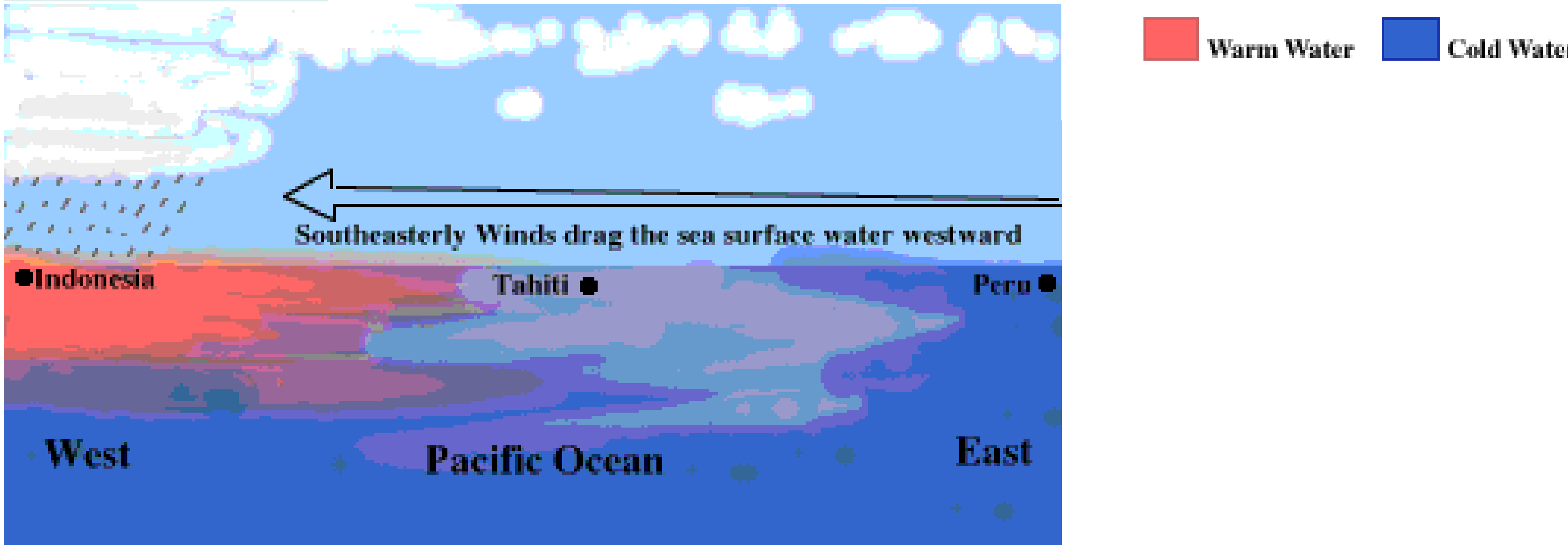


Non El Niño Years
colder water in the eastern tropical Pacific

- The plot of average sea surface temperatures from 1949-1993 shows that the average December SSTs were much cooler in the eastern Pacific (less than 22 degrees Celsius) than in the western Pacific (greater than 25 degrees Celsius), gradually decreasing from west to east.

EL NINO

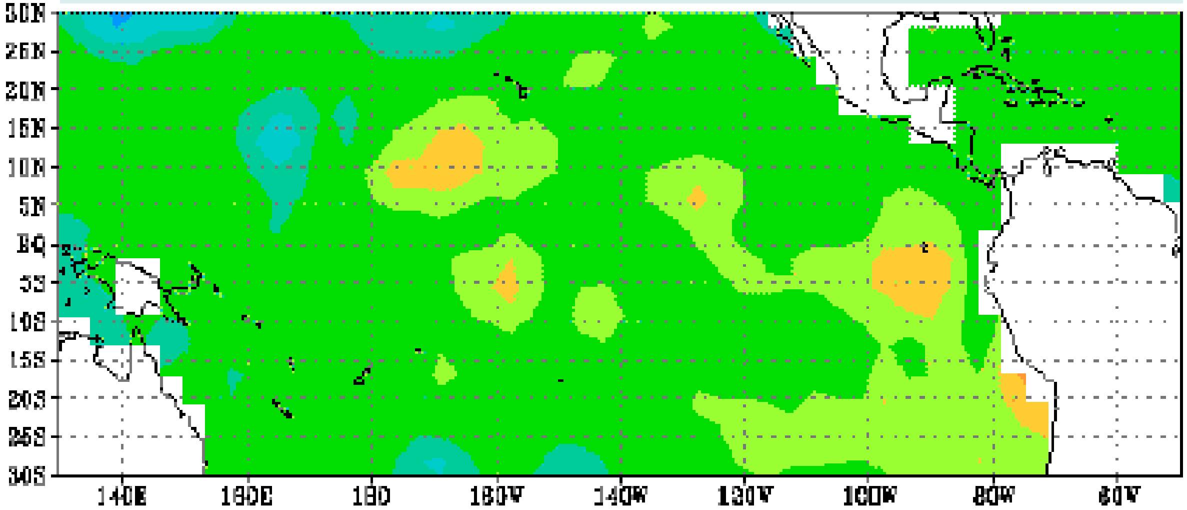
Non-El Nino Year



- This is why during most non El Niño years, heavy rainfall is found over the warmer waters of the western Pacific (near Indonesia) while the eastern Pacific is relatively dry.

EL NINO

SST Anomaly in July 1982

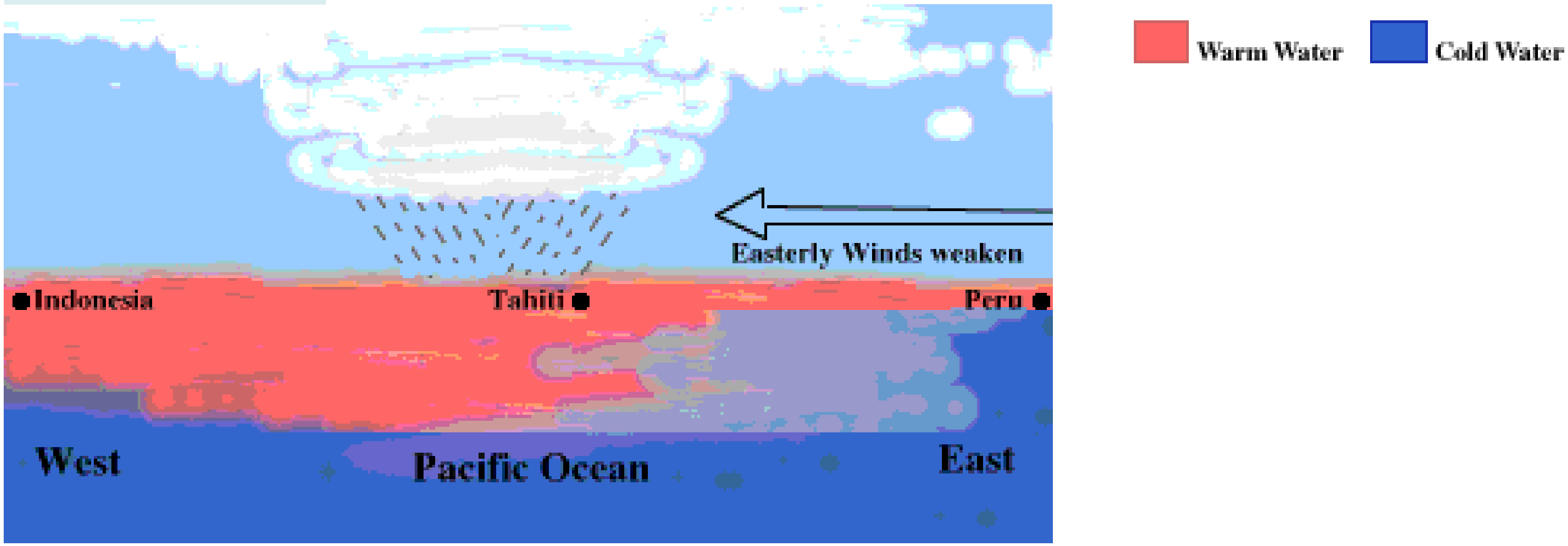


El Niño Events
results from weakening easterly trade winds

- The deeper thermocline limits the amount of nutrient-rich deep water tapped by upwelling processes.

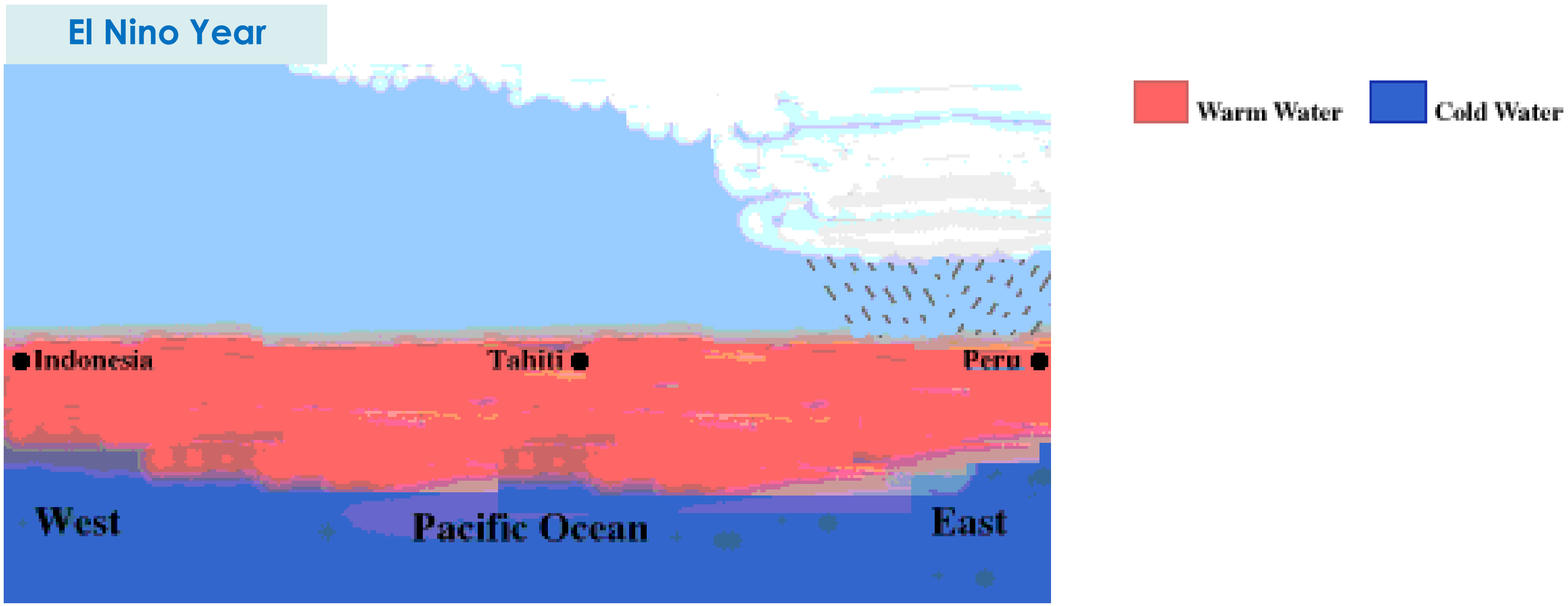
EL NINO

El Nino Year



- As the warmer water shifts eastward, so do the clouds and thunderstorms associated with it, resulting in dry conditions in Indonesia and Australia while more flood-like conditions exist in Peru and Ecuador.

EL NINO



- El Niño causes all sorts of unusual weather, sometimes bringing rain to coastal deserts of South America which never see rain during non-El Niño years.

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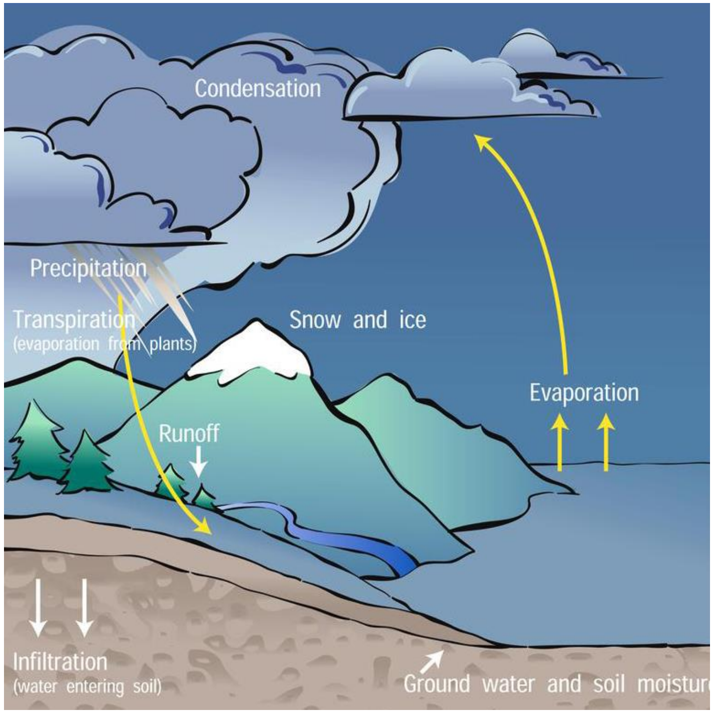

LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

Atmospheric Water

- Atmospheric Circulation
- Water Vapor
- Precipitation
- Evaporation
- Evapotranspiration

HYDROLOGIC CYCLE

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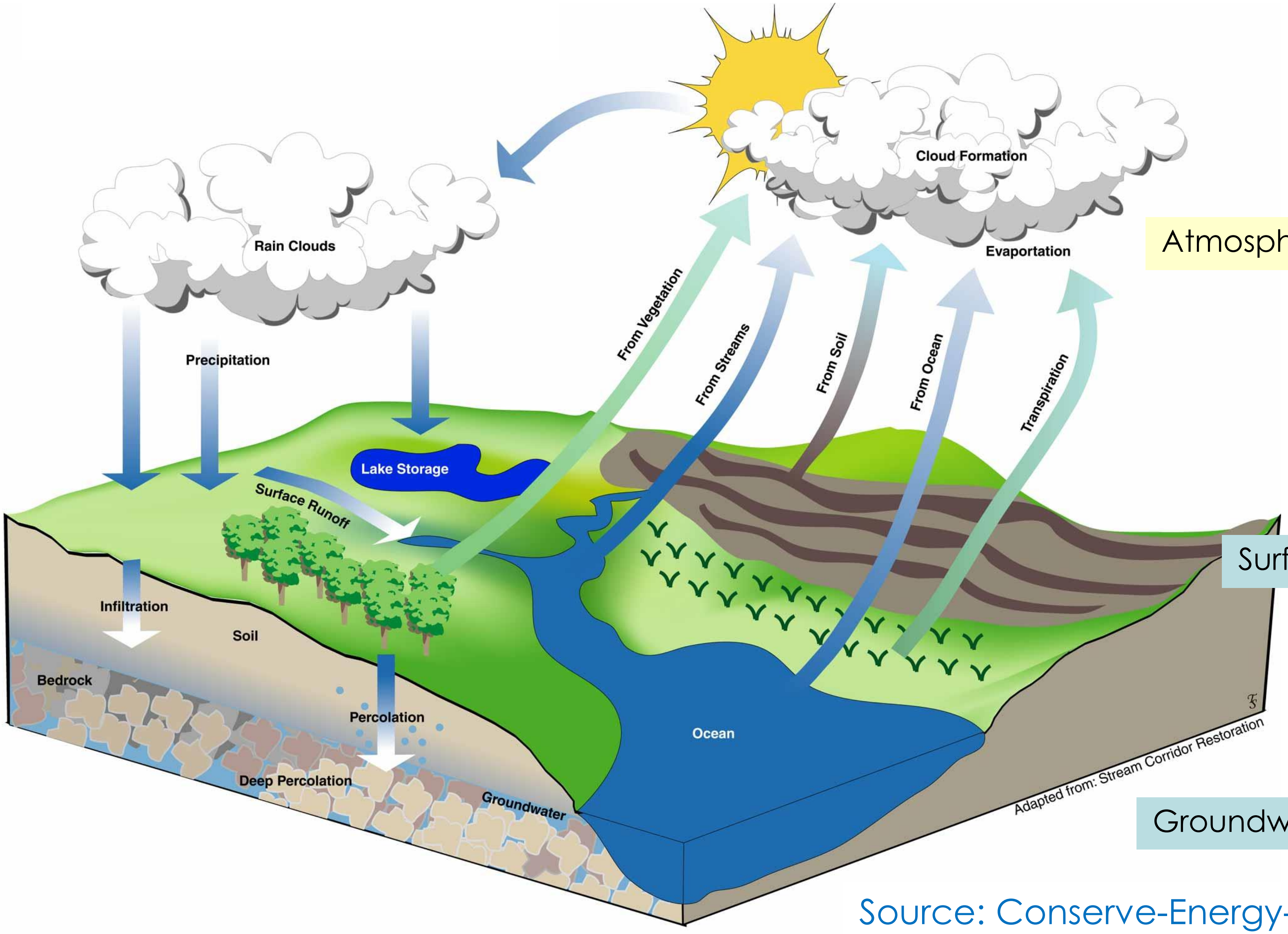

Atmospheric Subsystem

Surface Subsystem

Groundwater Subsystem

Source: Conserve-Energy-Future (2018)

ATMOSPHERIC WATER:

Atmospheric Water

- Many meteorological processes occur continuously within the atmosphere.
- The processes of precipitation and evaporation are the most important for hydrology.
- Much of the water precipitated on the land surface is derived from moisture evaporated from the ocean and transported long distances by **Atmospheric Circulation**.
- The two basic driving forces of atmospheric circulation result from

  (1) The rotation of the earth

  (2) The transfer of heat energy between equator and the poles

ATMOSPHERIC WATER: ATMOSPHERIC CIRCULATION

Atmospheric Circulation

- Atmospheric circulation is the large-scale movement of air, and together with ocean circulation is the means by which thermal energy is redistributed on the surface of the Earth.

- The Earth's atmospheric circulation varies from year to year, but the large-scale structure of its circulation remains fairly constant.

- The smaller scale weather systems – mid-latitude depressions, or tropical convective cells – atmospheric circulation occurs "randomly.

ATMOSPHERIC WATER: ATMOSPHERIC CIRCULATION

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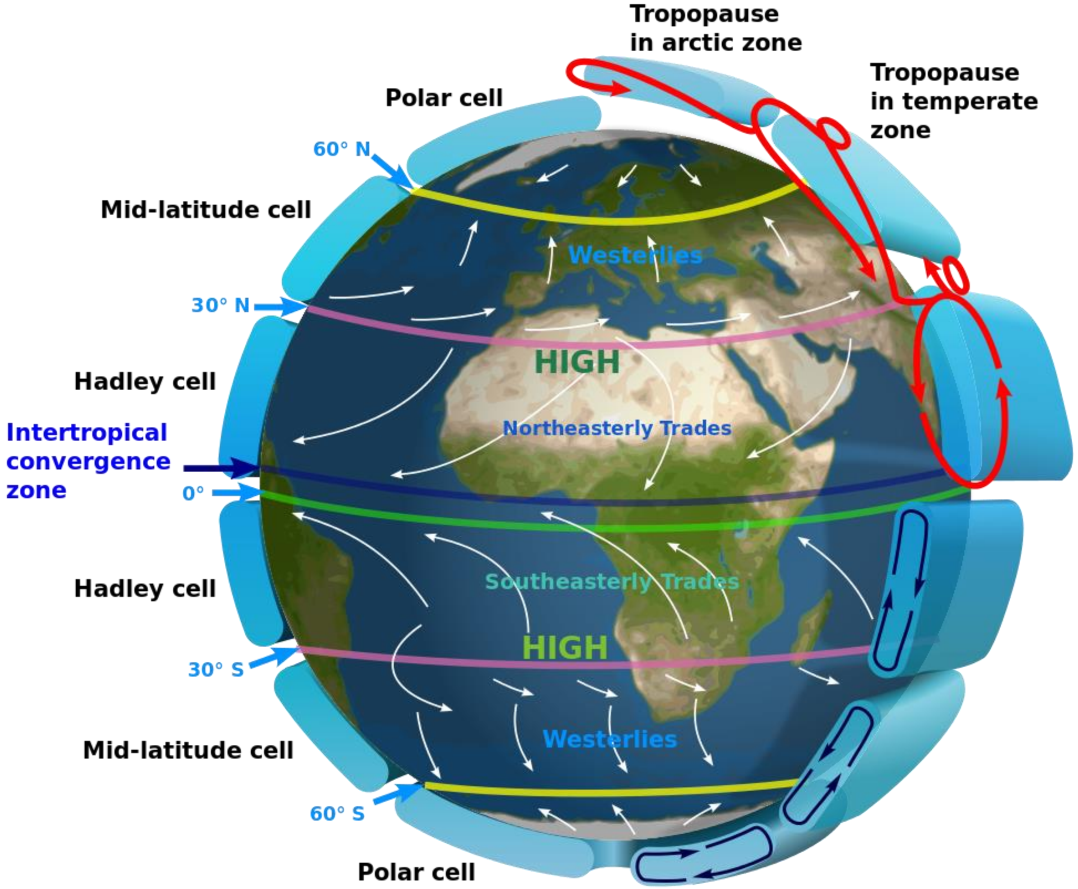
Large Scale Atmospheric Circulation on Earth



Source: Wikipedia (2018a)

- The atmospheric circulation occurs in the troposphere.
- The troposphere ranges in height from about 8 km at the poles to 16 km at the equator.
- The temperature in the troposphere decreases with altitude at a rate varying with the moisture content of the atmosphere.

ATMOSPHERIC WATER: WATER VAPOR

Atmospheric Water

- Atmospheric water mostly exists as a gas or vapor.
- Briefly and locally, it becomes a liquid in rainfall and in water droplets in clouds. or
- It becomes a solid in snowfall, in hail and in ice cystals in clouds.

Water Vapor

- Water vapor is the gaseous phase of water. It is one state of water within the hydrosphere. Water vapor can be produced from the evaporation or boiling of liquid water or from the sublimation of ice.
- The amount of water vapor in the atmosphere is less than 1/100,000 of all waters of the earth, but it plays a vital role in the hydrologic cycle.

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Vapor Transport Equation

The Reynolds transport equation is the continuity equation for water vapor transport.

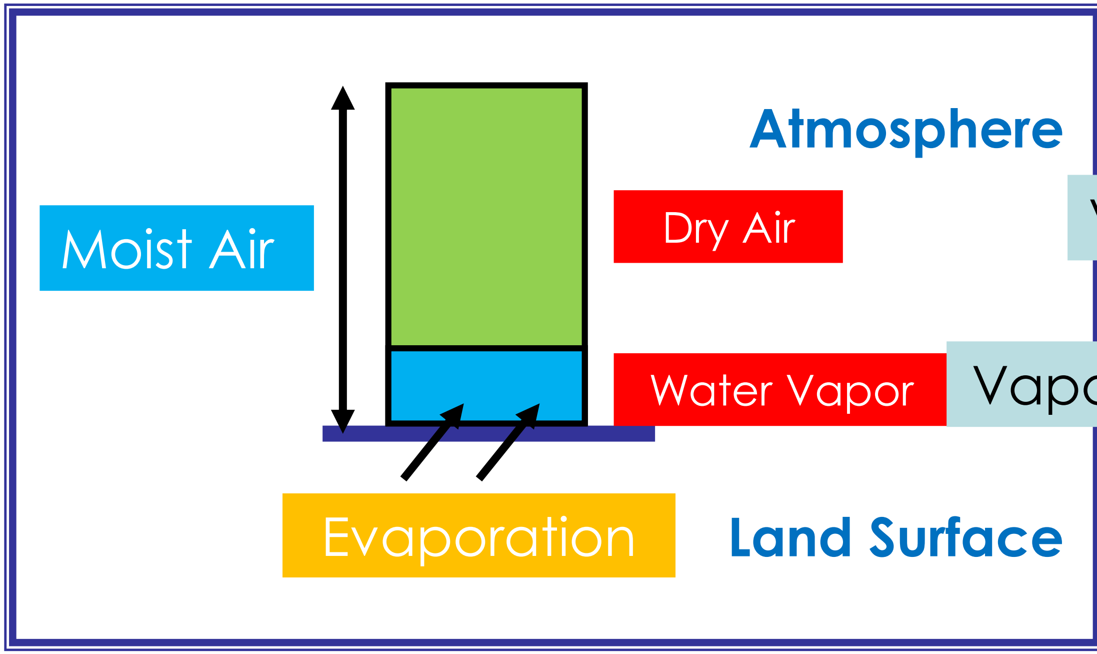
$$m_v = \frac{d}{dt}\iiint_{c.v.} q_v \rho_a d\forall + \iint_{c.s.} q_v \rho_a V.dA$$

$$q_v = \frac{\rho_v}{\rho_a}$$

When

m_v= Mass Flow Rate of Water Vapor Transport
q_v = Specific Humidity
ρ_v = Density of Water Vapor
ρ_a = Density of Moist Air

WATER VAPOR: VAPOR PRESSURE



Vapor Pressure of Dry Air, p-e

$$p - e = \rho_d R_d T$$

Vapor Pressure of Water Vapor, e

$$e = \rho_v R_v T$$

Vapor Pressure of Moist Air, p

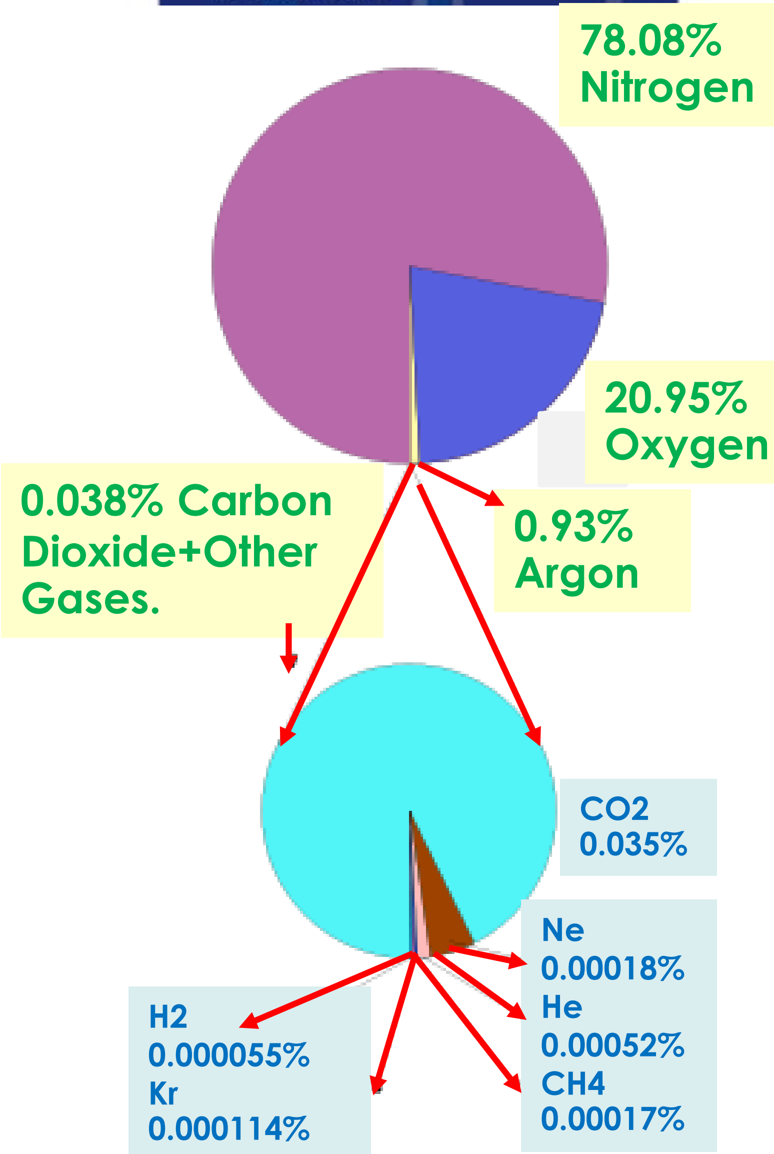
$$p = \rho_a R_a T$$

Dalton's Law of Partial Pressure

WATER VAPOR: DRY AIR

Atmosphere Gas Proportions

- The Earth's atmosphere (or air) is a layer of gases surrounding the planet Earth that is retained by the Earth's gravity.
- It has a mass of about five quadrillion metric tons.
- **Dry air contains roughly (by volume) 78.08% nitrogen, 20.95% oxygen, 0.93% argon, 0.038% carbon dioxide, and trace amounts of other gases.** Air also contains a variable amount of water vapor, on average around 1%.



WATER VAPOR: VAPOR PRESSURE

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Vapor Pressure of Water Vapor, e

$$e = \rho_v R_v T$$

T = Absolute Temperature in K
R_v = Gas Constant for Water Vapor
ρ_v = Density of Water Vapor

Vapor Pressure of Dry Air, p-e

$$p - e = \rho_d R_d T$$

T = Absolute Temperature in K
R_d = Gas Constant for Dry Air = 287 J/kg.K
ρ_d = Density of Dry Air

Total Vapor Pressure, p

$$p = e + (p - e) = \rho_v R_v T + \rho_d R_d T$$

$$= \rho_v \left(\frac{R_d}{0.622}\right) T + \rho_d R_d T$$

$$= \left[\frac{\rho_v}{0.622} + \rho_d\right] R_d T$$

When

$$\rho_a = \rho_d + \rho_v$$

$$R_v = \frac{R_d}{\mathbf{0.622}}$$

$$p = \rho_a R_a T$$

Rewritten in terms of gas constant for moist air, R_a

WATER VAPOR:
SATURATION VAPOR PRESSURE

The relationship between the gas constants for moist air and dry air is given by;

$$R_a = R_d(1 + 0.608 q_v)$$
$$= 287(1 + 0.608 q_v) \ \ J/kg.K$$

For a given air temperature, there is a maximum moisture content in the air can hold, the corresponding vapor pressure is called "**Saturation Vapor Pressure, e_s**"

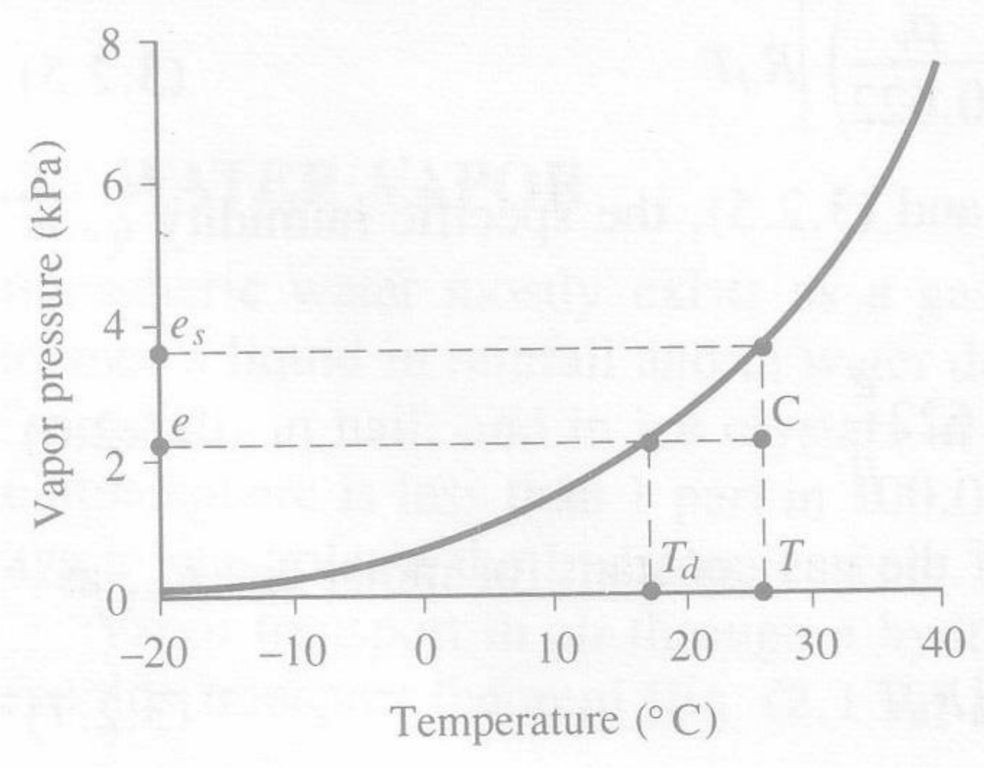
At this vapor pressure, the rate of evaporation and condensation are equal.

$$e = 611 \exp\left(\frac{17.27T}{237.3 + T}\right)$$

When
T = Absolute Temperature in degree Celcious
e_s = Saturation Vapor Pressure in Pascals

WATER VAPOR: SATURATION VAPOR PRESSURE

Over a water surface the saturation vapor pressure is related to the air temperature.



The gradient of the saturated vapor pressure curve, (in Pascals per degree celcius)

$$\Delta = \frac{de_s}{dT}$$

$$\Delta = \frac{4098 e_s}{(237.3 + T)^2}$$

WATER VAPOR: SATURATION VAPOR PRESSURE

Saturated vapor pressure of water vapor over liquid water

Temperature (^0C)	Saturated Vapor Pressure (kPa)
-20	125
-10	286
0	611
5	872
10	1227
15	1704
20	2337
25	3167
30	4243
35	5624
40	7378

Source: Chow et al. (1988)

WATER VAPOR: RELATIVE HUMIDITY & SPECIFIC HUMIDITY

The "**Relative Humidity, Rh**" is the ratio of the actual vapor pressure to its saturation value at a given air temperature.

$$R_h = \frac{e}{e_s}$$

The temperature at which air would just become saturated at a given specific humidity is its "**Dew-Point Temperature, Td**"

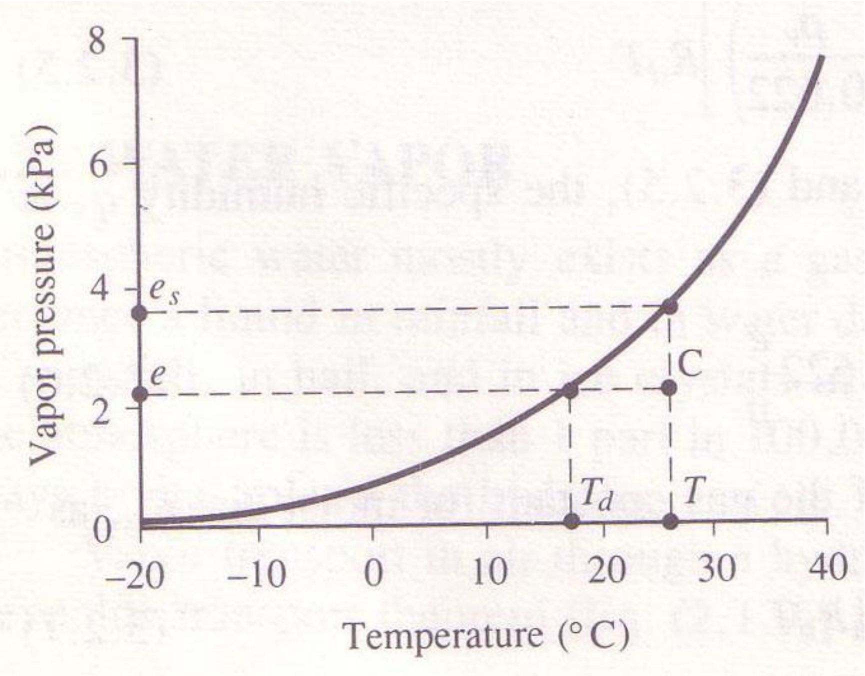
"**Specific Humidity, qv**" is approximated by;

$$q_v = 0.622\frac{e}{p}$$

WATER VAPOR: EXAMPLE 1

At a climate station, air pressure is measured as 100 kPa, air temperature as 20°C, and the wet-bulb or dew point temperature as 16°C. Calculate

(a) vapor pressure
(b) relative humidity
(c) specific humidity
(d) air density



WATER VAPOR: EXAMPLE 1

Solution

Saturated vapor pressure at T=20°C :

$$e_s = 611\exp\left(\frac{17.27T}{237.3+T}\right)$$

$$= 611\exp\left(\frac{17.27x20}{237.3+20}\right)$$

$$= 2{,}339 \text{ Pa}$$

The actual vapor pressure, e (T=Td=16°C) :

$$e = 611\exp\left(\frac{17.27T_d}{237.3+T_d}\right)$$

$$= 611\exp\left(\frac{17.27x16}{237.3+16}\right)$$

$$= 1{,}819 \text{ Pa}$$

WATER VAPOR: EXAMPLE1

The relative humidity :

$$R_h = \frac{e}{e_s} = \frac{1{,}819}{2{,}339} = 0.78 = 78\%$$

The specific humidity :

$$q_v = 0.622\frac{e}{p} = 0.622\left(\frac{1{,}819}{100\text{x}10^3}\right)$$
$$= 0.0113 \text{ kg water/kg moist air}$$

Air density :

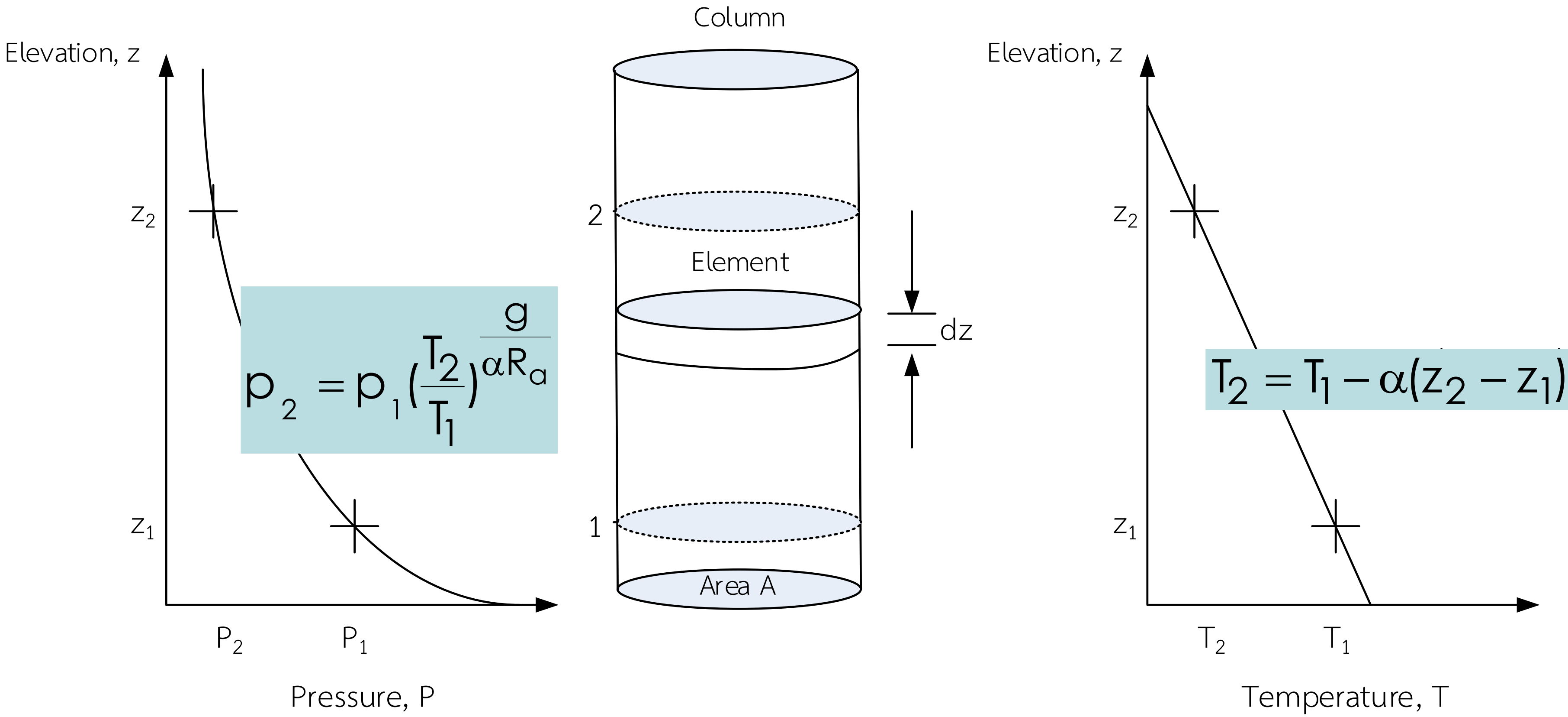
$$\rho_a = \frac{p}{R_a T} = \frac{100\text{x}10^3}{289\text{x}293} = 1.18\ \frac{\text{kg}}{\text{m}^3}$$

When

$$R_a = 287(1 + 0.608 q_v)$$
$$qv = 0.0113$$
$$T = 20^\circ C = 20 + 273\ K = 293\ K$$

WATER VAPOR
IN A STATIC ATMOSPHERIC COLUMN

$$\alpha = -\frac{dT}{dz} = \text{Lapse Rate}$$



WATER VAPOR
IN A STATIC ATMOSPHERIC COLUMN

Precipitable Water

- Precipitable water is the depth of water in a column of the atmosphere, if all the water in that column were precipitated as rain. As a depth, the precipitable water is measured in millimeters or inches. Often abbreviated as "**TPW" = Total Precipitable Water**.

- In other words, the amount of moisture in an atmospheric column is called "**Precipitable Water**"

WATER VAPOR
IN A STATIC ATMOSPHERIC COLUMN

Consider an element of height dz in a column of horizontal cross-sectional area A.

- The mass of air = $\rho_a A dz$
- The mass of water = $\rho_v \rho_a A dz$

The total mass of precipitable water in the column between elevations z1 and z2 is

$$m_p = \int_{z_1}^{z_2} q_v \rho_a A d_z$$

The integral mp is calculated using intervals of height Δz, each with an incremental mass of precipitable water

$$\Delta m_p = \bar{q}_v \bar{\rho}_a A \Delta z$$

$\bar{q}_v$, $\bar{\rho}_v$= the average values of specific humidity and air density over the interval.

PRECIPITABLE WATER: EXAMPLE2

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Calculate the precipitable water in a saturated air column 10 km high above 1 m^2 of ground surface. The surface pressure is 101.3 kPa, the surface air temperature is 30°C, and the lapse rate is 6.5°C/km.

Elev.	Temperature		Air Pressure	Density		Vapor Pressure	Specific Humidity	Avg. over Increment	Incremental Mass	% of Total Mass
z (km)	T (°C)	T (°K)	p (kPa)	ρ_a (kg/m³)	$\bar{\rho}_a$ (kg/m³)	e (kPa)	qv (kg/kg)	$\bar{q}_v$ (kg/kg)	Δm (kg)	(%)
0	30	303	101.3	1.16		4.24	0.0261			
2	17	290	80.4	0.97	1.07	1.94	0.0150	0.0205	43.7	57
4	4	277	63.2	0.79	0.88	0.81	0.0080	0.0115	20.2	26
6	-9	264	49.1	0.65	0.72	0.31	0.0039	0.0060	8.6	11
8	-22	251	37.6	0.52	0.59	0.10	0.0017	0.0028	3.3	4
10	-35	238	28.5	0.42	0.47	0.03	0.0007	0.0012	1.1	2
								Total	77.0	100

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$$

$$T_2 = T_1 - \alpha(z_2 - z_1)$$

PRECIPITABLE WATER: EXAMPLE 2

Find temperature at z_2 :

$$T_2 = T_1 - \alpha(z_2 - z_1) = 30 - 0.0065(2000 - 0) = 17^{\circ}C = 290\ K$$

$$z_1 = 0\ m, T_1 = 30 + 273 = 303\ K$$

$$z_2 = 2{,}000\ m$$

Find pressure at z_2 :

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}} = 101.3\left(\frac{290}{303}\right)^{5.26} = 80.3\ kPa$$

$$\frac{g}{\alpha R_a} = \frac{9.81}{0.0065x287} = 5.26$$

Find air density at z_1 (at the ground) :

air density at z_2 = 0.97 kg/m^3

$$\rho_a = \frac{p}{R_a T} = \frac{101.3x10^3}{287x303} = 1.16\ \frac{kg}{m^3}$$

$$\overline{\rho}_a = \left(\frac{1.16 + 0.97}{2}\right) = 1.07\ \frac{kg}{m^3}$$

PRECIPITABLE WATER: EXAMPLE 2

Find saturated vapor pressure at z_1 (at the ground) :

$$e = 611\exp\left(\frac{17.27T}{237.3+T}\right) = 611\exp\left(\frac{17.27x30}{237.3+30}\right) = 4{,}244\ Pa = 4.24kPa$$

The saturated vapor pressure at z_2 =2,000 m where T=17°C, e = 1.94 kPa. The specific humidity at the ground, z_1 surface is

$$q_v = 0.622\frac{e}{p} = 0.622x\frac{4.24}{101.3} = 0.026\ \frac{kg}{kg}$$

Specific humidity at z_2, q_v =0.015 kg/kg

$$\overline{q}_v = \left(\frac{0.026+0.015}{2}\right) = 0.0205\ \frac{kg}{kg}$$

The mass of precipitable water in the first 2 km increment is

$$\Delta m_p = \overline{q}_v \overline{\rho}_a A\Delta z = 0.0205x1.07x1x2{,}000 = 43.7\ kg$$

PRECIPITABLE WATER: EXAMPLE 2

The total mass of precipitable water in the column is found to be mp = 77kg. The equivalent depth pf liquid water is

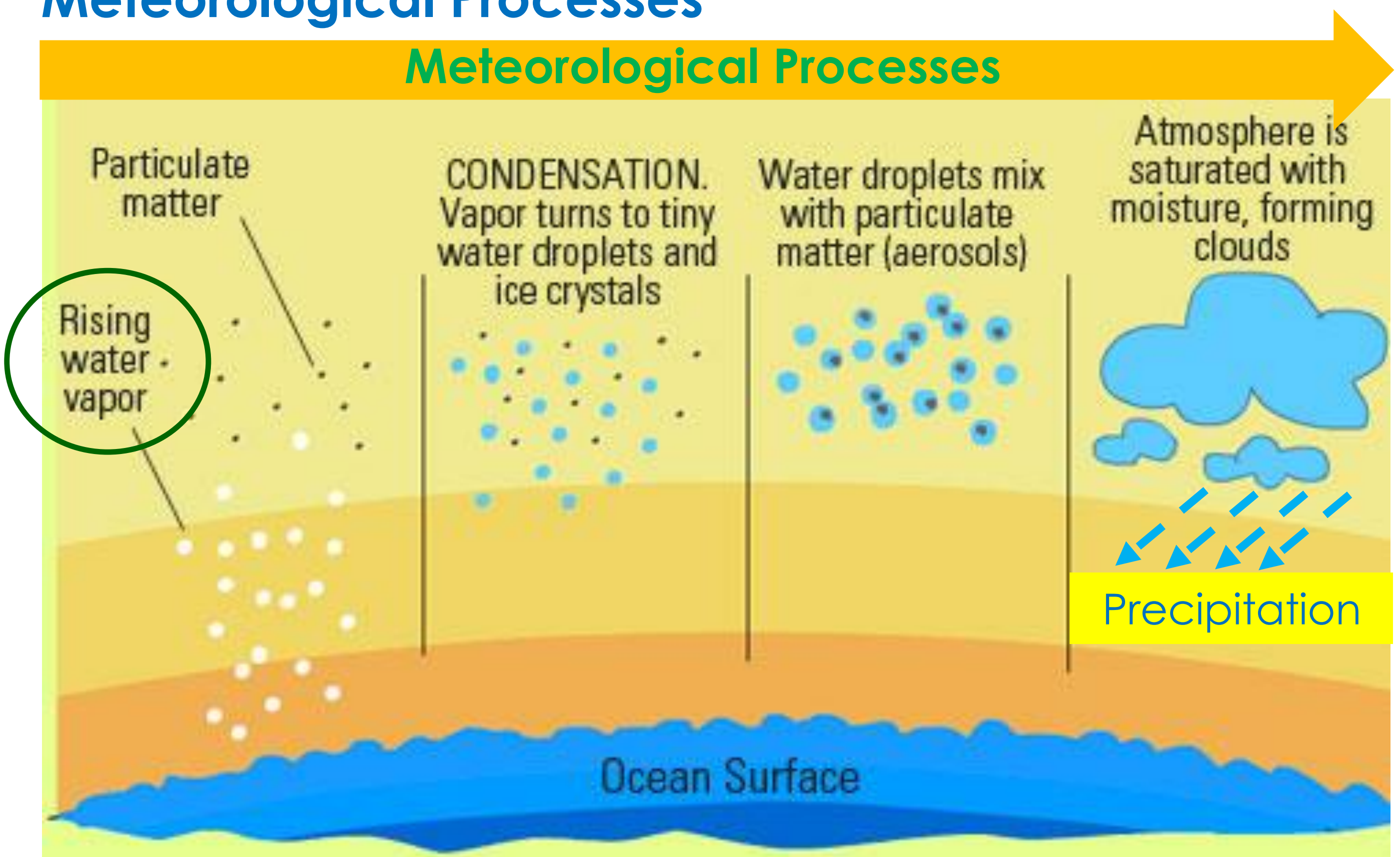
$$\frac{m_p}{\rho_w A} = \frac{77}{(1{,}000 \mathrm{x} 1)} = 0.077 \text{ m} = 77 \text{ mm}$$

PRECIPITATION

Precipitation

In meteorology, precipitation is any product of the condensation of atmospheric water vapor that falls under gravity.

Meteorological Processes



Source: eschooltoday (2018)

Forms of Precipitation

Drizzle/Mist

- Drizzle is a light liquid precipitation consisting of liquid water drops smaller than those of rain – generally smaller than 0.5 mm.

Rain

- Rain is liquid water in the form of droplets that have condensed from atmospheric water vapor and then becomes heavy enough to fall under gravity.
- Water drops of size between 0.5-0.6 mm.

Source: Wikipedia (2018b)

Forms of Precipitation

Snow

- Snow refers to forms of ice crystals that precipitate from the atmosphere (usually from clouds).
- Diameter is 1-2 mm.
- Average specific gravity is 0.1.

Sleet [Rain and snow mixed]

- Sleet is precipitation composed of rain and partially melted snow.
- Diameter is 0.5-5 mm.

Source: Wikipedia (2018b)

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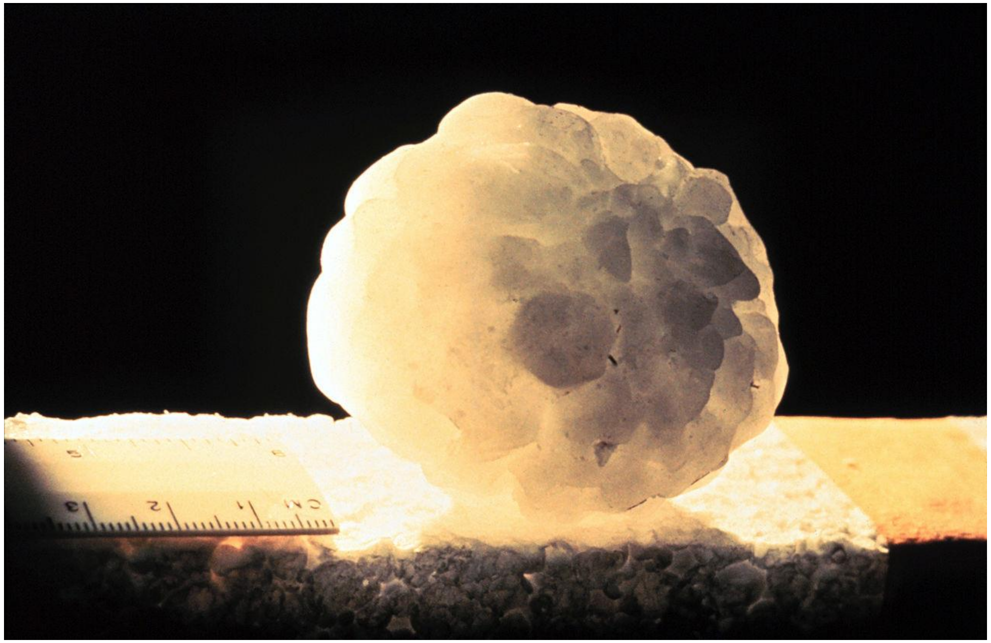
Forms of Precipitation



Hail

- Hail is the precipitation in the form of ice balls of diameter more than about 8 mm.

Source: Wikipedia (2018)

RAINFALL DATA: RAINFALL MEASUREMENT

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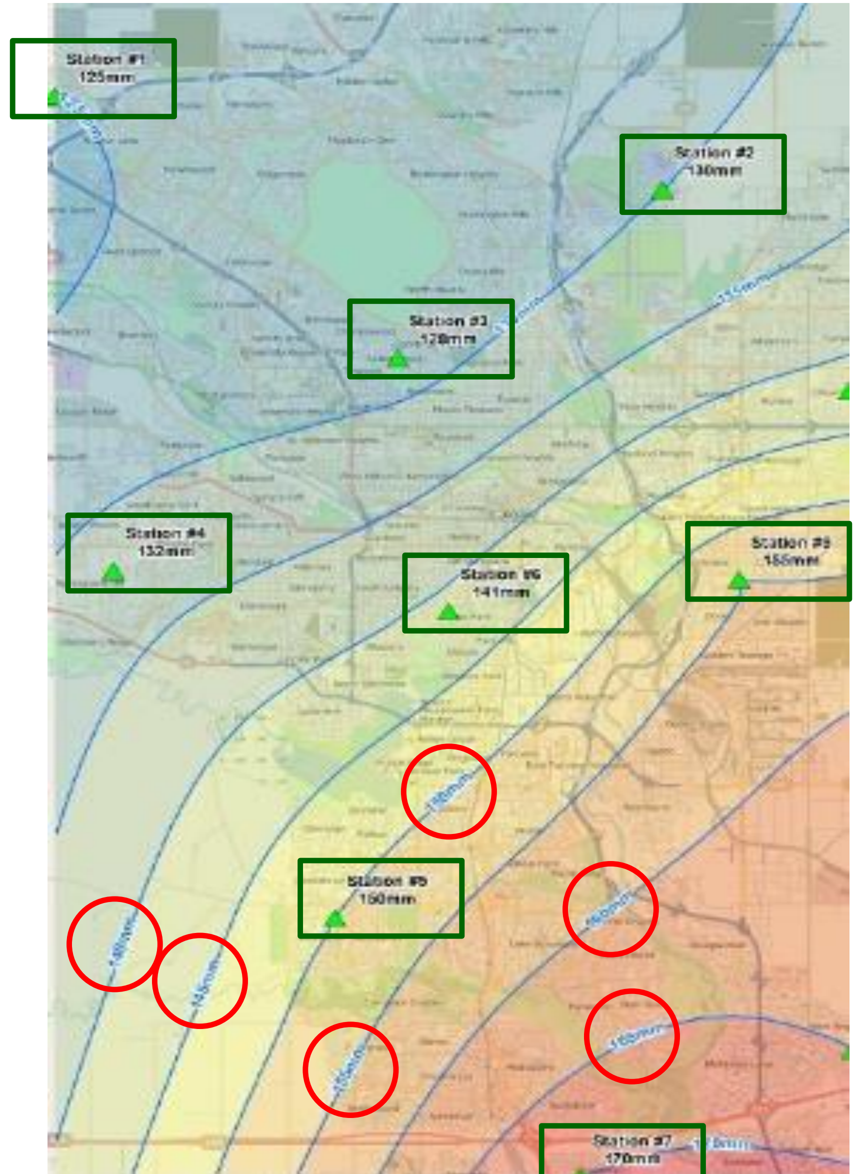
Rain Guage

Tipping Bucket Rain Guage

RAINFALL DATA:
RAINFALL ISOHYETAL MAP

Rainfall Isohyetal Map

- Rainfall data varies greatly in space and time.
- Rainfall can be represented by "**Isohyetal Map**".
- **Isohyet** is a contour of constant rainfall.



RAINFALL DATA: RAINFALL HYETOGRAPH

Time (min)	Rainfall (in)	Cumulative rainfall	Running Totals 30 min	Running Totals 1 h	Running Totals 2 h
0		0.00			
5	0.02	0.02			
10	0.34	0.36			
15	0.10	0.46			
20	0.04	0.50			
25	0.19	0.69			
30	0.48	1.17	1.17		
35	0.50	1.67	1.65		
40	0.50	2.17	1.81		
45	0.51	2.68	2.22		
50	0.16	2.84	2.34		
55	0.31	3.15	2.46		
60	0.66	3.81	2.64	3.81	
65	0.36	4.17	2.50	4.15	
70	0.39	4.56	2.39	4.20	
75	0.36	4.92	2.24	4.46	
80	0.54	5.46	2.62	4.96	
85	0.76	6.22	*3.07*	5.53	
90	0.51	6.73	2.92	*5.56*	
95	0.44	7.17	3.00	5.50	
100	0.25	7.42	2.86	5.25	
105	0.25	7.67	2.75	4.99	
110	0.22	7.89	2.43	5.05	
115	0.15	8.04	1.82	4.89	
120	0.09	8.13	1.40	4.32	8.13
125	0.09	8.22	1.05	4.05	*8.20*
130	0.12	8.34	0.92	3.78	7.98
135	0.03	8.37	0.70	3.45	7.91
140	0.01	8.38	0.49	2.92	7.88
145	0.02	8.40	0.36	2.18	7.71
150	0.01	8.41	0.28	1.68	7.24
Max. depth	0.76		3.07	5.56	8.20
Max. intensity (in/h)	9.12		6.14	5.56	4.10

=1.17-0.00
=1.67-0.02
=3.81-0.00
=4.17-0.02
=3.81-0.00
=8.22-0.02
=0.76/(5/60)
=8.20/2

Source: Chow et al. (1988)

Computation of rainfall depth and intensity at a point

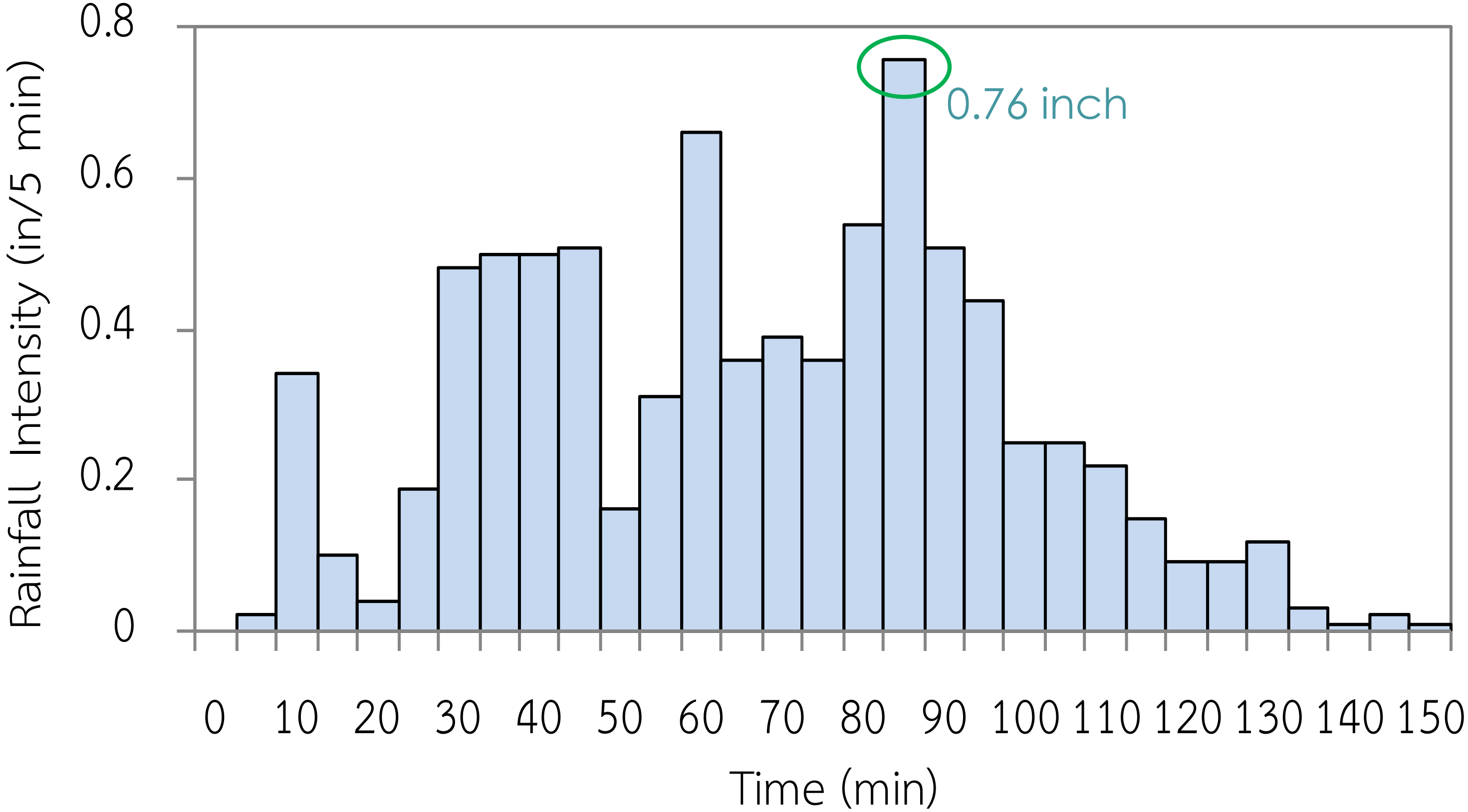
The rainfall data in 5-minute increments from gage 1-Bee in the Austin storm.

Computations of max rainfall depth and intensity give index of how severe a particular storm is, compared to other storms recorded at the same location, and they provide useful data for design of control structures.

RAINFALL DATA: RAINFALL HYETOGRAPH

Rainfall Hyetograph

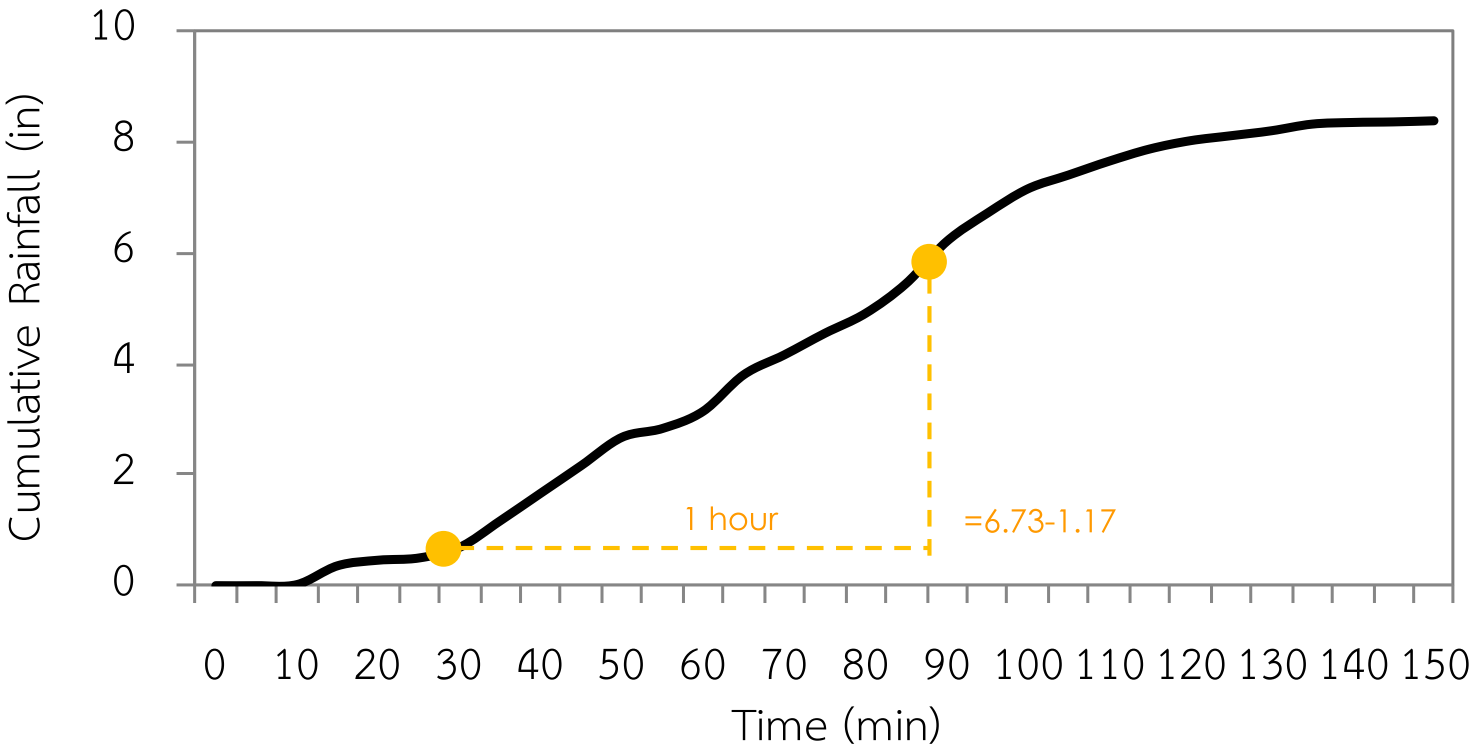
Rainfall hyetograph is a plot of rainfall depth or intensity as a function of time.



RAINFALL DATA: CUMULATIVE RAINFALL HYETOGRAPH

Cumulative Rainfall Hyetograph/Rainfall Mass Curve

Cumulative rainfall hyetograph is a plot of cumulative rainfall as a function of time.



RAINFALL DATA: AREAL RAINFALL

Areal Rainfall

In general, for water resources planning purposes, knowledge is required of **the average rainfall depth over a certain area**. This is called the "**Areal Rainfall**".

Some examples where the areal rainfall is required include;

- Design of a culvert or bridge draining a certain catchment area.
- Design of a pumping station to drain an urbanized area.
- Design of a structure to drain a polder.

Source: Chow et al. (1988)

RAINFALL DATA:
AREAL RAINFALL ESTIMATION METHODS

Estimation Methods

There are various methods to estimate the average rainfall over an area, (areal rainfall) with area A from n Point-measurements, P_i.

- **Arithmetic-Mean Method**
- **Thiessen Polygon Method**
- **Isohyetal Method**
- **Grid Point Method**
- **Kriging Method**

Arithmetic-Mean Method

- It is the simplest method in which average depth of rainfall is obtained by obtaining the sum of the depths of rainfall (P_1, P_2, P_3,...., P_n) measured at stations 1, 2, 3,...., n and dividing the sum by the total number of stations.

$$\overline{P} = \frac{P_1 + P_2 + P_3 + ... + P_n}{n} = \frac{1}{n}\sum_{i=1}^{n} P_i$$

- This method is suitable if the rain gage stations are uniformly distributed over the entire area and the rainfall variation in the area is not large.

RAINFALL DATA:
AREAL RAINFALL ESTIMATION METHODS

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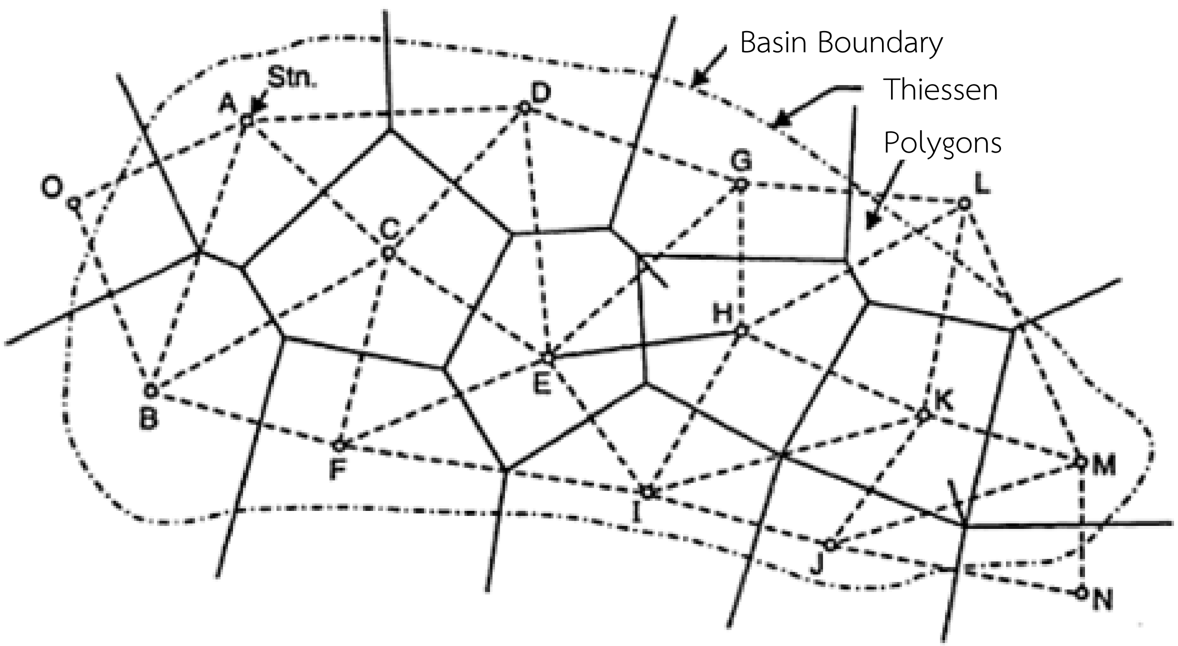
Thiessen Polygon Method

- Thiessen polygon method takes into account the non-uniform distribution of the gages by assigning a weightage factor for each rain gage.
- The entire area is divided into number of triangular areas by joining adjacent rain gage stations with the straight lines.



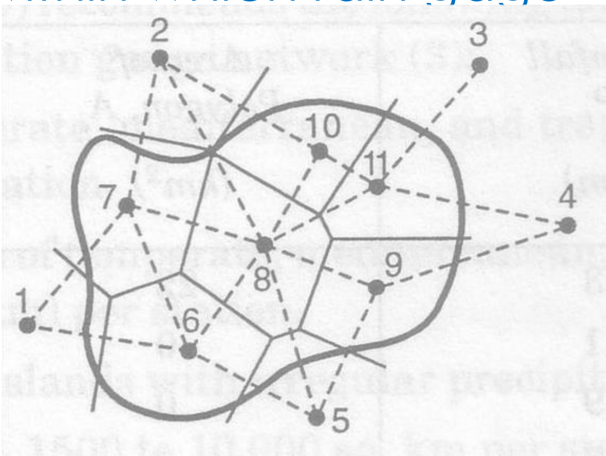
RAINFALL DATA: AREAL RAINFALL ESTIMATION METHODS

Thiessen Polygon Method

- Assuming that rainfall P_i recorded at any stations i representative rainfall of the area A_i of the polygon i within which rain gage station is located.

$$\bar{P} = \frac{1}{A}\sum_{i=1}^{n} P_i A_i$$

$$A = \sum_{i=1}^{n} A_i = A_1 + A_2 + A_3 + \dots + A_n$$



- The method is better than the arithmetic mean method since it assigns some weightage to all rain gages on area basis.
- The rain gage stations outside the catchment can also be used effectively.
- Once the weightage factors for all the rain gage stations are computed, the calculation of the average rainfall depth P is relatively easy for given network of stations.

RAINFALL DATA:
AREAL RAINFALL ESTIMATION METHODS

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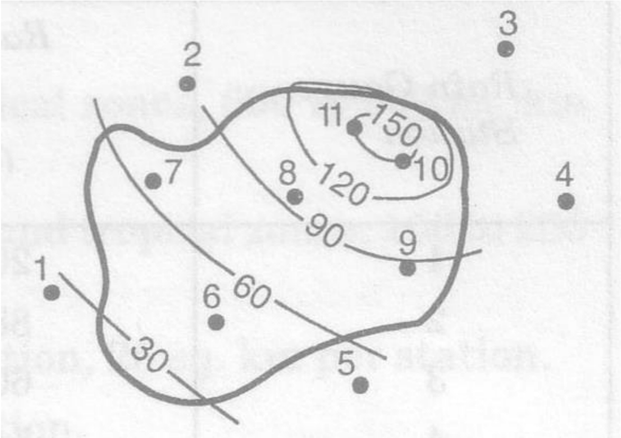
Isohyetal Method

- An isohyet is a contour of equal rainfall.
- Knowing the depths of rainfall at each rain gage station of an area, assuming **linear variation of rainfall between any two adjacent stations**, one can draw a smooth curve passing through all points indicating the same value of rainfall.
- The area between two adjacent isohyets is measured with the help of a **"Planimeter"**.
- Average depth of rainfall, P

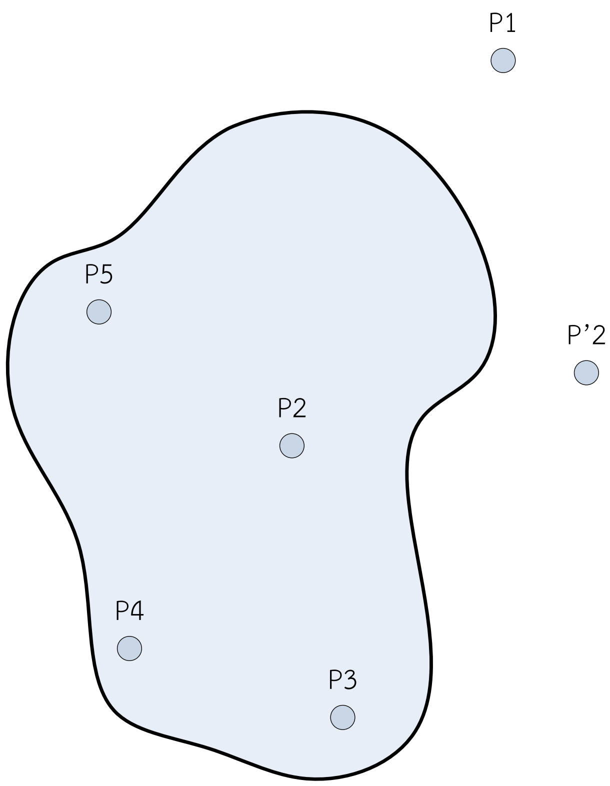


$$\overline{P} = \frac{1}{A}\sum \frac{[\text{Area between two adjacent isohyets}] \text{x}}{[\text{mean of the two adjacent isohyte values}]}$$

- Since this method considers actual spatial variation of rainfall, it is considered as **the best method for computing average depth of rainfall**.

ARITHMETIC-MEAN METHOD: EXAMPLE 3

Compute the areal average rainfall by arithmetic-mean method.



Station	Observed rainfall within the area (mm)
P2	20
P3	30
P4	40
P5	50
Total	140

Average Rainfall = 140/4 = 35 mm

THIESSEN POLYGON METHOD: EXAMPLE 4

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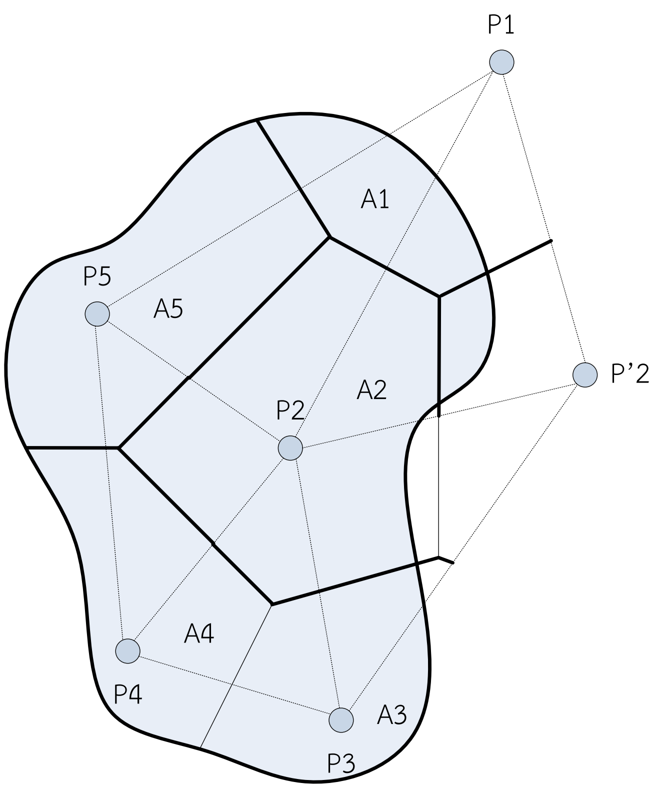
Compute the areal average rainfall by Thiessen polygon method.

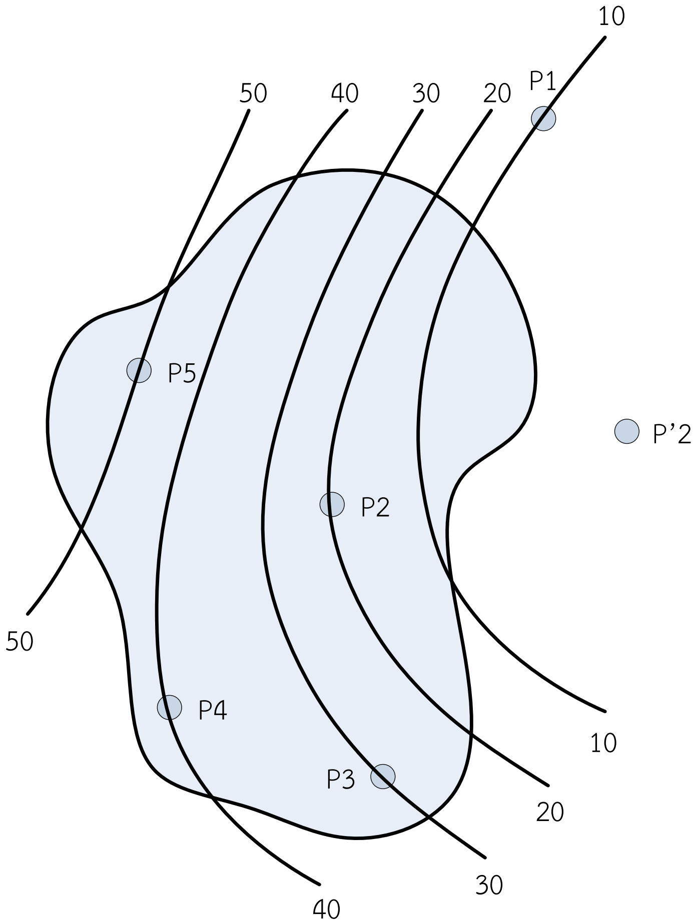


Station	Observed rainfall (mm)	Area (km^2)	Weighted rainfall (mm)
P1	10	0.22	2.2
P2	20	4.02	80.4
P3	30	1.35	40.5
P4	40	1.60	64.0
P5	50	1.95	97.5
Total		9.14	284.6

Average Rainfall = 284.6/9.14
= 31.1 mm

ISOHYETAL METHOD: EXAMPLE 5

Compute the areal average rainfall by isohyetal method.

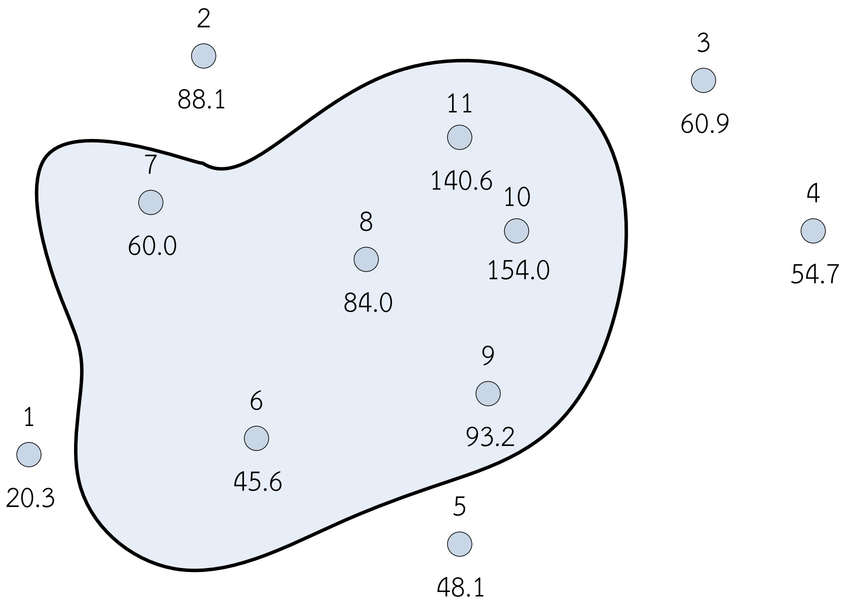


Isohyets	Area Enclosed (km^2)	Average Rainfall (mm)	Rainfall Volume
	0.88	5*	4.4
10	1.59	15	23.9
20	2.24	25	56.0
30	3.01	35	105.4
40	1.22	45	54.9
50	0.20	53*	10.6
Total		9.14	255.2

Average Rainfall = 255.2/9.14 = 27.9 mm

ARITHMETIC-MEAN METHOD: EXAMPLE 6

The average depth of annual rainfall precipitation as obtained at the rain gage stations for a specified area are as shown in figure. The values are in cm. Determine the average depth of annual precipitation using the arithmetic-mean method.



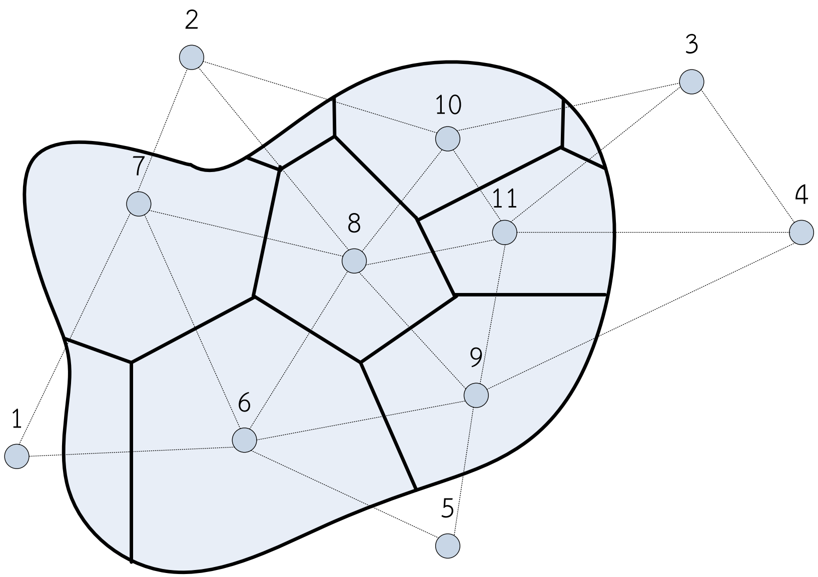
Solution

$$\overline{P} = \frac{1}{11}[20.3 + 88.1 + 60.9 + 54.7 + 48.1 + 45.6 + 60.0 + 84.0 + 93.2 + 140.6 + 154.0]$$

$$= \frac{1}{11}(849.5) = 77.23 \text{ cm}$$

THIESSEN POLYGON METHOD: EXAMPLE 7

The average depth of annual rainfall precipitation as obtained at the rain gage stations for a specified area are as shown in figure. The values are in cm. Determine the average depth of annual precipitation using the Thiessen polygon method.



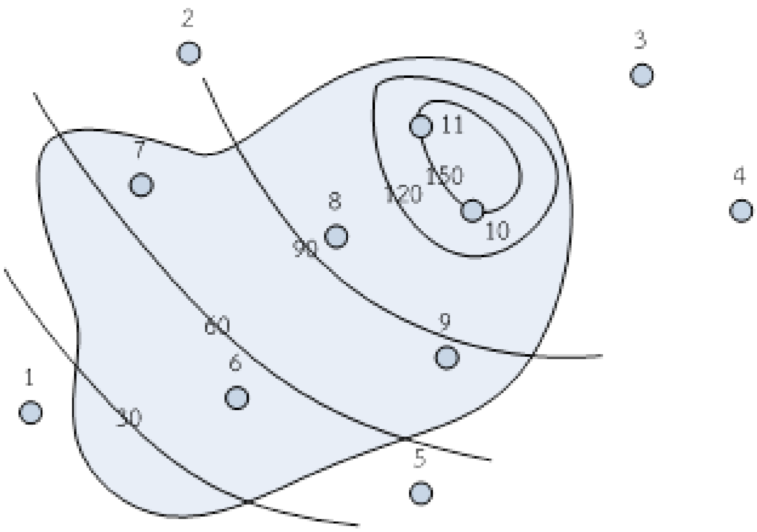
THIESSEN POLYGON METHOD: EXAMPLE 7

Rainfall Guage Station	Rainfall, Pi (cm)	Area of Polygon, Ai (km^2)	Weightage Factor (%), $A_i/\Sigma A_i x100$	$P_iA_i/\Sigma A_i$
1	20.3	22	1.13	0.23
2	88.1	0	0	0
3	60.9	0	0	0
4	54.7	0	0	0
5	48.1	62	3.19	1.53
6	45.6	373	19.19	8.75
7	60.0	338	17.39	10.43
8	84.0	373	19.19	16.12
9	93.2	286	14.71	13.71
10	140.6	236	12.41	17.07
11	154.0	254	13.07	20.13
Total		1,944	100.01	87.97

$$\text{Average Annual Precipitation} = \Sigma P_i \frac{A_i}{\Sigma A_i} = 87.97 \text{ cm}$$

ISOHYETAL METHOD: EXAMPLE 8

The average depth of annual rainfall precipitation as obtained at the rain gage stations for a specified area are as shown in figure. The values are in cm. Determine the average depth of annual precipitation using the isohyetal method.



ISOHYETAL METHOD: EXAMPLE 8

Isohyets (cm)	Net Area, Ai (km^2)	Average Precipitaiton, Pi (cm)	P_iA_i
<30	96	25	2,400
30-60	600	45	27,000
60-90	610	75	45,750
90-120	360	105	37,800
120-150	238	135	32,130
>150	40	160	6,400
Total	1,944		151,480

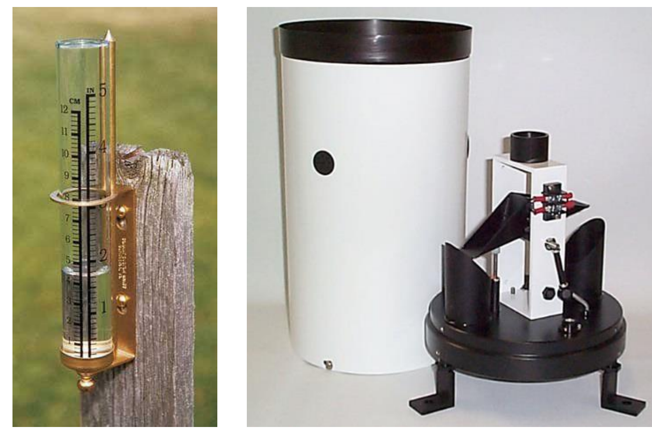
$$\text{Average Annual Precipitation for the basin} = \frac{151{,}480}{1{,}944}$$

$$= 77.92 \text{ mm}$$

RAINFALL DATA: CONTINUITY AND CONSISTENCY CHECK

Continuity and Consistency

- Rainfall data must be checked for **continuity and consistency** before they are analyzed for any purpose.
- Changes in the catchment rainfall are caused by the changes in relevant conditions of the rain gauge;
    - Guage location
    - Observation technique
    - Surrounding
    - etc.

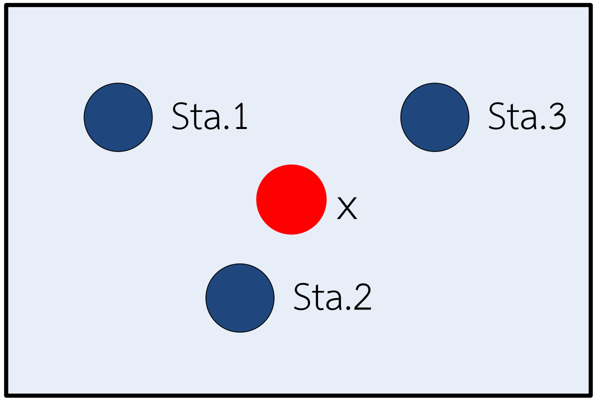


Rain Guage & Tipping Bucket Rainguage

Surrounding

Estimation of Rainfall Missing Data

The missing annual rainfall, Px



$$P_x = \frac{1}{M}(P_1 + P_2 + ... + P_m)$$

$$P_x = \frac{N_x}{M}[\frac{P_1}{N_1} + \frac{P_2}{N_2} + ... + \frac{P_m}{N_m}]$$

Multiple Linear Regression

$$P_x = a + b_1 P_1 + b_2 P_2 + ... + b_m P_m$$

When

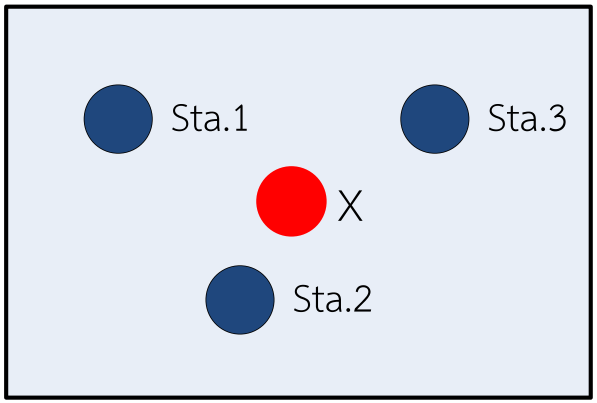
1, 2, 3,….,M = neighbouring rainfall stations

P_1, P_2, P_3,….,P_m = annual rainfall values

N_1, N_2, N_3,….,N_m = average rainfall values

ESTIMATION OF RAINFALL MISSING DATA: EXAMPLE 9

Fill up the missing annual rainfall at station X.

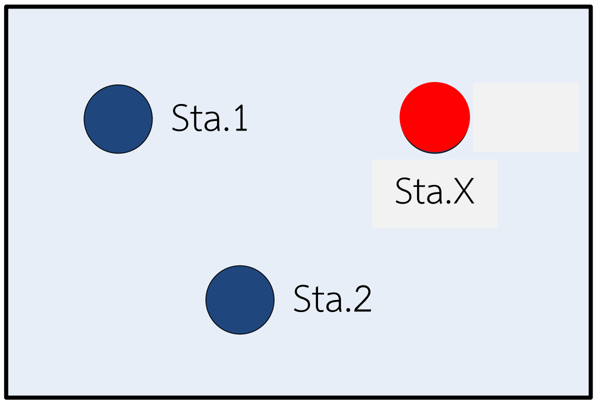


Station	Annual Rainfall (mm/yr)
1	1,000
2	1,300
3	1,000
X	-
Px	1,100

$$P_x = \frac{1}{M}(P_1 + P_2 + ... + P_m)$$

ESTIMATION OF RAINFALL MISSING DATA: EXAMPLE 10

Fill up the missing annual rainfall at station X.



$N_1 = N_A = 1{,}000$ (Avg.10)
$N_2 = N_B = 1{,}200$ (Avg.10)
$N_X = 1{,}000$ (Avg.9)
$M = 2$
$P_X = 1{,}000$ mm

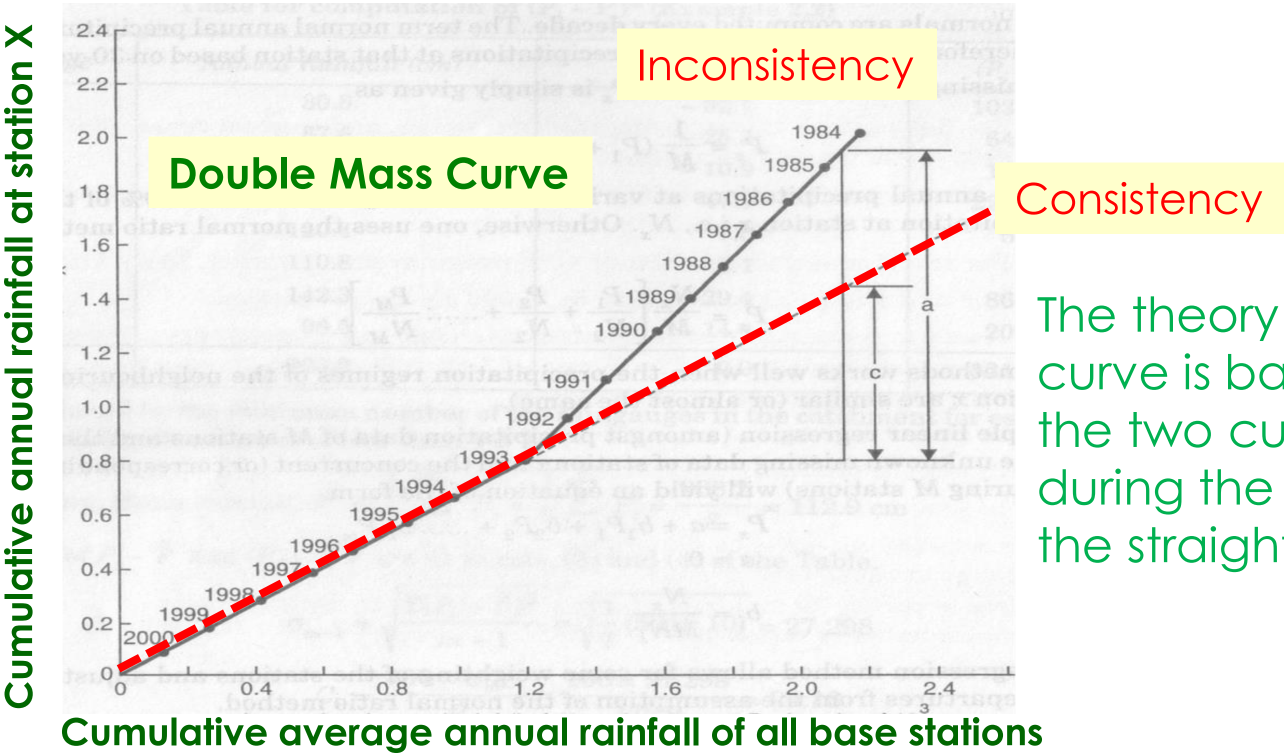
Year	Station A	Station B	Station X
1	1,000	1,200	1,000
2	1,000	1,200	1,000
3	1,000	1,200	1,000
4	1,000	1,200	1,000
5	1,000	1,200	1,000
6	1,000	1,200	1,000
7	1,000	1,200	Px

$$P_x = \frac{N_x}{M}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \dots + \frac{P_m}{N_m}\right]$$

RAINFALL DATA: CONTINUITY AND CONSISTENCY CHECK

Double Mass Curve

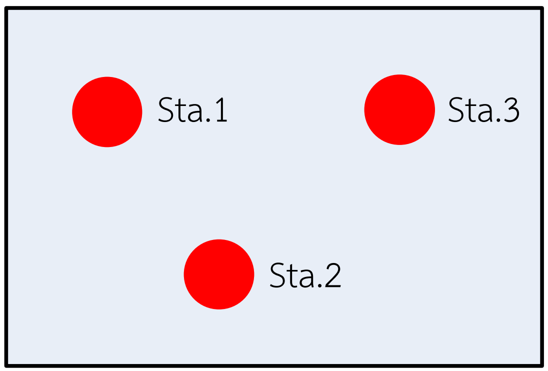
Double mass curve compares the accumulated annual rainfall at a given station with the concurrent accumulated values of average rainfall for a group of the surrounding stations.



The theory of double mass curve is based on the fact that the two cumulative quantities during the same period exhibit the straight line.

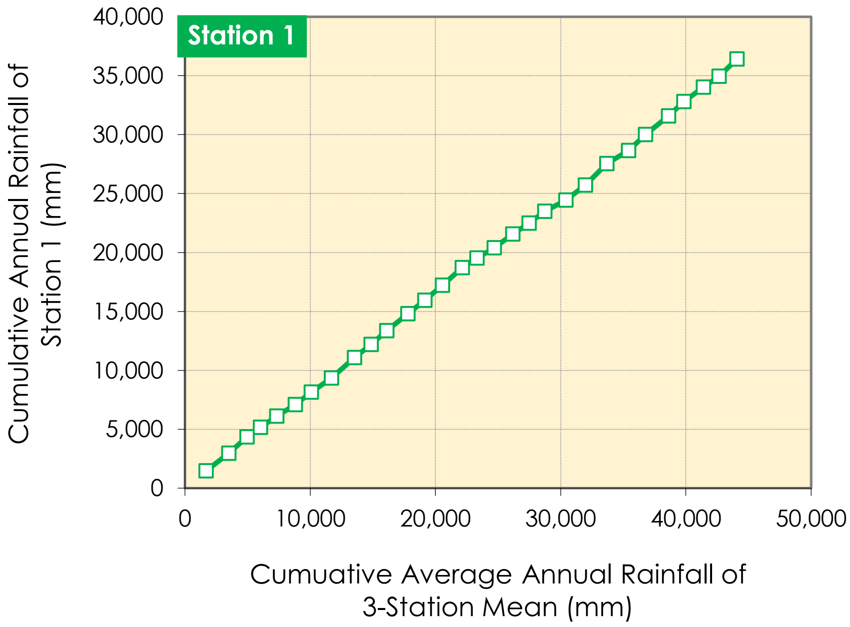
DOUBLE MASS CURVE: EXAMPLE 10

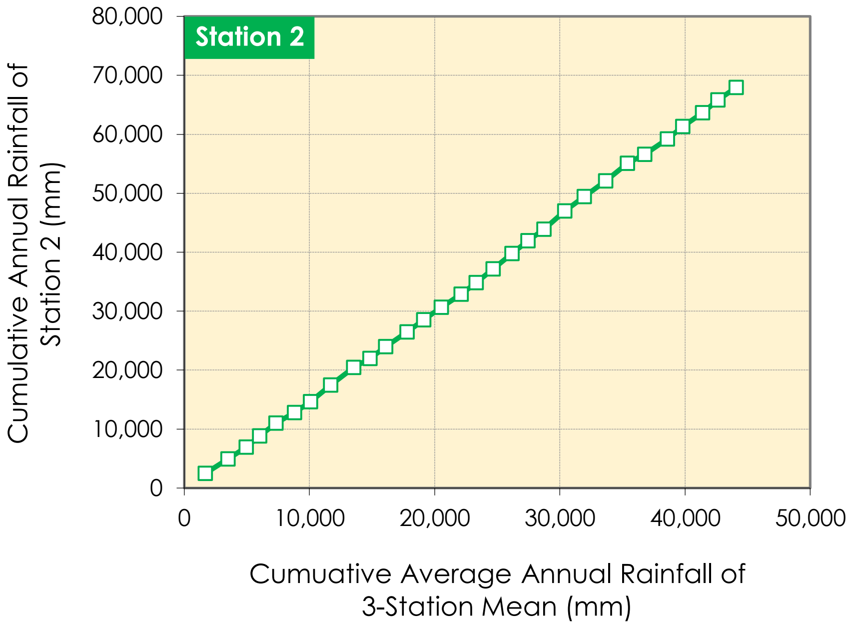
Construct the double mass curves of rainfall data as tabulated.

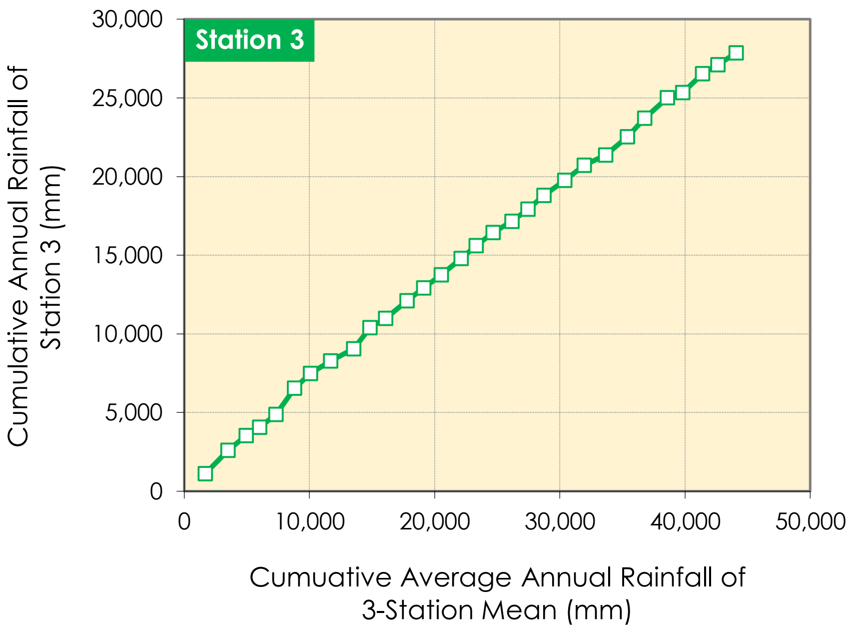


	Annual Rainfall (mm/yr)			Cumulative Annual Rainfall (mm/yr)			Avg. Value
No	Annual Rainfall (mm/yr)			Cumulative Rainfall			
	Station 1	Station 2	Station 3	Station 1	Station 2	Station 3	Average
1	1,486.20	2,472.20	1,113.40	1,486.20	2,472.20	1,113.40	1,690.60
2	1,475.70	2,468.80	1,482.90	2,961.90	4,941.00	2,596.30	3,499.73
3	1,403.80	2,001.30	953.10	4,365.70	6,942.30	3,549.40	4,952.47
4	793.80	1,917.50	521.20	5,159.50	8,859.80	4,070.60	6,029.97
5	962.60	2,130.90	812.70	6,122.10	10,990.70	4,883.30	7,332.03
6	964.20	1,819.80	1,673.60	7,086.30	12,810.50	6,556.90	8,817.90
7	1,056.90	1,851.80	925.10	8,143.20	14,662.30	7,482.00	10,095.83
8	1,217.00	2,783.00	794.10	9,360.20	17,445.30	8,276.10	11,693.87
9	1,737.00	3,034.00	775.20	11,097.20	20,479.30	9,051.30	13,542.60
10	1,096.90	1,492.50	1,355.40	12,194.10	21,971.80	10,406.70	14,857.53
11	1,165.90	2,020.00	575.90	13,360.00	23,991.80	10,982.60	16,111.47
12	1,458.00	2,504.70	1,126.60	14,818.00	26,496.50	12,109.20	17,807.90
13	1,132.60	2,042.90	819.20	15,950.60	28,539.40	12,928.40	19,139.47
14	1,272.80	2,115.30	827.50	17,223.40	30,654.70	13,755.90	20,544.67
15	1,484.10	2,236.60	1,036.00	18,707.50	32,891.30	14,791.90	22,130.23
16	818.60	1,922.30	826.60	19,526.10	34,813.60	15,618.50	23,319.40
17	865.40	2,379.40	835.20	20,391.50	37,193.00	16,453.70	24,679.40
18	1,168.70	2,609.80	710.00	21,560.20	39,802.80	17,163.70	26,175.57
19	939.30	2,177.40	776.40	22,499.50	41,980.20	17,940.10	27,473.27
20	984.10	1,928.60	874.80	23,483.60	43,908.80	18,814.90	28,735.77
21	969.80	3,088.20	943.60	24,453.40	46,997.00	19,758.50	30,402.97
22	1,275.90	2,443.40	961.30	25,729.30	49,440.40	20,719.80	31,963.17
23	1,810.20	2,689.40	646.50	27,539.50	52,129.80	21,366.30	33,678.53
24	1,110.70	2,938.30	1,170.40	28,650.20	55,068.10	22,536.70	35,418.33
25	1,349.70	1,571.10	1,174.60	29,999.90	56,639.20	23,711.30	36,783.47
26	1,580.10	2,568.70	1,312.10	31,580.00	59,207.90	25,023.40	38,603.77
27	1,214.10	2,127.00	322.80	32,794.10	61,334.90	25,346.20	39,825.07
28	1,229.30	2,317.20	1,195.00	34,023.40	63,652.10	26,541.20	41,405.57
29	926.70	2,199.00	566.40	34,950.10	65,851.10	27,107.60	42,636.27
30	1,464.70	2,117.90	773.10	36,414.80	67,969.00	27,880.70	44,088.17

DOUBLE MASS CURVE: EXAMPLE 10







RAINFALL-RUNOFF RELATION: EMPIRICAL FORMULA/WATER BUDGET/ANNS

Empirical Formula

- Barlow's Formula

$$R = K_b P$$

- Strange's Formula

$$R = K_s P$$

- Khosla analyzed monthly rainfall data (P_m), runoff (R_m), and Temperature (T_m) for various catchments of India and US.

$$R_m = P_m - L_m$$

L_m represents monthly losses

Hydrologic Water Budget Equation

$$R = P - ET - G - \Delta S$$

Artificial Neural Network (ANNs) Technique

RAINFALL-RUNOFF RELATION: EMPIRICAL FORMULA/WATER BUDGET/ANNS

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Barlow's runoff coefficient Kb in percent

Class	Description of Catchment	Values of Kb (Percent)		
		Season 1	Season 2	Season 3
A	Flat, cultivated and absorbent soils	7	10	15
B	Flat, partly cultivated, stiff soils	12	15	18
C	Average catchment	16	20	32
D	Hills and plains with little cultivation	28	35	60
E	Very hilly, steep and hardly any cultivation	36	45	81

Season 1: Light rain, no heavy downpour
Season 2: Average or varying rainfall, no continuous downpour
Season 3: Continuous downpour

Developed for use in UP and **catchment is less than about 150 km^2**.

RAINFALL-RUNOFF RELATION: EMPIRICAL FORMULA/WATER BUDGET/ANNS

Strange's runoff coefficient Ks in percent

Total Monsoon Rainfall (cm)	Runoff Coefficient Ks (Percent)		
	Good Catchment	Average Catchment	Bad Catchment
25	4.3	3.2	2.1
50	15.0	11.3	7.5
75	26.3	19.7	13.1
100	37.3	28.0	18.7
125	47.6	35.7	23.8
150	58.9	44.1	29.4

Developed for use in **border areas of Maharashtra and Karnataka**.

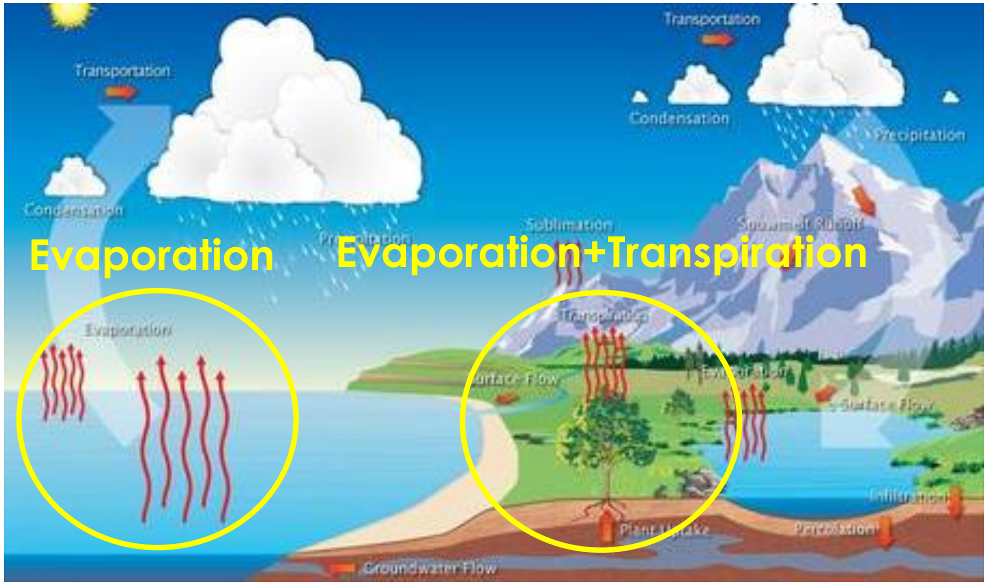
EVAPORATION

Evaporation

- Evaporation is the process by which **water changes from a liquid to a gas or vapor**.
- Evaporation is the primary pathway that water moves from the liquid state back into the water cycle as atmospheric water vapor.

Evaporation from **open water surface**.
Evaporation from **land surface** comprises.

- **Evaporation** directly from soil and vegetation surface.
- **Transpiration** through plant leaves.

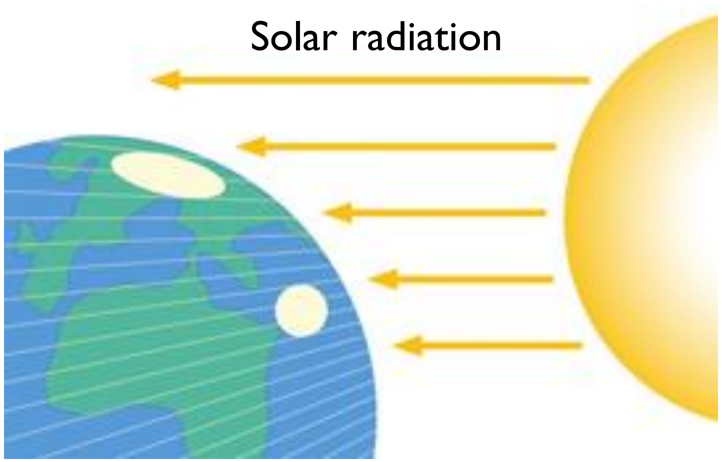


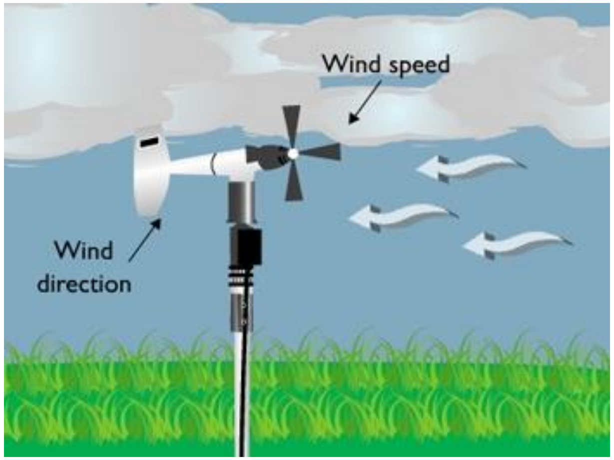
EVAPORATION

Factors influencing Evaporation Loss

The two main factors influencing evaporation from an open water surface are :

- The supply of energy to provide latent heat of vaporization. (**Solar radiation is the main source of heat energy**).
- The ability to transport vapor away from evaporative surface. It depends on **wind velocity over the surface and specific humidity gradient** in the air above it.





EVAPORATION DATA:
EVAPORATION MEASUREMENT

Evaporation Pan

Evaporation Pan Method, E = Kp.Ep

E = Evaporation Rate (mm/day)
Kp = Pan Coefficient
Ep = Measured Evaporation (mm/day)

EVAPORATION DATA:
ESTIMATION OF EVAPORATION DATA

Data needed
- Temperature
- Wind Velocity
- Relative Humidity

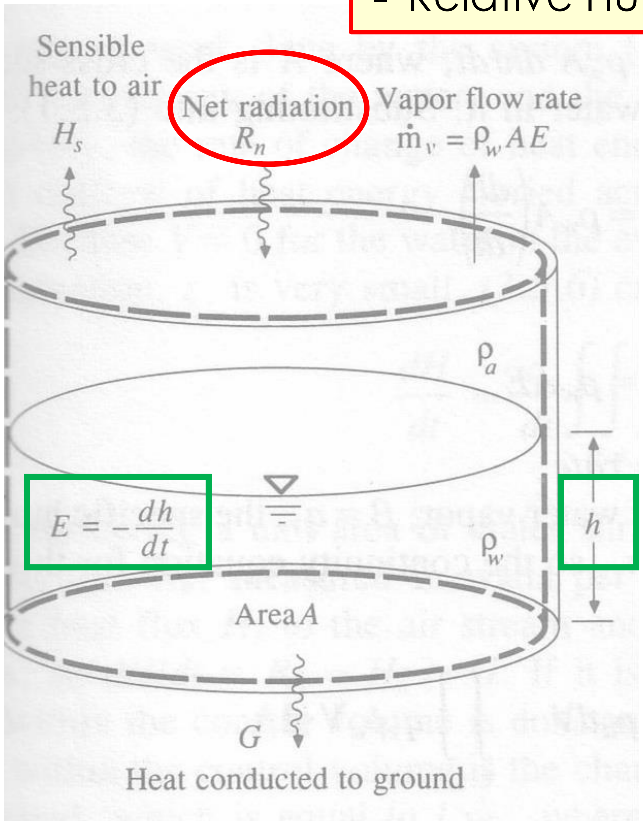
Estimation of Evaporation Data

Consider the evaporation from an evaporation pan.

An evaporation pan is a circular tank containing water.

The rate of evaporation is measured by the rate of fall of water surface.

A control surface is drawn around the pan enclosing both the water in the pan and the air above it.



Evaporation Pan

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Estimation of Evaporation Data

- **Energy Balance Method**

$$E_r = \mathbf{0.0353} R_n \quad \left(\frac{mm}{day}\right)$$

When

$$R_n = \text{Net Radiation} \quad \left(\frac{W}{m^2}\right)$$

- **Aerodynamic Method**

$$E_a = B(e_{as} - e_a) \quad \left(\frac{mm}{day}\right)$$

When

$$B = \frac{\mathbf{0.102} u_2}{[\ln(\frac{z_2}{z_0})]^2} \quad \left(\frac{mm}{day}.Pa\right)$$

$$e_{as} = \mathbf{611} \exp\left(\frac{\mathbf{17.27} T}{\mathbf{237.3} + T}\right) \quad (Pa)$$

$$e_a = R_h e_{as} \quad (Pa)$$

EVAPORATION DATA: ESTIMATION OF EVAPORATION DATA

Estimation of Evaporation Data

- **Combined Method**

$$E = \frac{\Delta}{\Delta + \gamma} E_r + \frac{\gamma}{\Delta + \gamma} E_a \quad \left(\frac{mm}{day}\right)$$

$$\Delta = \frac{4098\, e_{as}}{(237.3 + T)^2} \quad \left(\frac{Pa}{^\circ C}\right)$$

$$\gamma = 66.8 \quad \left(\frac{Pa}{^\circ C}\right)$$

- **Evaporation Pan Method**

$$E = K_p E_p$$

- **Priestley-Taylor Method**

$$\Delta = \alpha \frac{\Delta}{(\Delta + \gamma)} E_r$$

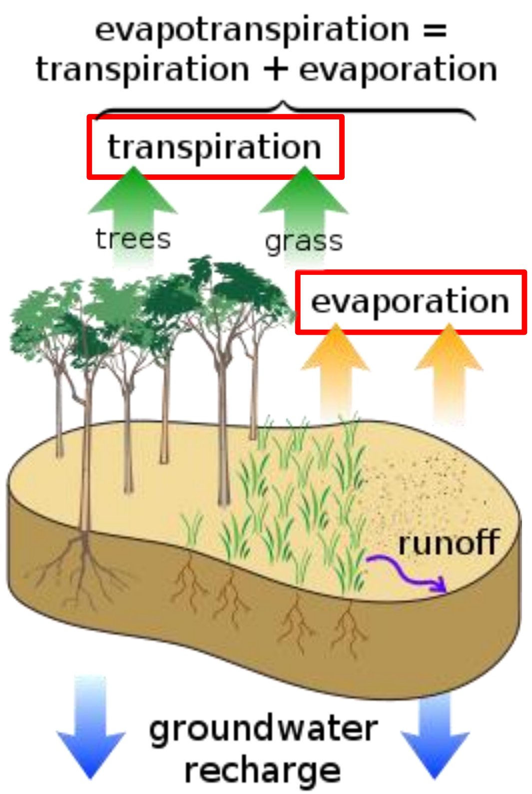
When

$$\alpha = 1.3$$

EVAPOTRANSPIRATION

Evapotranspiration (ET)

The processes of evaporation from land surface and transpiration from vegetation are collectively termed "**Evapotranspiration**".



Source: Wikipedia (2018c)

The term **Evapotranspiration** combines two words:

- **Evaporation** of water from the soil.
- **Transpiration** of water from plants into the air.

Evapotranspiration means the total loss of water from a crop into the air.

ET = ETc = Crop Water

Crop water is important because it determines how much water must be provided by irrigation or rain.

EVAPOTRANSPIRATION:
ET CALCULATION

Crop ET (ETc) vs Reference Crop ET (ETo)

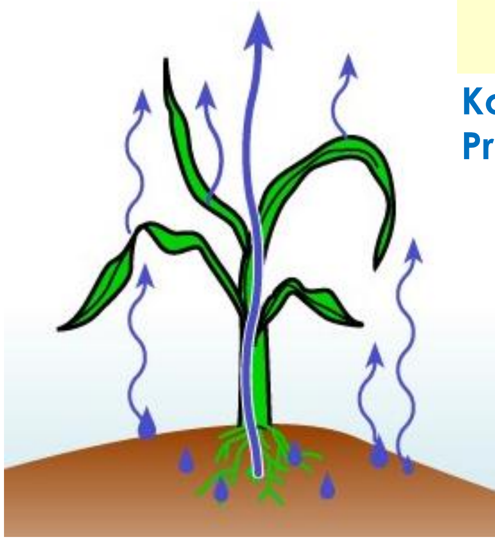
Crop of Interest

Grass Reference Crop (Alfalfa Grass)

Climatic Data:
- Radiation
- Temperature
- Wind Speed
- Humidity

ETc = Kc.ETo

Kc = Crop Coefficient Provided by RID



Reference Crop Evapotranspiration Equations:
- Penman Equation
- Penman-Monteith Equation
- FAO-Penman Monteith Equation
- Doorenbos-Pruitt Equation
- Bladney-Criddle Equation
- others

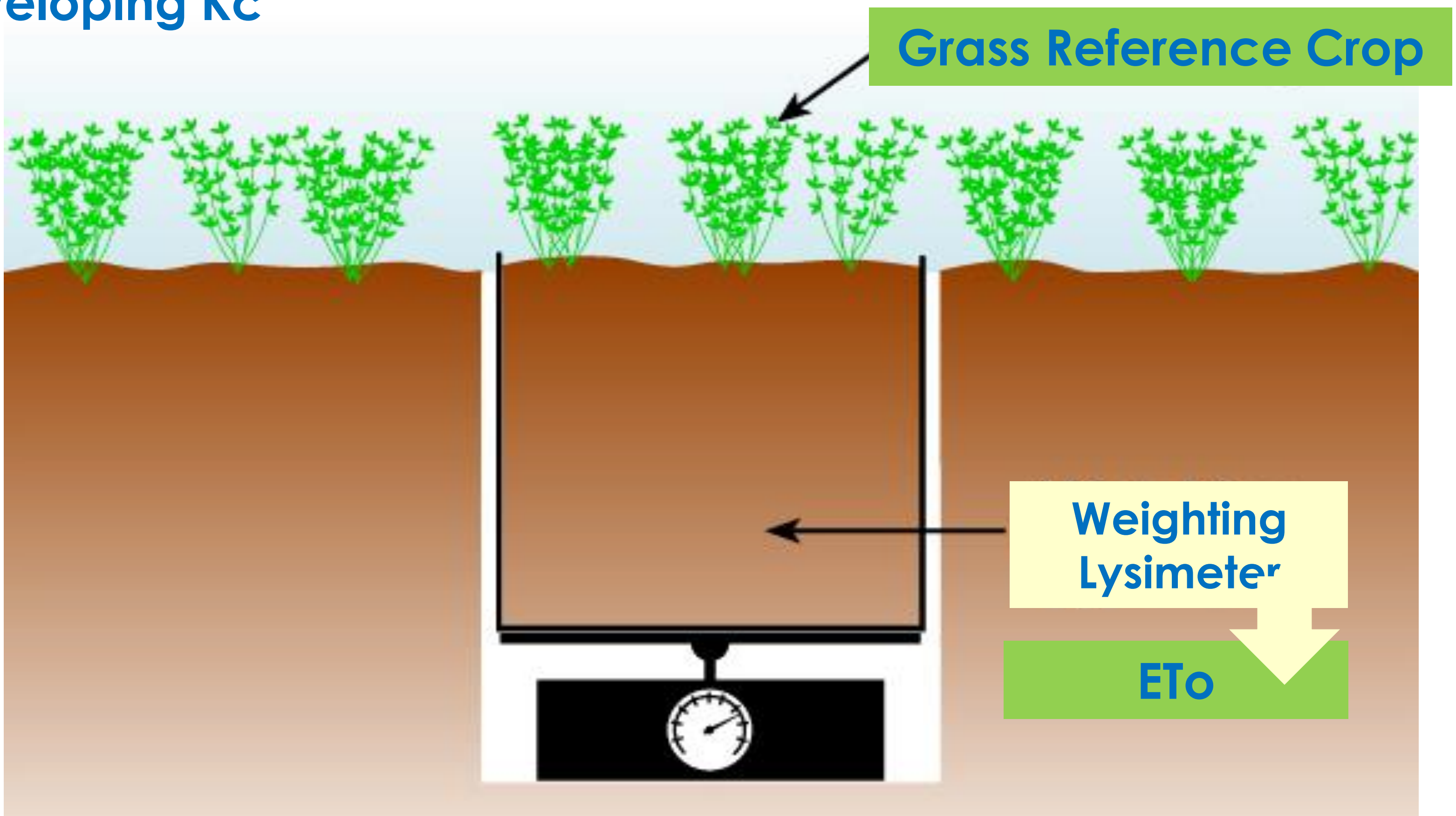
Direct Measurement by Lysimeter

ETc

ETo

EVAPOTRANSPIRATION: DEVELOPING CROP COEFFICIENT, KC

Developing Kc

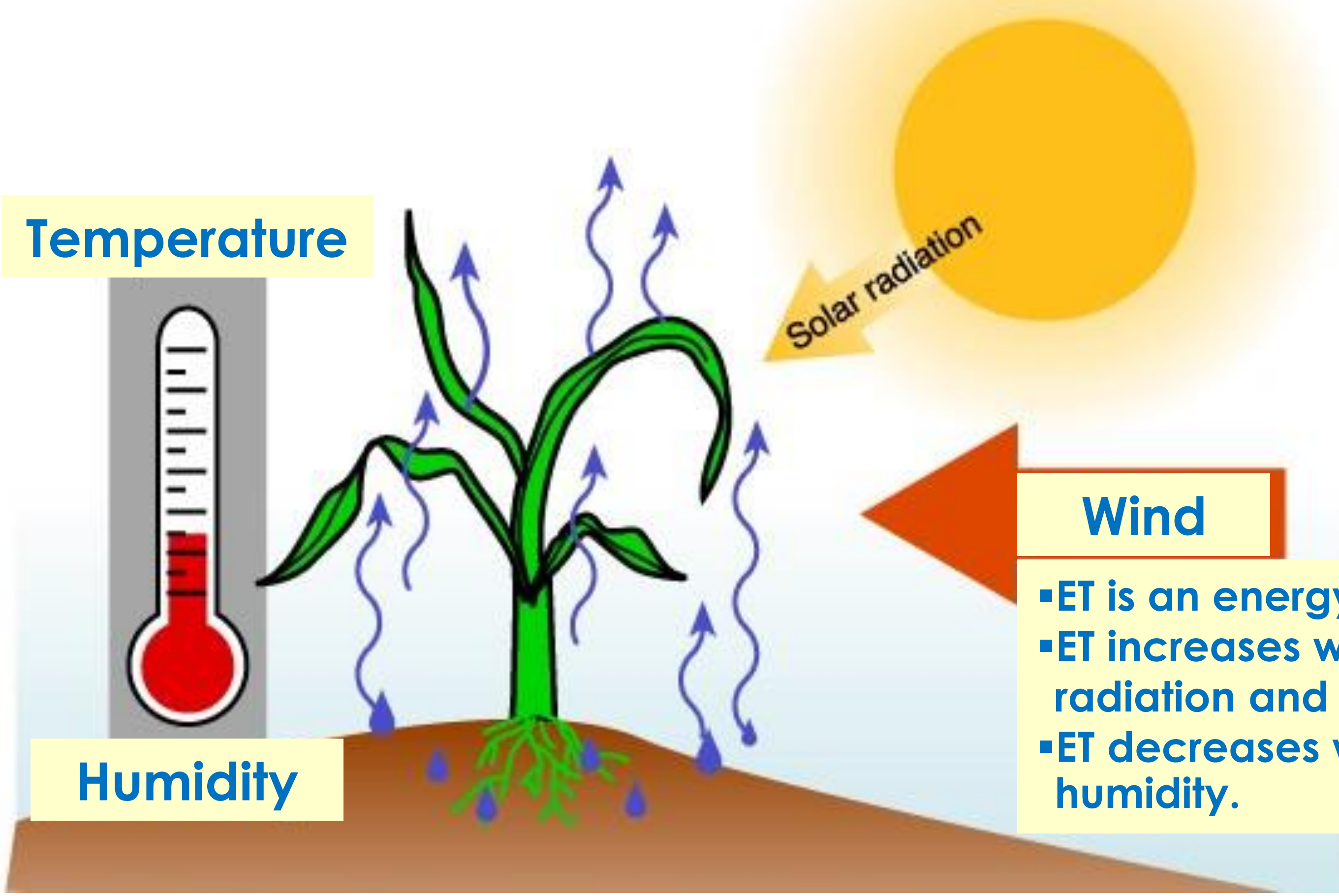


Steps:
1. Measure climatic conditions
2. Measure water use
3. Calibrate formula to calculate ETo

Penman-Monteith Formula

EVAPOTRANSPIRATION: DEVELOPING CROP COEFFICIENT, KC

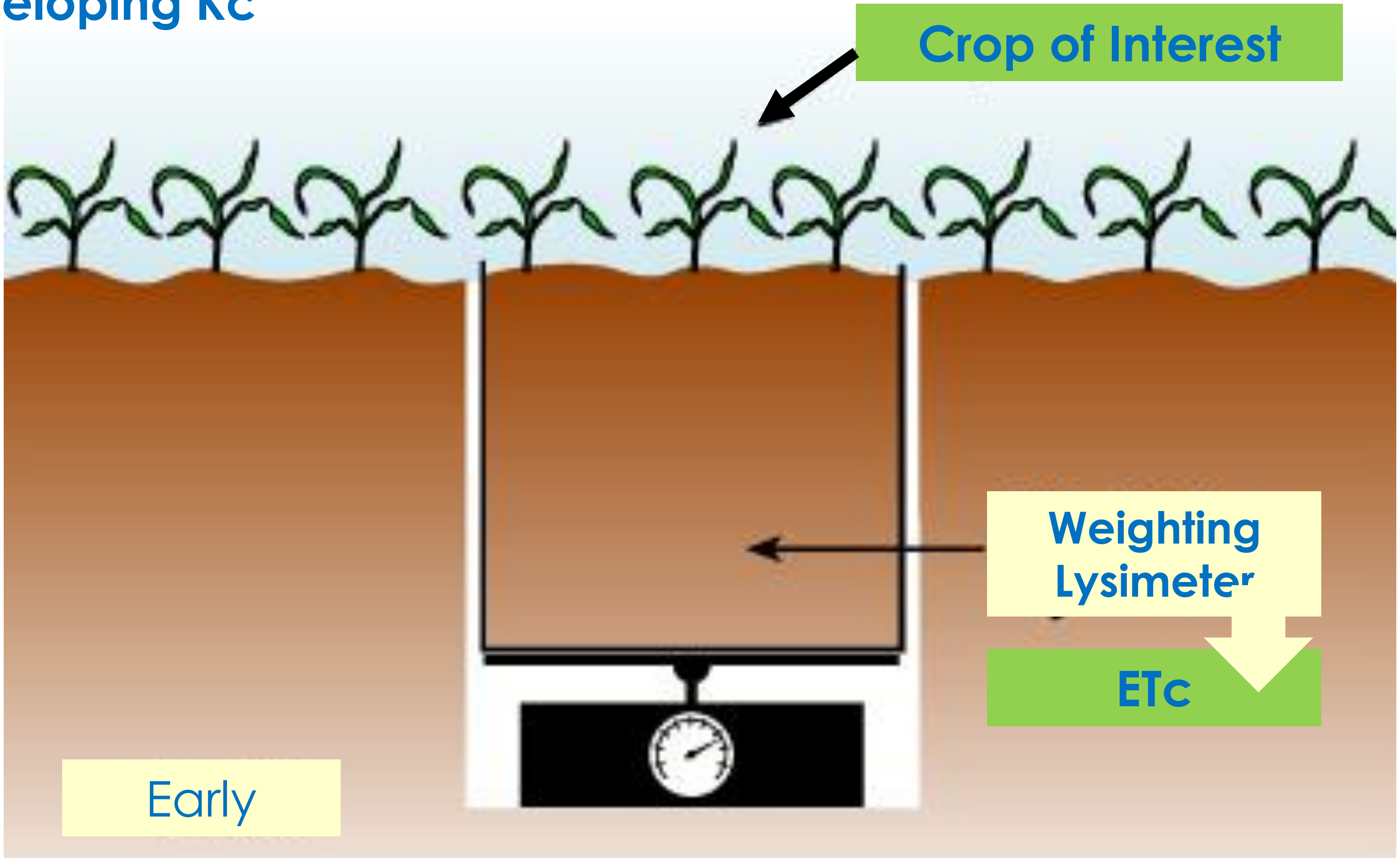
Measure climatic conditions and calculate ETo



- ET is an energy driven process.
- ET increases with temperature, solar radiation and wind.
- ET decreases with increasing humidity.

EVAPOTRANSPIRATION: DEVELOPING CROP COEFFICIENT, KC

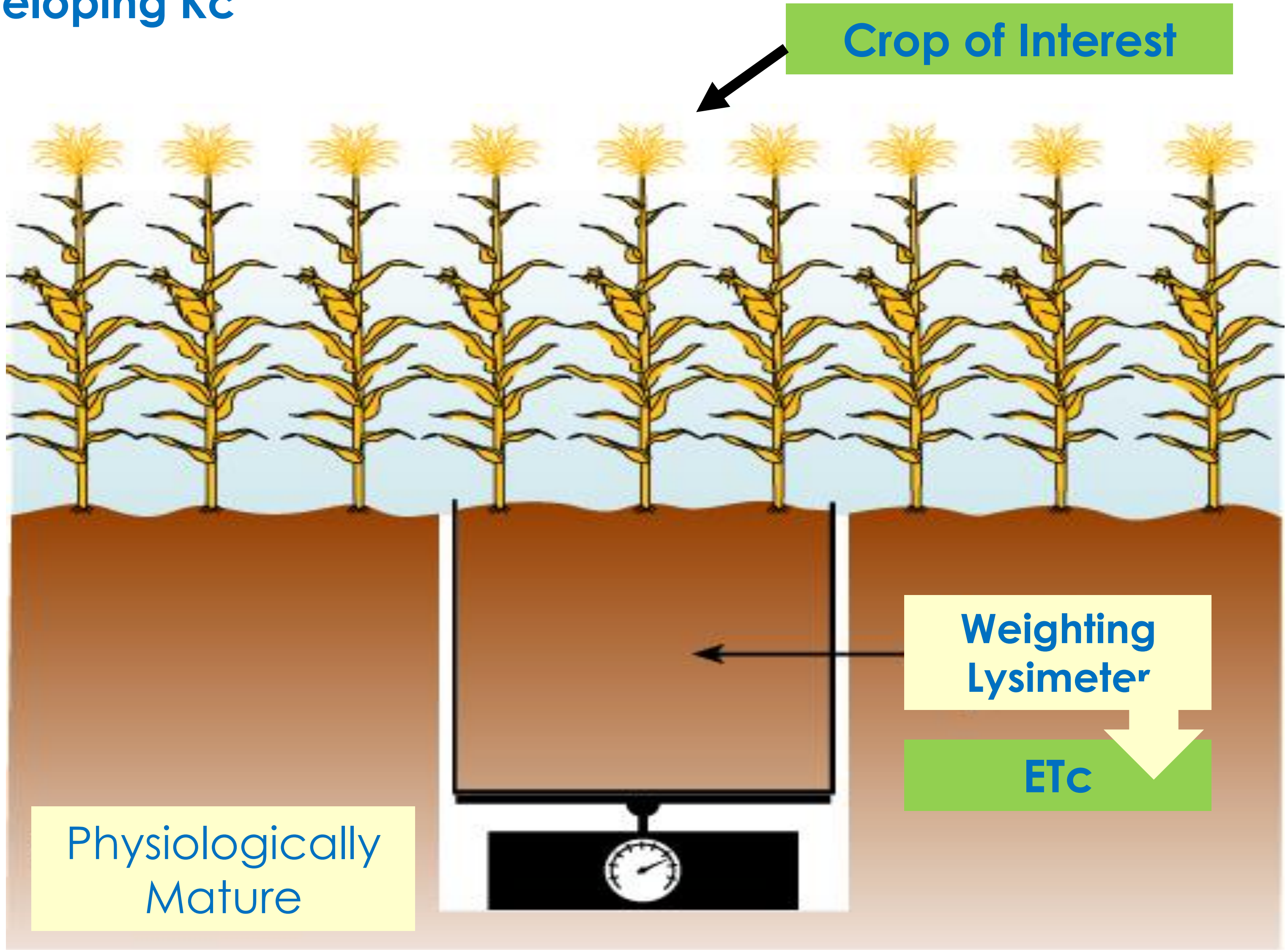
Developing Kc



Measure ETc for the crop stage of growth.

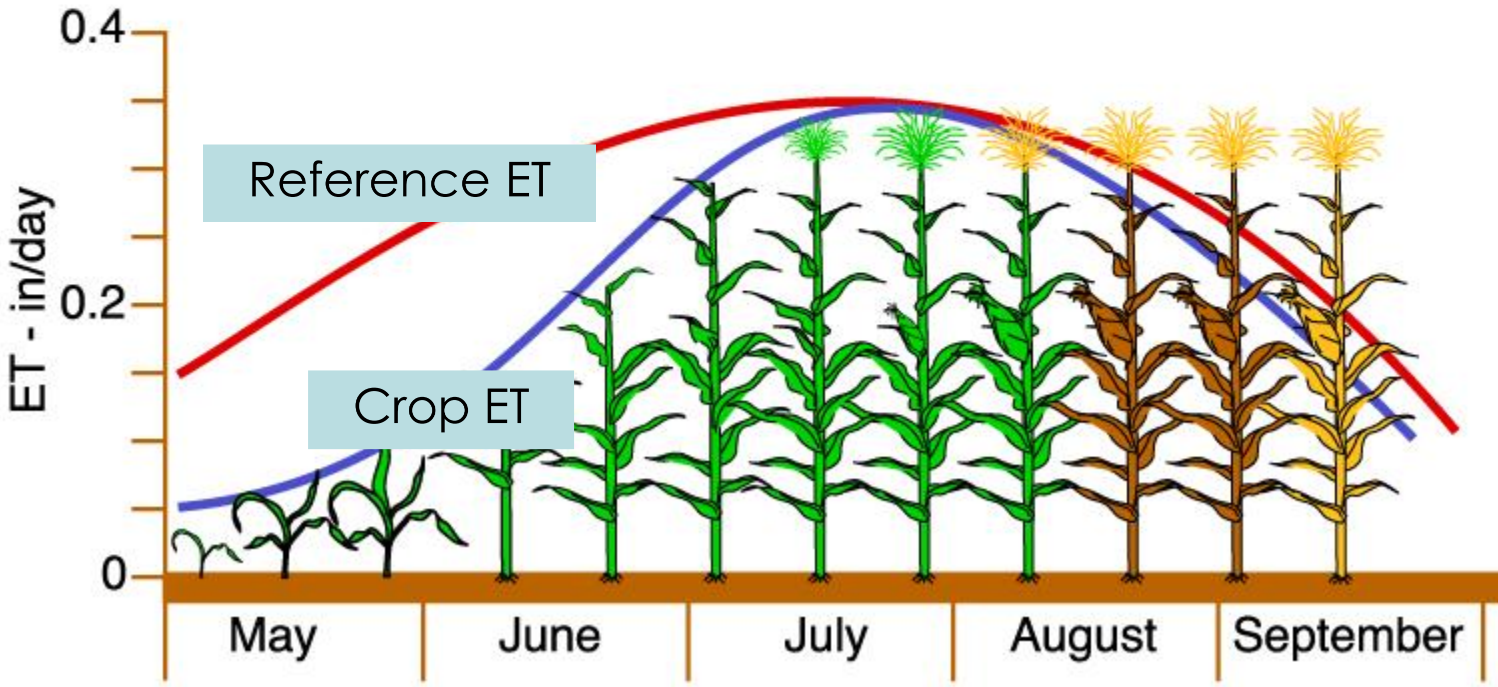
EVAPOTRANSPIRATION: DEVELOPING CROP COEFFICIENT, KC

Developing Kc



EVAPOTRANSPIRATION: DEVELOPING CROP COEFFICIENT, KC

Crop ET vs Reference ET



Kc=ETc/ETo

ETO CALCULATION:
EXAMPLE 10

Calculation of ETo by Pan Evaporation Method in Excel Spreadsheet

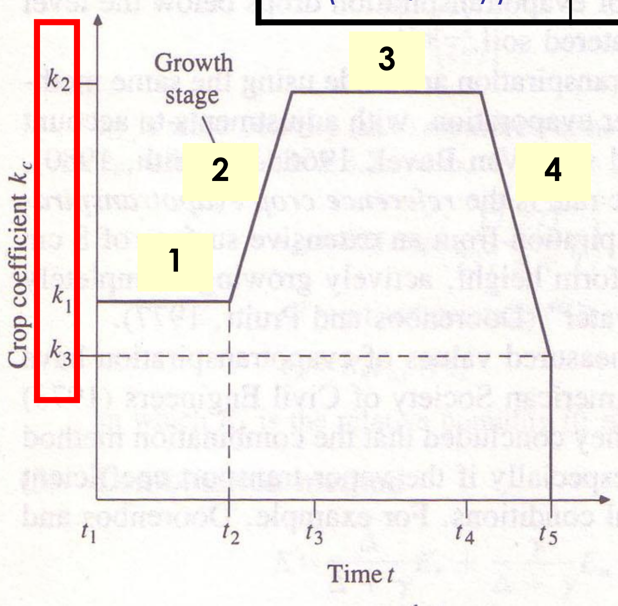
ETO CALCULATION: EXAMPLE 11

Calculation of ETo by Meteorological Method in Excel Spreadsheet

EVAPOTRANSPIRATION CALCULATION: EXAMPLE 12

The monthly values of reference crop evapotranspiration Etr, calculated using the combination method for average condition in Silistra, Bulgaria.

	t1		t2	t3		t4	t5	
Month	Apr	May	Jun	Jul	Aug	Sep	Oct	Apr-Oct total
ETo (mm/day)	4.14	5.45	5.82	6.60	5.94	4.05	2.34	34.3 mm
Kc	**0.38**	0.38	0.69	**1.00**	1.00	0.78	**0.55**	
ET (mm/day)	1.57	2.07	4.02	6.60	5.94	3.16	1.29	24.7 mm



Crop coefficient for corn, **k1=0.38, k2=1.00, k3=0.55**
t1=Apr, t2=Jun, t3=Jul,t4=Sep, t5=Oct
Calculate the actual evapotranspiration from this crop assuming a well-watered soil.

1-Initial stage (less than 10% ground cover).
2-Development state (from initial stage to attainment of effective full ground cover (70-80%)).
3-Mid-season stage (from full ground cover to maturation).
4-Late season state (full maturity and harvest).

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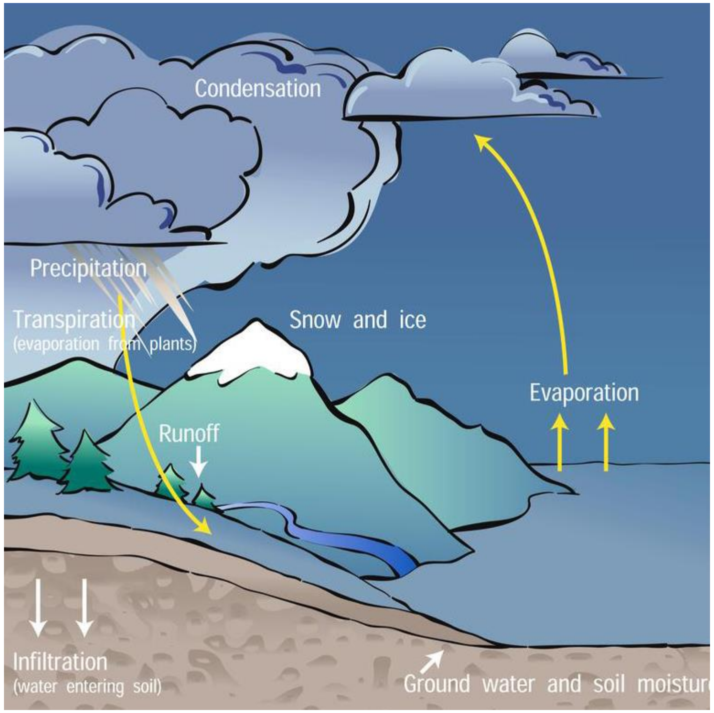

LECTURE NOTES
EGCE 323 HYDROLOGY

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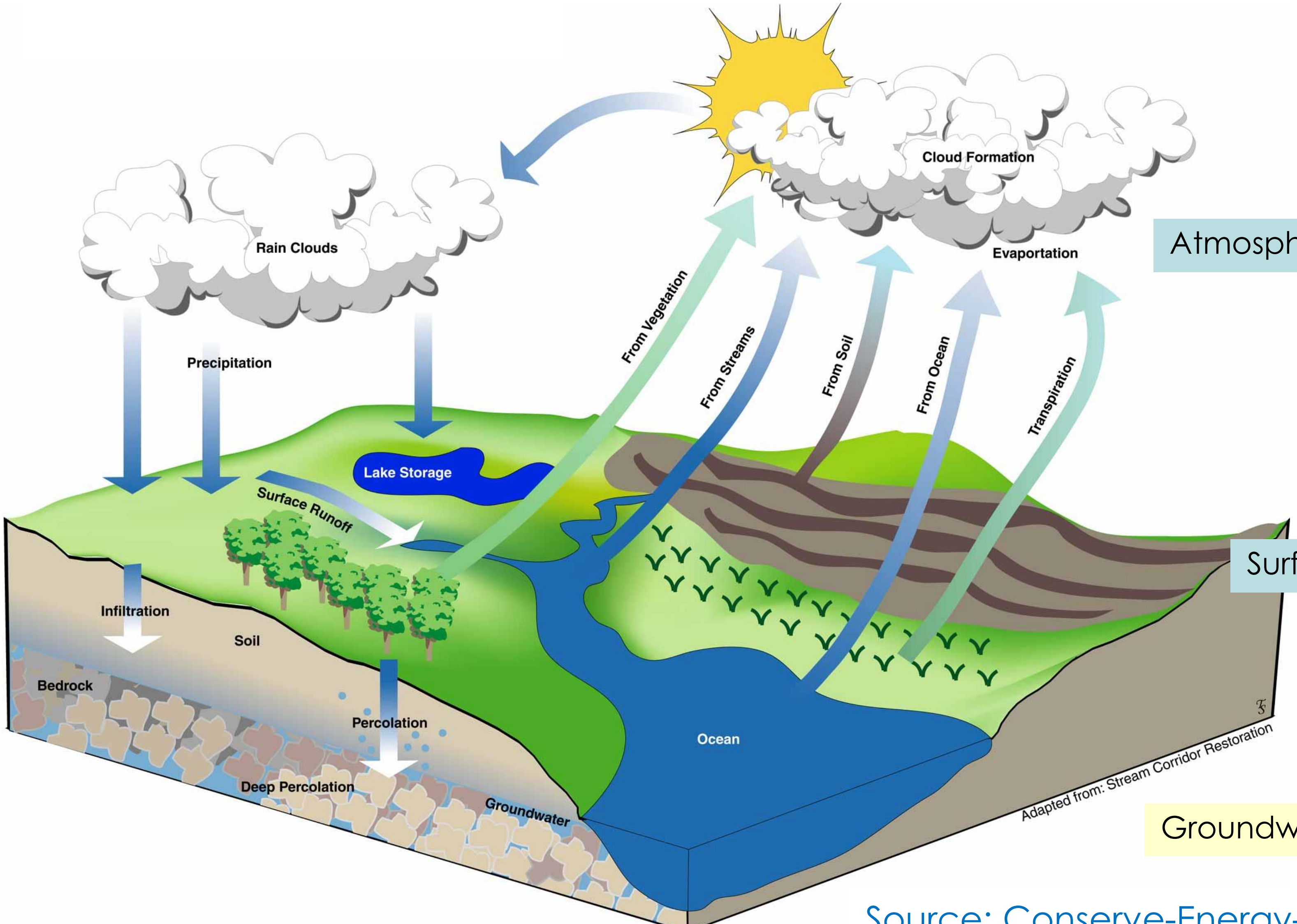
Revised in 2018

Groundwater

- Groundwater
- Groundwater Flow Processes

HYDROLOGIC CYCLE



Atmospheric Subsystem

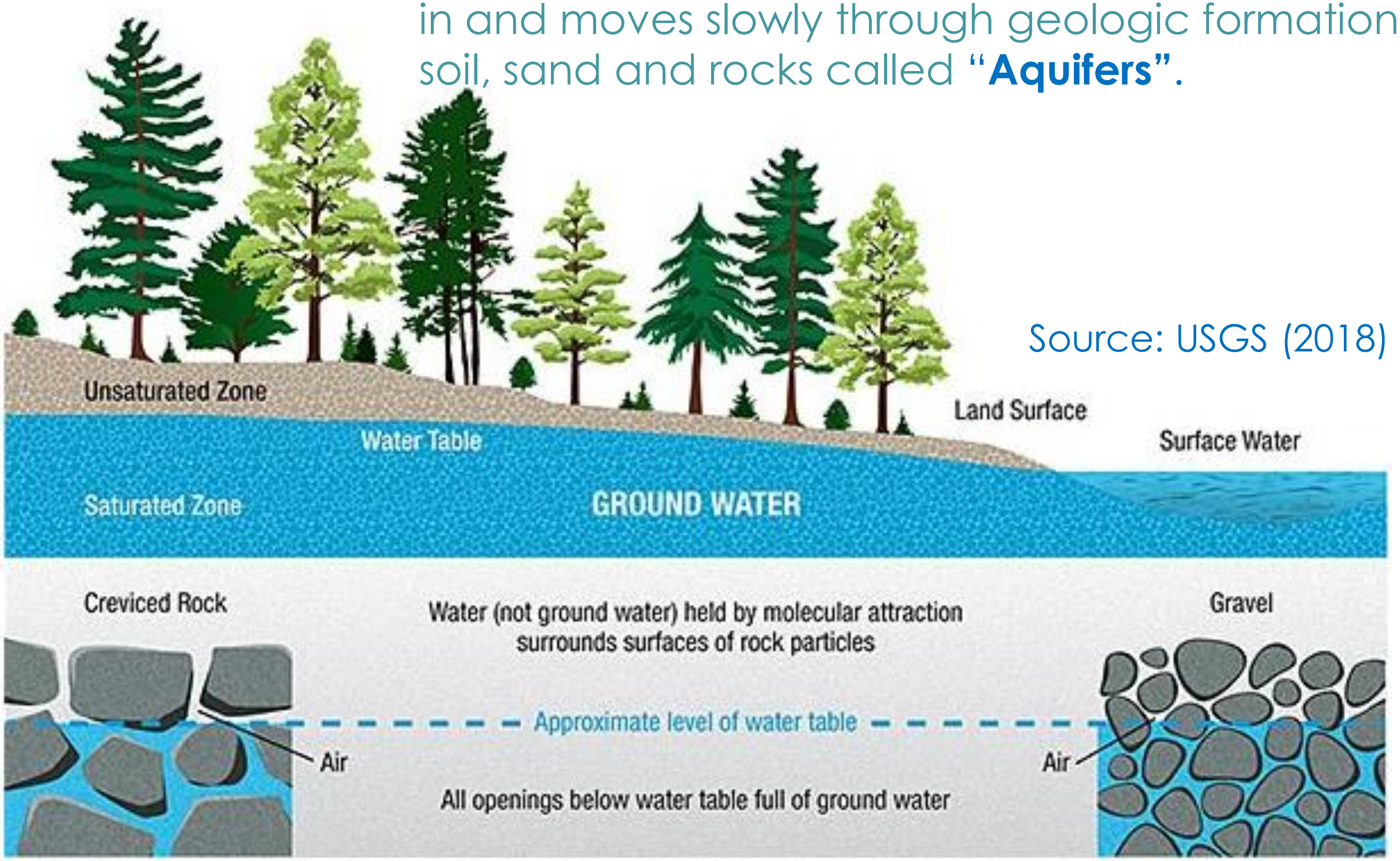
Surface Subsystem

Groundwater Subsystem

Source: Conserve-Energy-Future (2018)

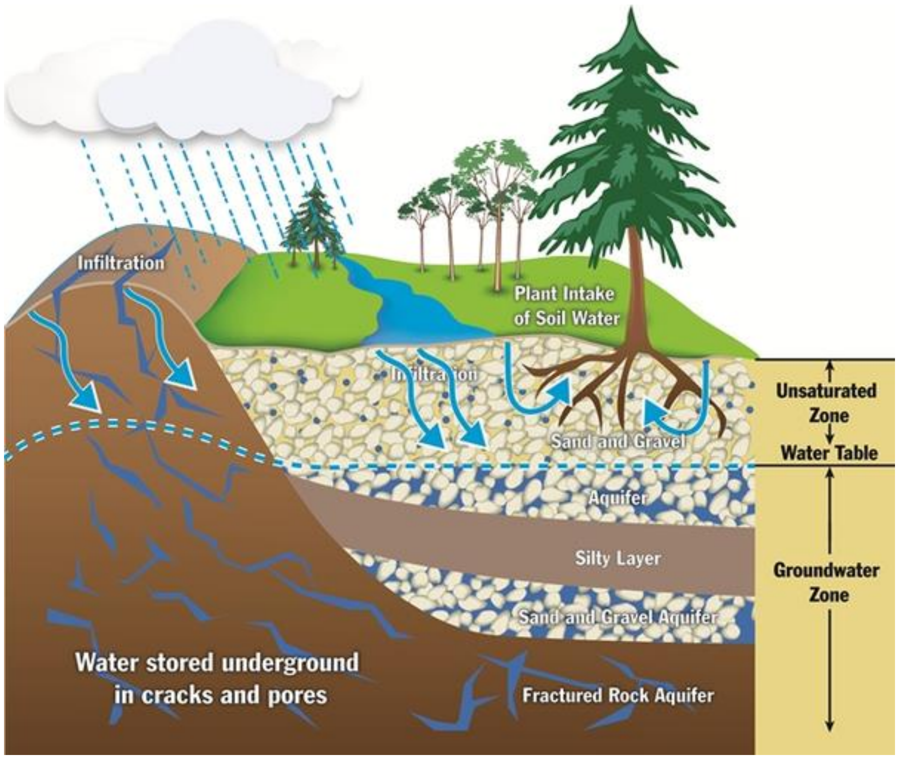
GROUNDWATER

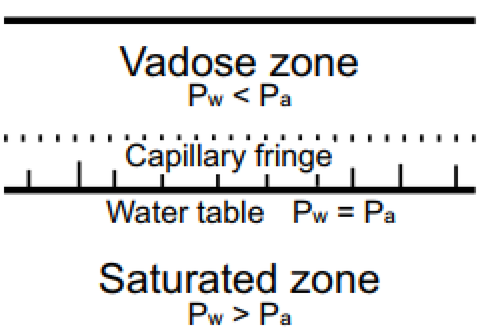
Groundwater is the water found underground in the cracks and spaces in soil, sand and rock. It is stored in and moves slowly through geologic formations of soil, sand and rocks called "**Aquifers**".



Source: USGS (2018)

GROUNDWATER ZONE





Groundwater and soil water together make up approximately 0.5% of all water in the hydrosphere.

Water beneath the surface can essentially be divided into three zones:

- **Soil Water Zone (Vadose Zone)**
- **Intermediate Zone (Capillary Fringe)**
- **Groundwater Zone (Saturated Zone)**

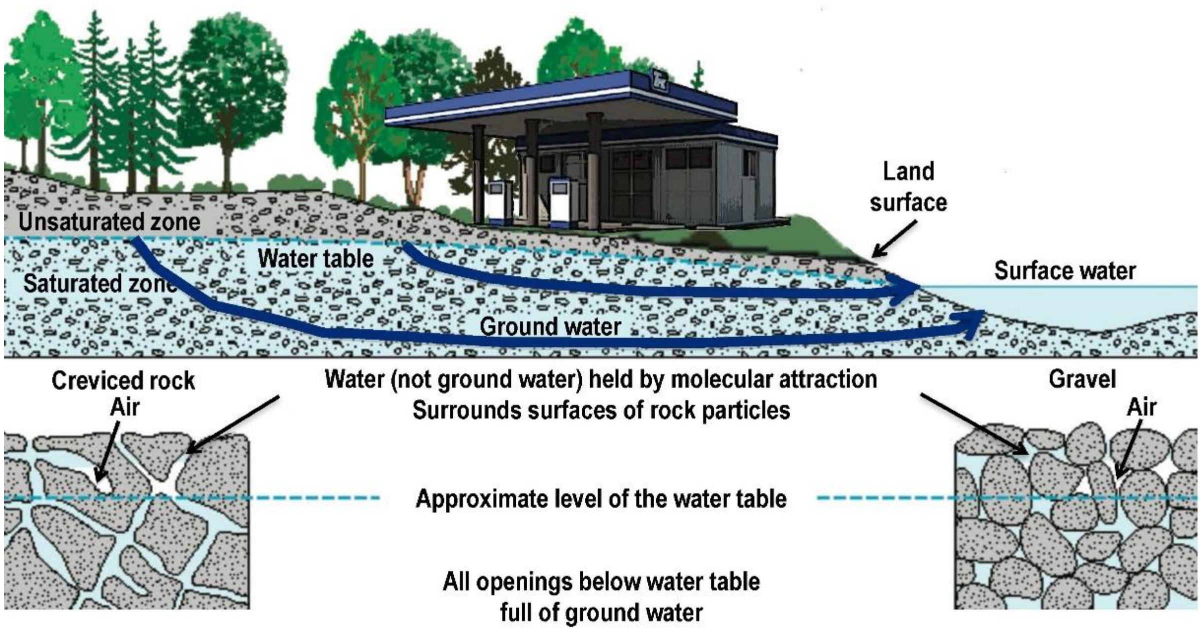
The top two zones, the vadose zone and capillary fringe, can be grouped into the zone of aeration (Unsaturated Zone), where during the year air occupies the pore spaces between earth materials.

GROUNDWATER FLOW

Groundwater Flow

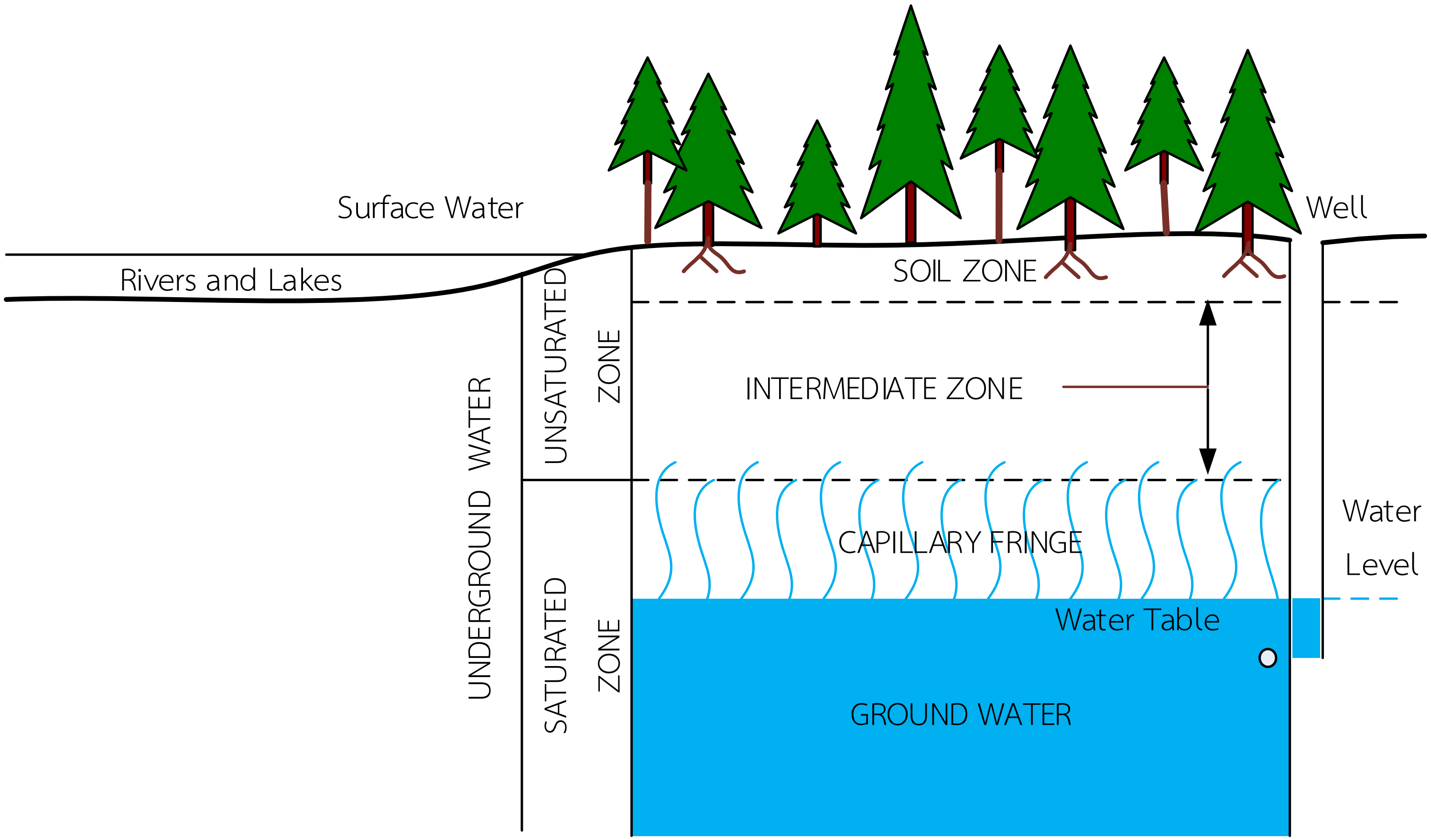
Three important processes are;

- **Infiltration** of surface water into the soil to become soil moisture.
- **Subsurface flow** (Unsaturated flow through the soil).
- **Groundwater flow** (Saturated flow through the soil/rock strata).



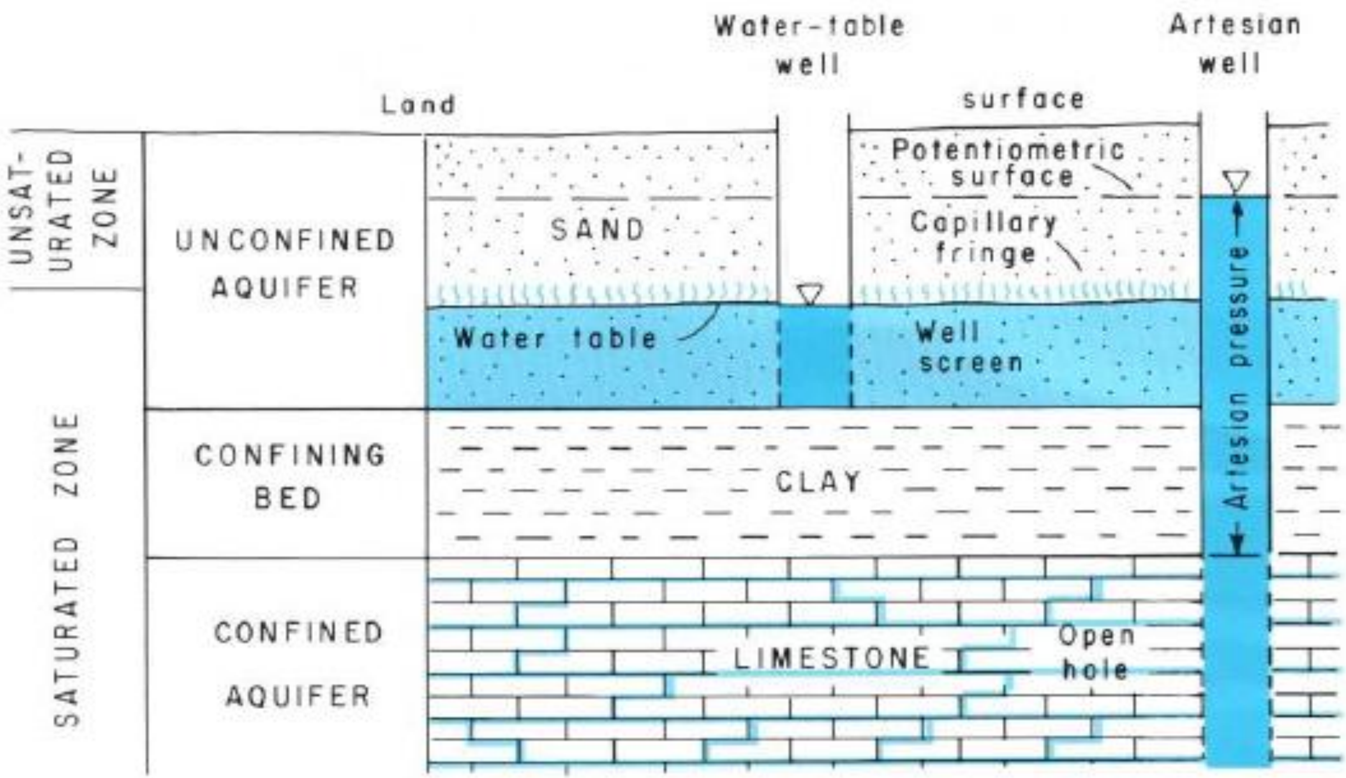
Subsurface Water Zone and Processes

GROUNDWATER FLOW ZONE



Subsurface Water Zone

GROUNDWATER FLOW ZONE

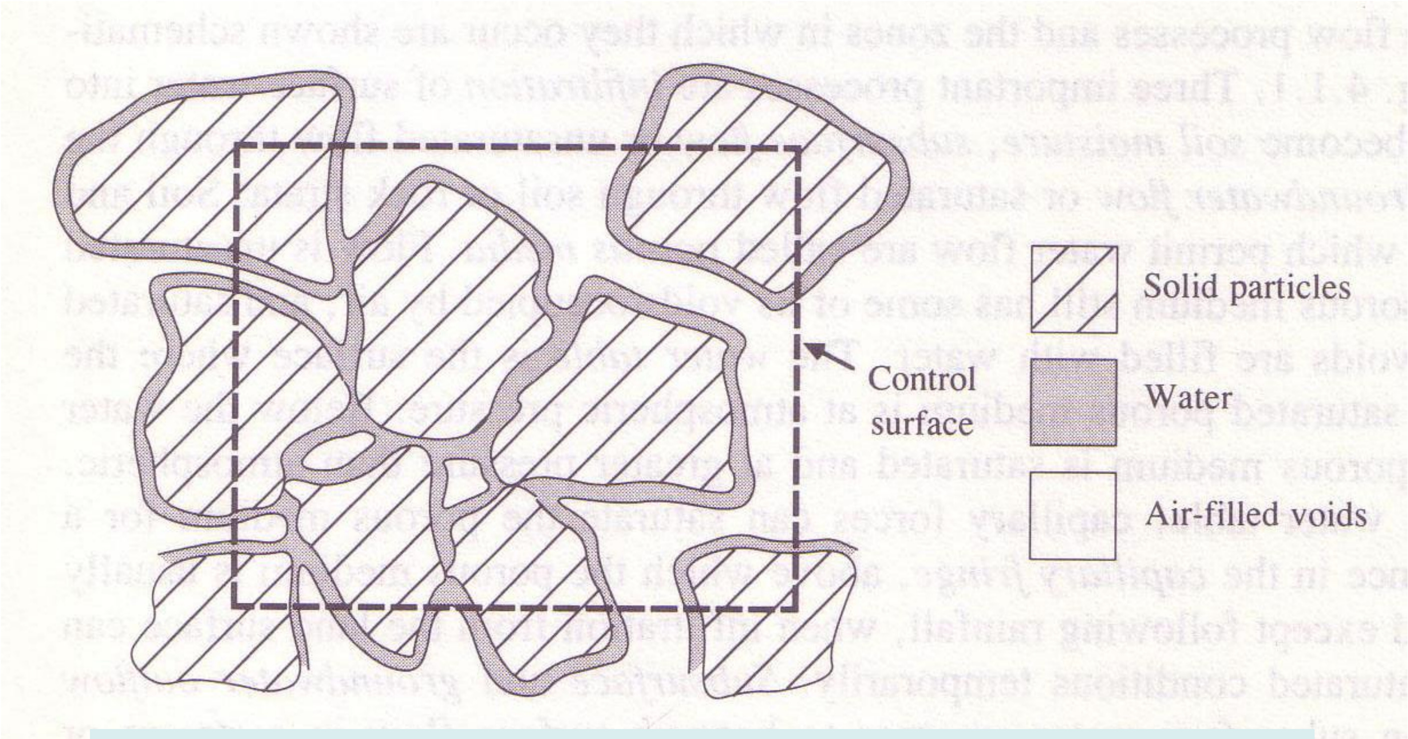


Types of Aquifer

UNSATURATED FLOW

- Soil and rock strata which permit water flow are called "**Porous Media**".
- **Flow is unsaturated** when the porous medium still has some of its voids occupied by air, and **saturated** when the voids are filled with water.
- The water table is the surface where the water in a saturated porous medium is at atmospheric pressure.
- Below the water table, the porous medium is saturated and at greater pressure than atmosphere.
- Above the water table, the porous medium is usually unsaturated except following rainfall, when infiltration from the land surface can produce saturated conditions temporarily.
- Subsurface and groundwater outflow occur when subsurface water emerges to become surface flow in a stream.
- Soil moisture is extracted by ET as the soil dries out.

UNSATURATED FLOW



Cross section through unsaturated porous medium

A portion of cross section is occupied by **soil particles** and **voids** (air & water)

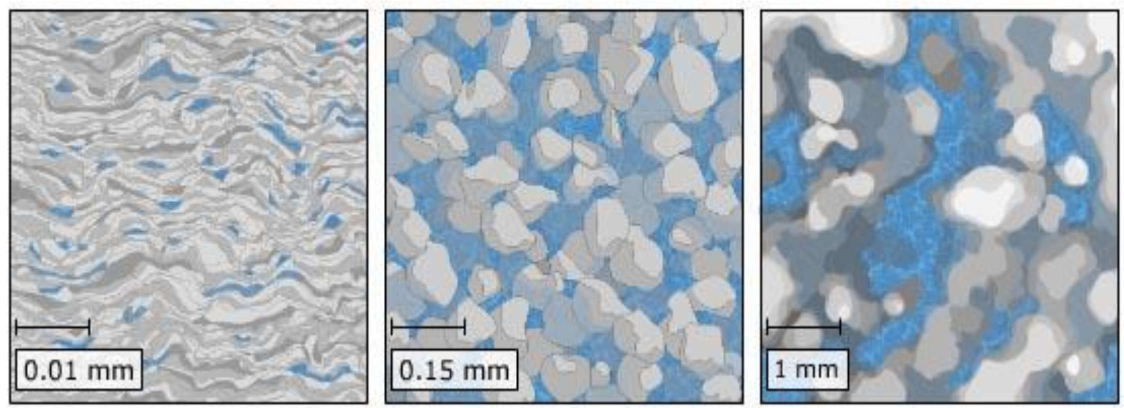
UNSATURATED FLOW

Porosity (η)

$$\eta = \frac{\text{Volume of Voids}}{\text{Total Volume}}$$

The range of η is approximately $0.25<\eta<0.75$ for soils, the value depending on the soil texture.

Porosity is a measure of how much of a rock/soil is open space. This space can be between grains or within cracks or cavities of the rock.



UNSATURATED FLOW

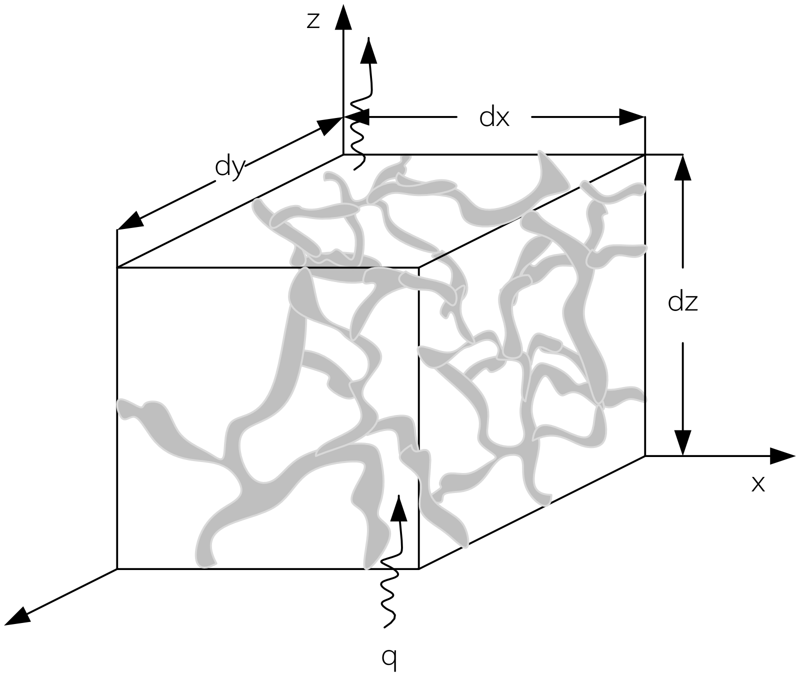
Soil Moisture Content (θ)

A part of the voids is occupied by water and the remainder by air, the volume occupied by water being measured by the **Soil Moisture Content (θ)**

$$\theta = \frac{\text{Volume of Water}}{\text{Total Volume}}$$

$0.25 < \theta < \eta$; the soil moisture content is equal to the porosity when the soil is saturated.

Darcy Flux



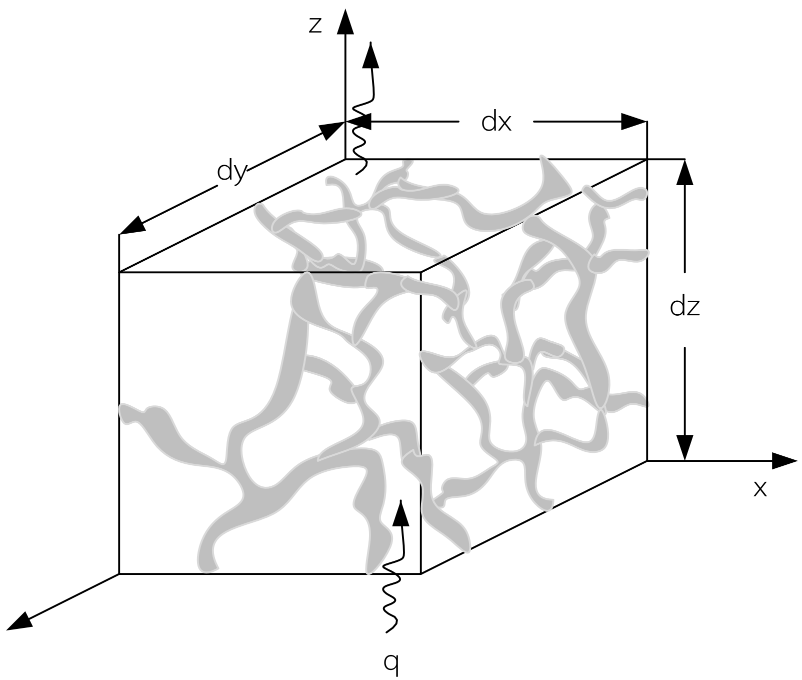
Control volume containing unsaturated soil

Its sides have length **dx**, **dy**, **dz** in the coordinate directions.
Volume = **dx.dy.dz**
The volume of water contained in the control volume = **θ.dx.dy.dz**
The flow of water through the soil is measured by the "**Darcy Flux**" (q)

$$q = \frac{Q}{A}$$

Source: Chow et al. (1988)

Darcy Flux



Control volume containing unsaturated soil

Darcy's Law was developed to relate the Darcy flux, **q** to the rate of head loss per unit length of medium, Sf

$$q = KS_f$$

Consider flow in the vertical direction and denote the total head of the flow by **h**

$$Sf = -\frac{\partial h}{\partial z}$$

$$q = -K\frac{\partial h}{\partial z}$$

The negative sign indicates that the total head is decreasing in the direction of flow because of friction,

Source: Chow et al. (1988)

INFILTRATION

Infiltration

Infiltration is a process of water penetrating from the ground surface into the soil.

The factor influences the infiltration rate;

- condition of soil surface and its vegetative cover
- properties of the soil: porosity and hydraulic conductivity
- current moisture content of the soil

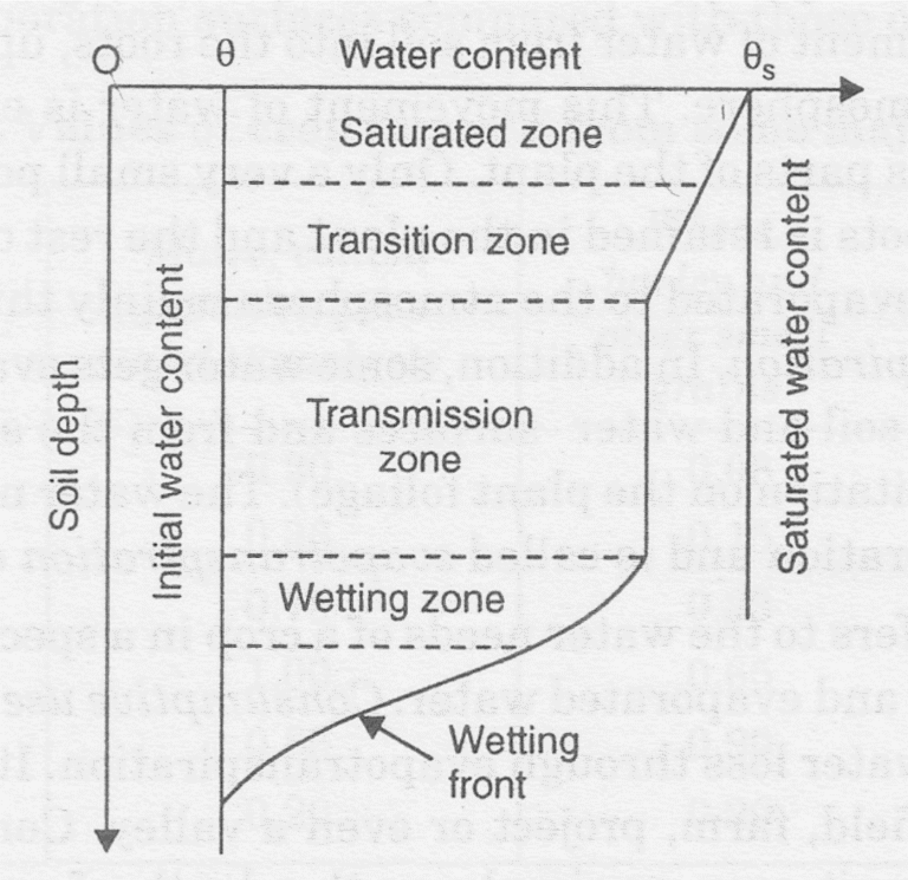
Hydraulic Conductivity/Permeability

Hydraulic conductivity is a measure of the ease with which a fluid (water in this case) can move through a porous media.

INFILTRATION

Moisture Zones during Infiltration
The distribution of soil moisture within the soil profile during the downward movement of water is illustrated in the figure.

There are 5 moisture zones
- Saturated Zone
- Transition Zone
- Transmission Zone
- Wetting Zone
- Wetting Front



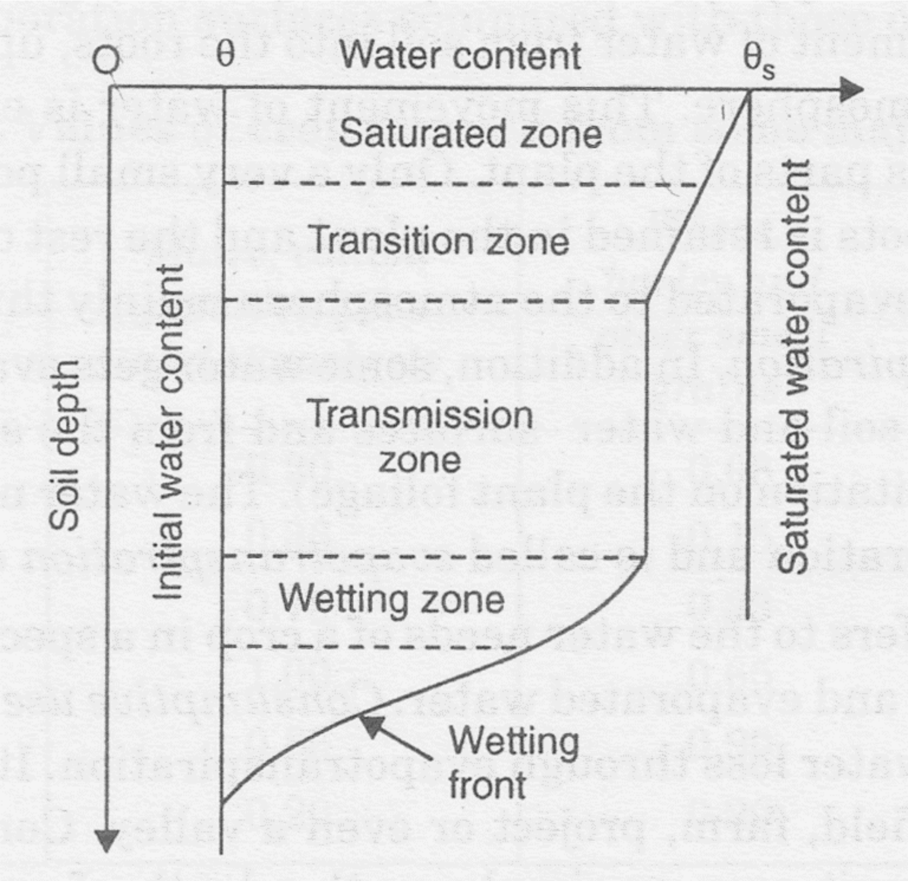
Moisture zones during infiltration

INFILTRATION

Saturated Zone
It is near the surface, extending up to about 1.5 cm below the surface and having a saturated water content.

Transition Zone
It is about 5 cm thick and is located below the saturated zone. In this zone, a rapid decrease in water content occurs.

Transmission Zone
The water content varies slowly with depth as well as time.



Moisture zones during infiltration

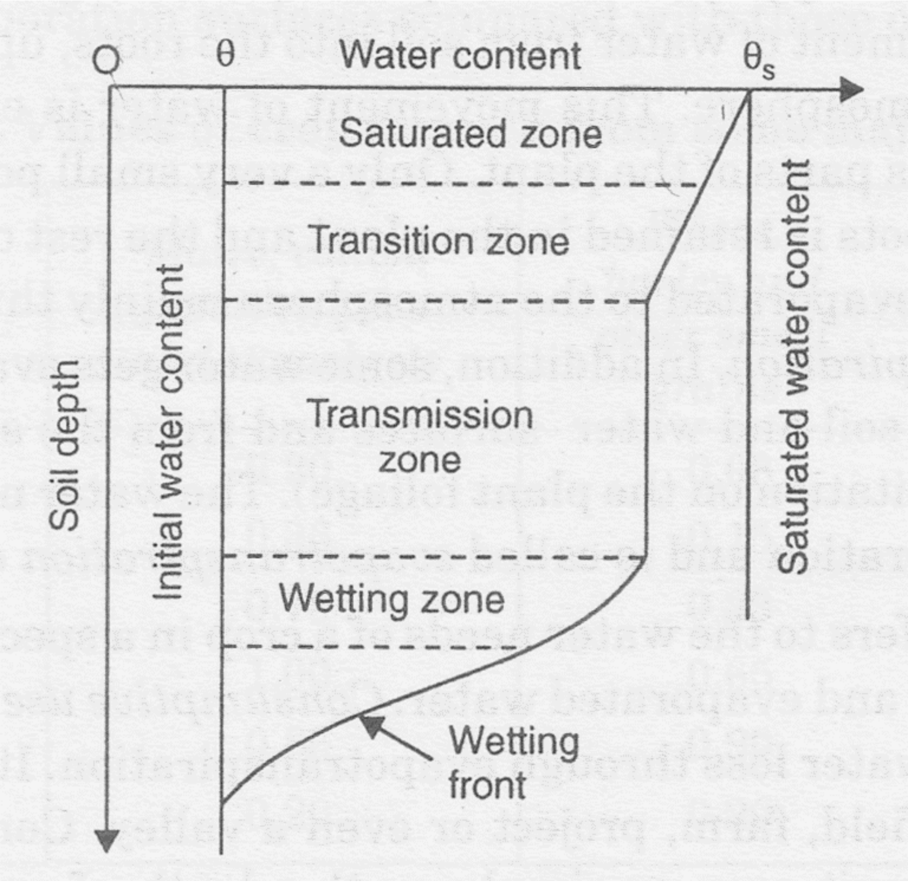
INFILTRATION

Wetting Zone

The sharp decrease in water content is observed.

Wetting Front

a region of very steep moisture gradient. This represents the limit of moisture penetration into the soil.



Moisture zones during infiltration

INFILTRATION MEASUREMENT

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Infiltration Rate, f expressed in inches per hour or centimeters per hour, is the rate at which water enters the soil at the surface.

If water is ponded on the surface, the infiltration occurs at the **Potential Infiltration Rate**.

Cumulative infiltration, F is the accumulated depth of water infiltrated during a given time period and is equal to the integral of the infiltration rate over that period.

$$F(t) = \int_0^t f(\tau)d\tau$$

τ is variable of time in the integration

$$f(t) = \frac{dF(t)}{dt}$$

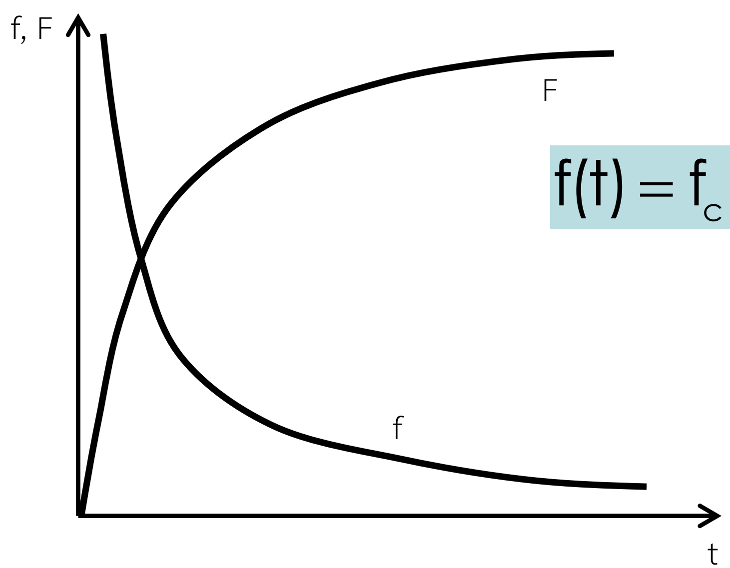
The infiltration rate is the time derivative of the cumulative infiltration.

INFILTRATION

Horton's Equation

One of the earliest infiltration equations was developed by Horton (1933, 1939) who observed that;

"Infiltration begins at some rate, fo and exponentially decreases until it reaches a constant rate, fc"



Infiltration by Horton's Equation

$$f(t) = f_c + (f_0 - f_c)e^{-kt}$$

K = a decay constant having dimensions $[T^{-1}]$

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Phillip's Equation

Philip (1957, 1969) solved the equation to yield an infinite series for cumulative infiltration, F(t), which is approximated by

$$F(t) = St^{1/2} + Kt$$

S = a parameter called sorptivity (which is a function of the soil suction potential)
K = hydraulic conductivity

By differentiation

$$f(t) = \frac{1}{2} St^{-1/2} + K$$

As $t \rightarrow \infty$, f(t) tends to K

For a horizontal column of soil, soil suction is the only force drawing water into the column

$$F(t) = St^{1/2}$$

INFILTRATION

Soil Texture	Porosity (%)	Basic Infiltration Rate (cm/hr)
Sand	32-42	2.5-25
Sandy Loam	40-47	1.3-7.6
Loam	43-49	0.8-2.0
Clay Loam	47-51	0.25-1.5
Silty Clay	49-53	0.03-0.5
Clay	51-55	0.01-0.1

INFILTRATION: EXAMPLE 1

A small tube with a cross-sectional area of 40 cm2 is filled with soil and laid horizontally. The open end of the tube is saturated, and after 15 minutes, 100 cm3 of water have infiltrated into the tube. If the saturated hydraulic conductivity of the soil is 0.4 cm/hr, determine how much infiltration would have taken place in 30 minutes if the soil column had initially been placed upright with its upper surface saturated.

The cumulative infiltration depth in the horizontal column is F = 100 $cm^3/40\ cm^2$ = 2.5 cm.

For horizontal infiltration, cumulative infiltration is a function of soil suction alone so that t = 15 min = 0.25 hr

$$F(t) = St^{1/2}$$

F(t) = 2.5 cm, t = 0.25 hr , therefore S = 5 $cm.hr^{-1/2}$

INFILTRATION: EXAMPLE 1

For infiltration down a vertical column, applies with K=0.4 cm/hr. Hence, with t = 30 min = 0.5 hr.

$$f(t) = \frac{1}{2} S t^{-1/2} + K$$

$S = 5$ cm.hr$^{-1/2}$, $t = 0.5$ hr, $K = 0.40$
$f(t) = 3.93$ cm

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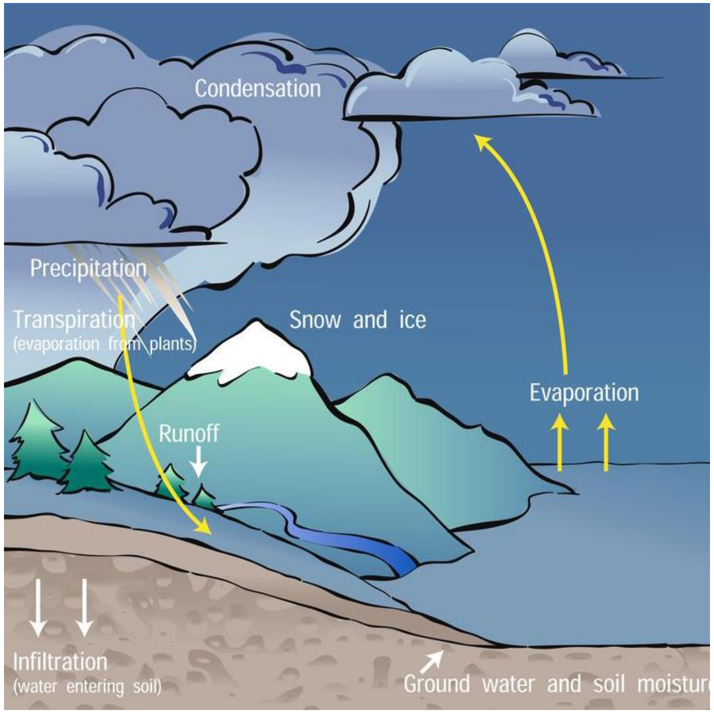

LECTURE NOTES
EGCE 323 HYDROLOGY

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LECTURE OUTLINE

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Surface Water

- Source of Streamflow
- Streamflow Characteristics
- Travel Time and Stream Networks

HYDROLOGIC CYCLE

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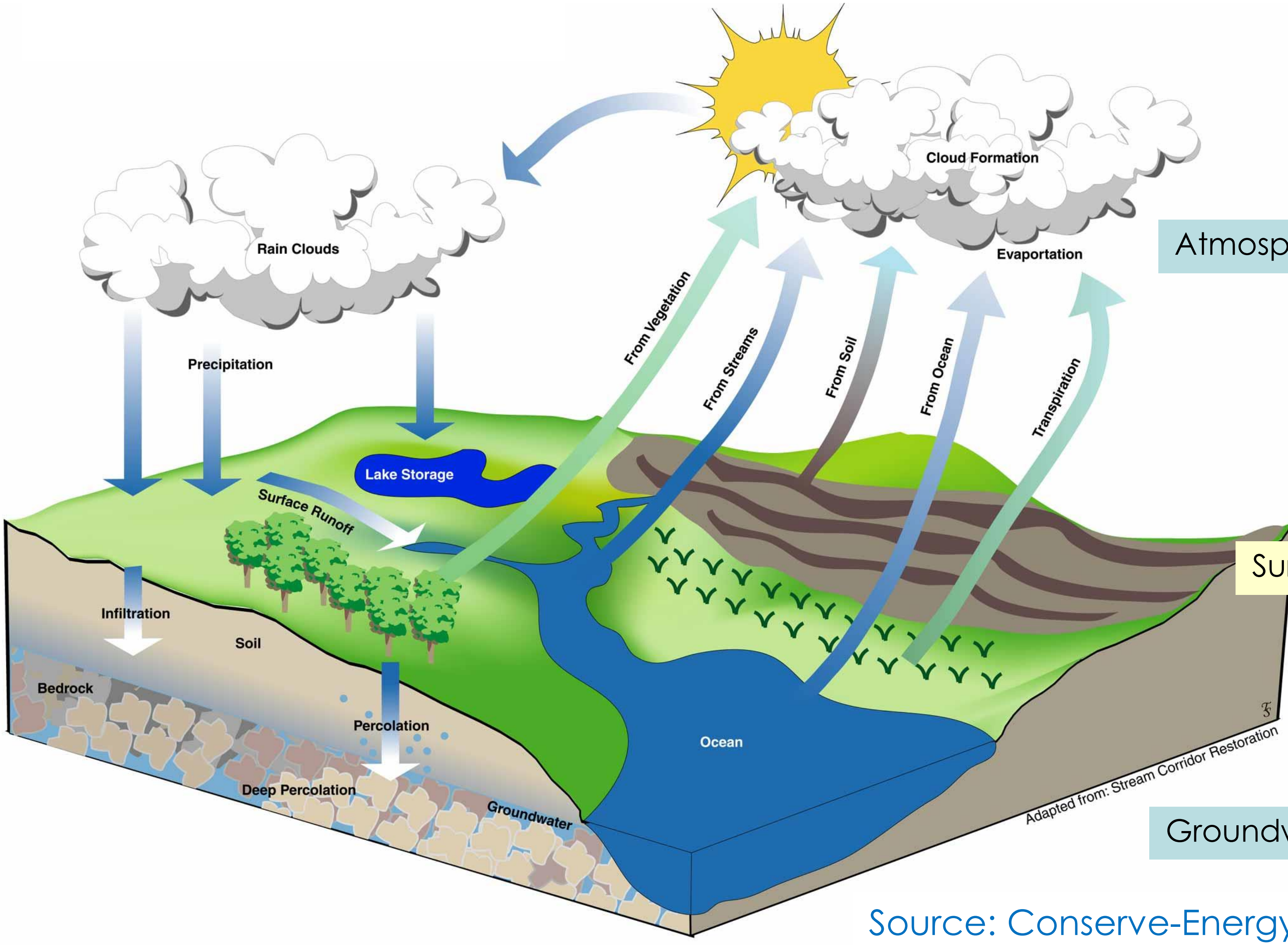


Atmospheric Subsystem

Surface Subsystem

Groundwater Subsystem

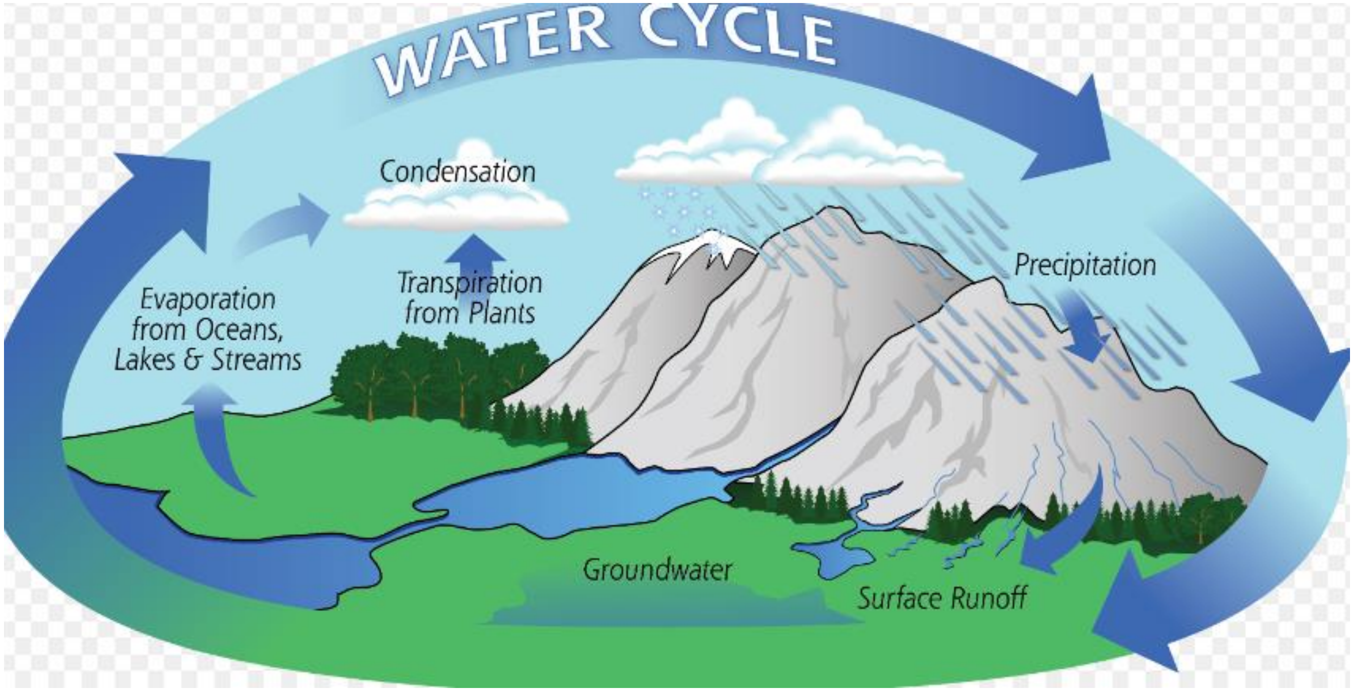
Source: Conserve-Energy-Future (2018)

SURFACE WATER

Surface Water

Surface water is water stored/flowing on the earth surface.

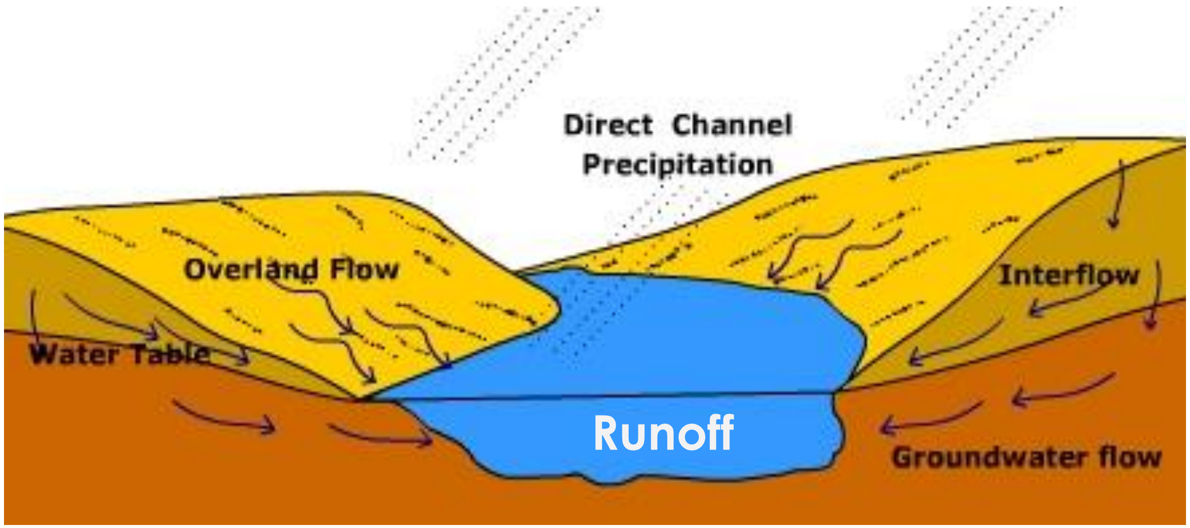
Surface Water
The surface water system continually interacts with the atmospheric and subsurface systems.



Source: NASA (2018)

SURFACE WATER: RUNOFF

Runoff



In hydrology, runoff is quantity of water discharged in surface streams.

- Runoff includes not only the waters that travel over the land surface and through channels to reach a stream but also interflow, the water that infiltrates the soil surface and travels by means of gravity toward a stream channel (always above the main groundwater level) and eventually empties into the channel.
- Runoff also includes groundwater that is discharged into a stream; streamflow that is composed entirely of groundwater is termed base flow, or fair-weather runoff, and it occurs where a stream channel intersects the water table.

Source: Earthonlinemedia (2018)

SURFACE WATER: STREAMFLOW

Streamflow/Channel Runoff

- Streamflow, or channel runoff, is the flow of water in streams, rivers, and other channels, and is a major element of the water cycle. It is one component of the runoff of water from the land to waterbodies, the other component being surface runoff.
- Channel flow is the main form of surface water flow.
- All the other surface flow processes contribute to it.
- Determining flow rates in stream channels is a central task of surface water hydrology.
- The precipitation which becomes streamflow may reach the stream by overland flow, subsurface flow, or both.

SURFACE WATER: SURFACE RUNOFF

Surface Runoff/Overland Flow

- Surface runoff (also known as overland flow) is the flow of water that occurs when excess stormwater, meltwater, or other sources flows over the Earth's surface.
- This might occur because soil is saturated to full capacity, because rain arrives more quickly than soil can absorb it, or because impervious areas (roofs and pavement) send their runoff to surrounding soil that cannot absorb all of it.
- Surface runoff is a major component of the water cycle. It is the primary agent in soil erosion by water.

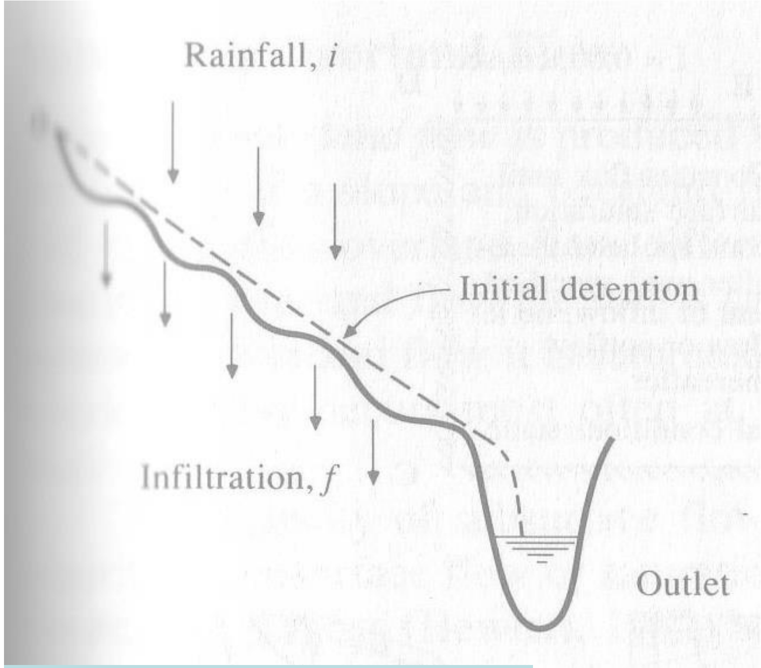
Runoff flowing into a stormwater drain

SURFACE WATER: HORTONIAN OVERLAND FLOW

Hortonian Overland Flow
Horton (1933) described overland flow as follows:

Neglecting interception by vegetation, **surface runoff is that part of rainfall which is not absorbed by the soil by infiltration**.

If the soil has an infiltration capacity, f, expressed in inches depth absorbed per hour, then when the rain intensity i is less than f, the rain is all absorbed and there is no surface runoff.



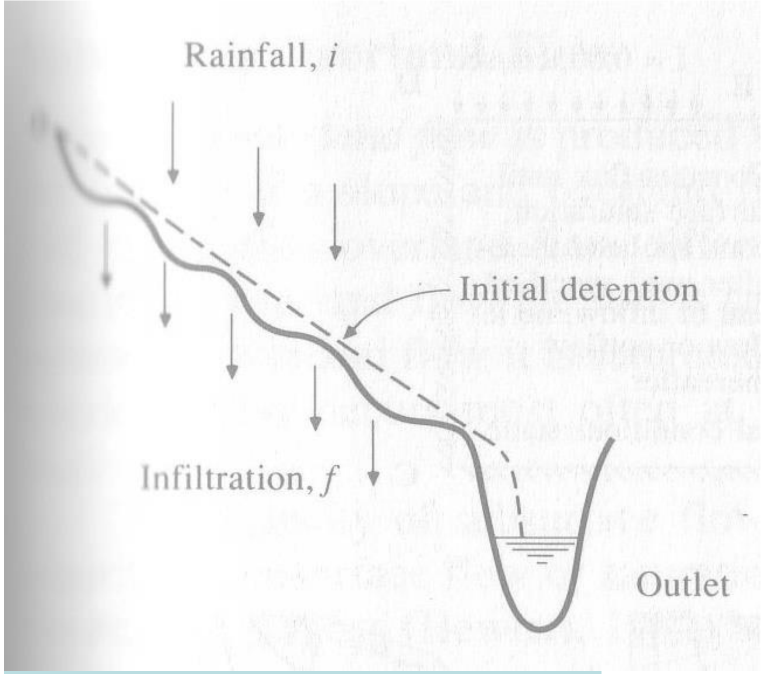
I < f No Surface runoff

SURFACE WATER: HORTONIAN OVERLAND FLOW

Hortonian Overland Flow

If i is greater than f, surface runoff will occur at the rate (i-f). Horton termed this difference (i-f) "**Rainfall Excess**".

Horton considered surface runoff to take the form of a sheet flow whose depth might be measured in fractions of an inch. As flow accumulates going down a slope, its depth increases until discharge into a stream channel occurs.



I > f Surface runoff = (i-f)
= Rainfall Excess

SURFACE WATER: HORTONIAN OVERLAND FLOW

Hortonian Overland Flow

- Hortonian overland flow is applicable for impervious surfaces in urban areas, and for natural surfaces with thin soil layers and low infiltration capacity as in semiarid and arid lands.

- Hortonian overland flow occurs rarely on vegetated surfaces in humid regions. Under these conditions, the infiltration capacity of the soil exceeds observed rainfall intensities for all except the most extreme rainfalls. Subsurface flow then becomes a primary mechanism for transporting stormwater to streams.

SURFACE WATER: STREAMFLOW

Streamflow Characteristics

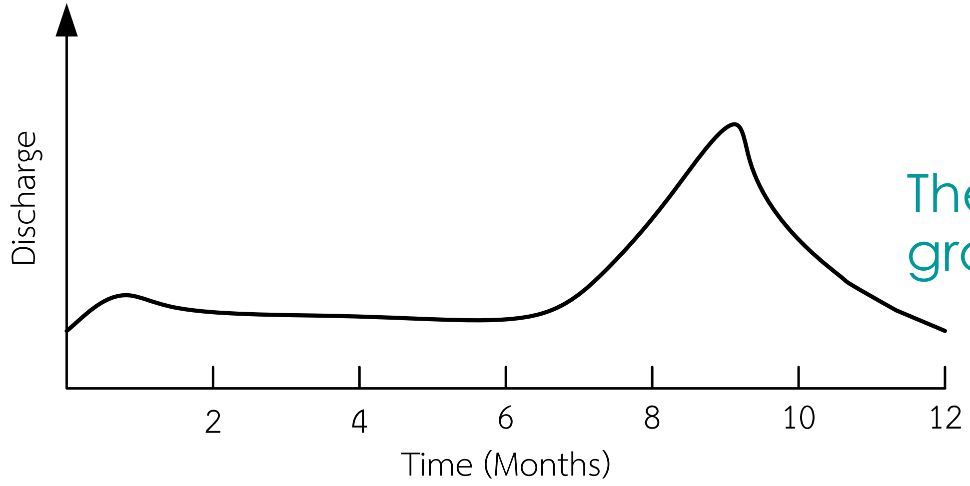
The flow characteristics of a stream depend upon;

- intensity and duration of rainfall
- shape, soil, vegetation, slope, and drainage network of the catchment basin
- climatic factors influencing evapotranspiration.

SURFACE WATER: STREAMFLOW

Perennial Streams

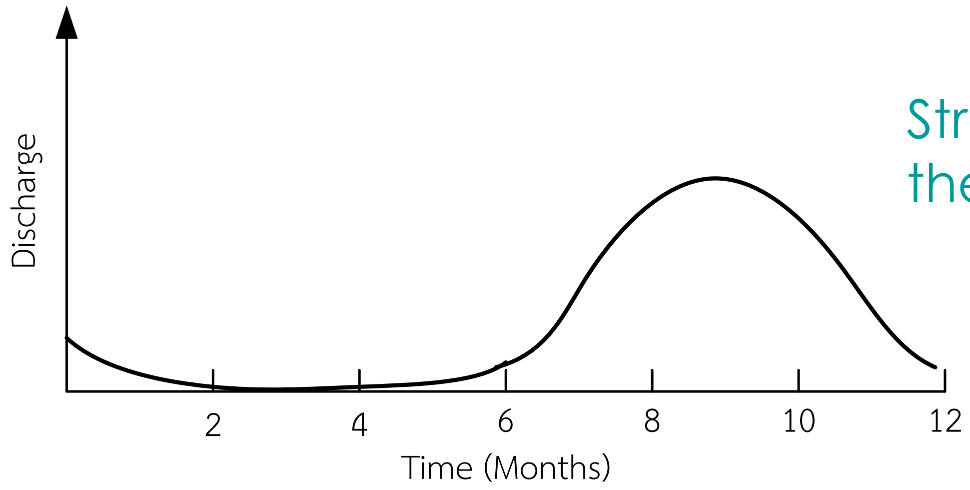
- Perennial streams have some flow at all times of a year due to considerable amount of base flow into the stream during dry periods of the year.



The stream bed is lower than the ground water table.

Intermittent Streams

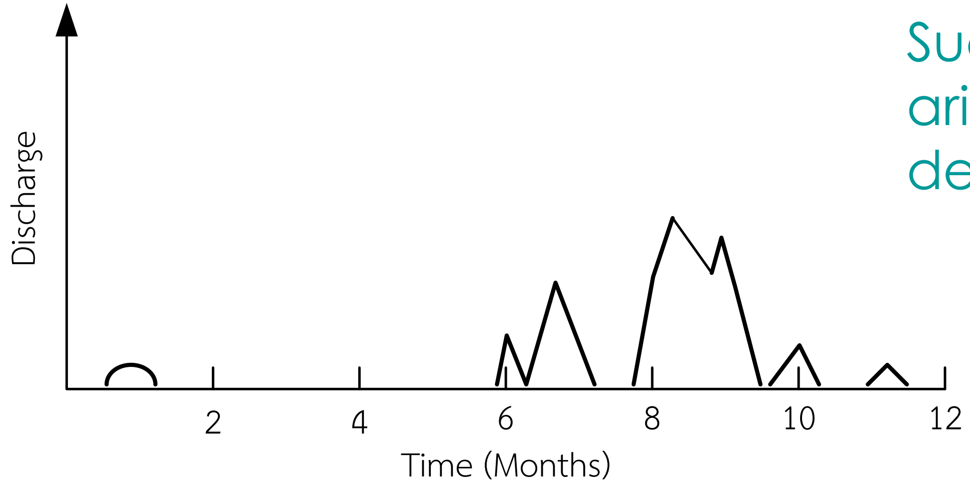
- Intermittent streams have limited contribution from the ground water.
- During the wet season when the ground water table is above the stream bed, there is a base flow contributing to the stream flow.



Streams remain dry for most of the dry season periods of a year.

Ephemeral Streams

- Ephemeral streams do not have any contribution from the base flow.
- The annual hydrograph of such a stream show series of short duration hydrographs indicating flash flows in response to the storm and the stream turning dry soon after the end of the storm.



Such streams generally found in arid zones, do not have well defined channel.

SURFACE WATER: STREAMFLOW

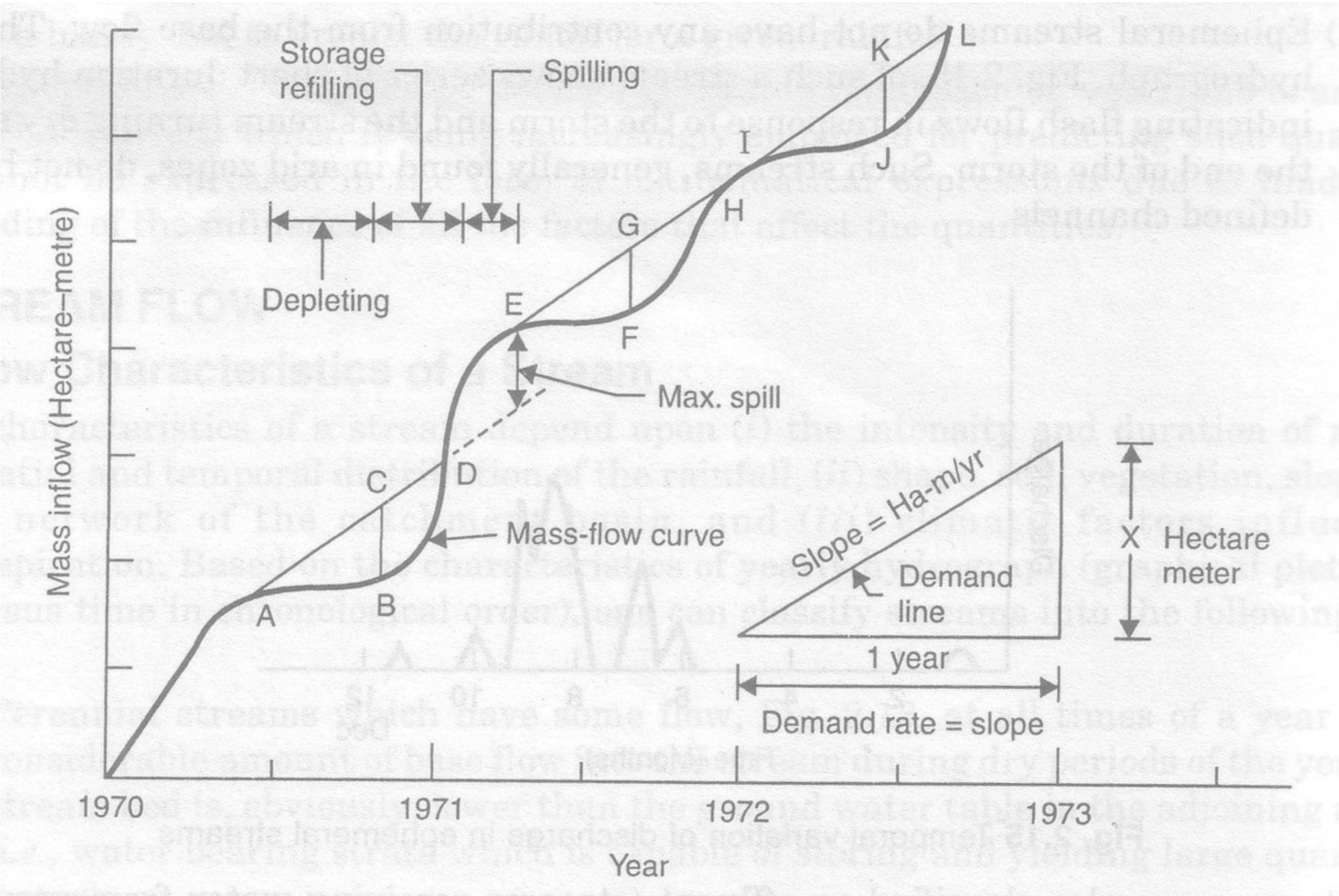
Streamflow Characteristics

Stream are also classified as

- Effluent: streams receiving water from ground water storage. [Perennial Streams]
- Influent: streams contributing water to the ground water storage. [Intermittent Streams/Ephemeral Streams]

Flow/Runoff Mass Curve

Flow mass curve is cumulative flow volume, V versus time curve.



SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

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Flow/Runoff Mass Curve

The mass curve ordinate, V at any time t is given as

$$V = \int_{t_o}^{T} Q\,dt$$

t_o = the time at the beginning of the curve.

The slope of the mass curve at any point on the plot, dV/dt equals the rate of streamflow at that time.

Mass curve is always rising curve or horizontal curve and is useful means by which one can calculate storage capacity of a reservoir to meet specified demand as well as safe yield of a reservoir of given capacity.

SURFACE WATER: GRAPHICAL REPRESENTATION OF STREAMFLOW EXAMPLE 1

Flow Mass Curve

The following table gives the mean monthly flows of a stream during a leap year. Determine the minimum storage required to satisfy a demand rate of 50 cms.

Month	Mean Monthly Flow (cms)	Days in Month	Monthly Flow Volume (cumec-day)	Accumulated Volume (cumec-day)
Jan	60	31	1,860	1,860
Feb	50	29	1,450	3,310
Mar	40	31	1,240	4,550
Apr	28	30	840	5,390
May	12	31	372	5,762
Jun	20	30	600	6,362
Jul	50	31	1,550	7,912
Aug	90	31	2,790	10,702
Sep	100	30	3,000	13,702
Oct	80	31	2,480	16,182
Nov	75	30	2,250	18,432
Dec	70	31	2,170	20,602

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GRAPHICAL REPRESENTATION OF STREAMFLOW

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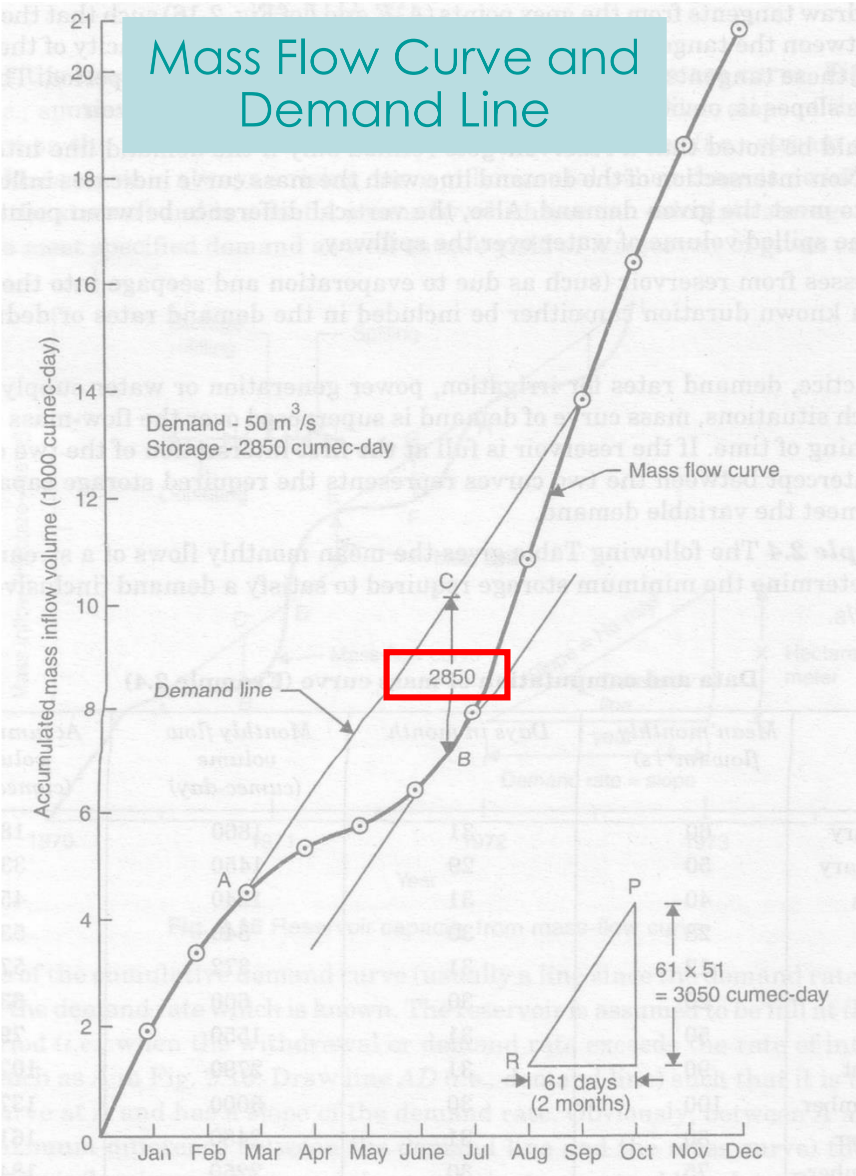
Mass Flow Curve and Demand Line



Mass curve of the accumulated flow versus time is shown in the figure.

For the mass curve and demand rate, all months are assumed to be of equal duration, 30.5 days.

A demand line with a slope of line PR is drawn tangential to the mass flow curve at A.

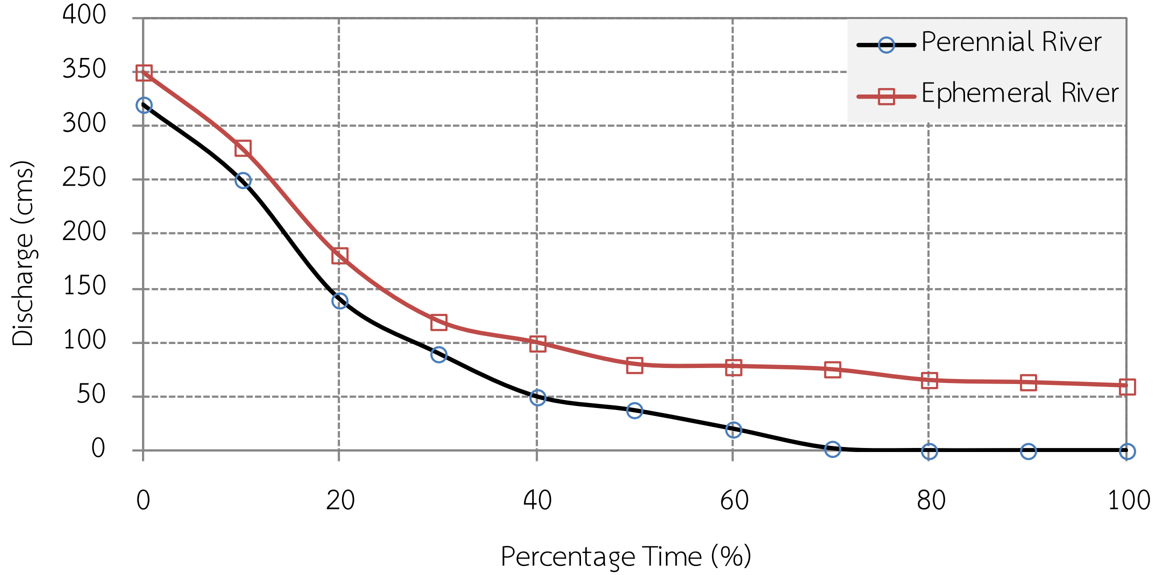
Another line parallel to this line is drawn so that it is tangential to the mass flow curve at B.

The vertical difference BC = 2,850 cumec-day is the required storage for satisfying the demand rate of 50 cms.

SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

Flow-Duration Curve/Discharge-Frequency Curve

Flow-Duration Curve of a stream is graphical plot of stream discharge against the corresponding percent of time that the stream discharge was equalled or exceeded.



The ordinate, Q at any percentage probability, Pp represents the flow magnitude in an average year that can be expected to be equalled or exceeded Pp percent of time and is termed as "**Pp% Dependable Discharge**".

SURFACE WATER: GRAPHICAL REPRESENTATION OF STREAMFLOW

Flow-Duration Curve/Discharge-Frequency Curve

The flow-duration curve describes the variability of the streamflow and is useful for;

- Determining dependable flow which information is required for planning of water resources and hydropower projects.
- Designing a drainage system
- Flood control studies

SURFACE WATER: GRAPHICAL REPRESENTATION OF STREAMFLOW

Preparing a Flow-Duration Curve

- The streamflow data is arranged in a descending order of stream discharges. If the number of such discharges is very large, one can use range of values as **class intervals**.

- Percentage probability, Pp of any flow magnitude, Q being equalled or exceeded is given as;

$$P_p = \frac{m}{N+1} x100(\%)$$

m = the order number of the discharge (or class interval)
N = the number of data points in the list

FLOW-DURATION CURVE: EXAMPLE 2

The observed mean monthly flows of a stream for a water year (June 01-May 31) are as given in the first two columns of the following table. Plot the flow-duration curve and estimate the flow that can be expected 75% of the time in a year and also the dependability of the flow of magnitude 30 cms.

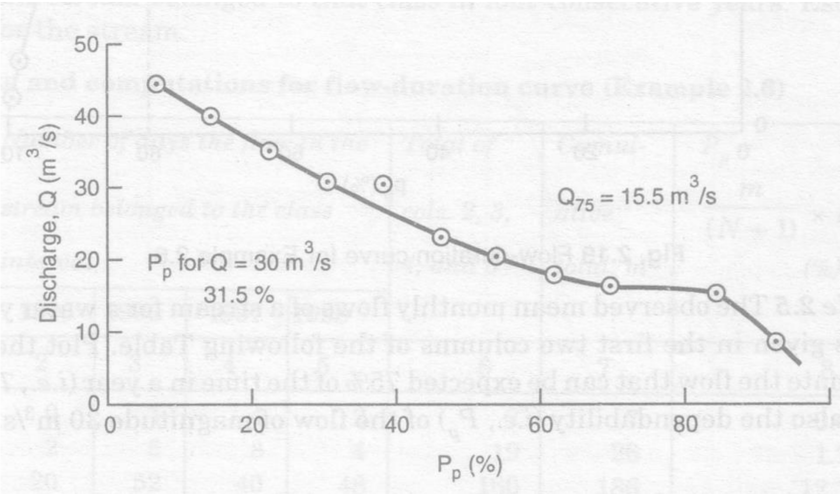
Month	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
Observed Flow	15	16	44	40	35	31	30	21	23	18	15	8

FLOW-DURATION CURVE: EXAMPLE 2

Month	Observed Flow, Q (cms)	Flow, Q arranged in descending order (cms)	Rank m	Pp (%)
Jun	15	44	1	7.7
Jul	16	40	2	15.4
Aug	44	35	3	23.1
Sep	40	31	4	30.8
Oct	35	30	5	38.5
Nov	31	23	6	46.2
Dec	30	21	7	53.8
Jan	21	18	8	61.5
Feb	23	16	9	69.2
Mar	18	15	10	84.6
Apr	15	15	11	84.6
May	8	8	N=12	92.3

Q_{75} =15.5 cms

Dependability of the flow of magnitude 30 cms = 31.5%



FLOW-DURATION CURVE: EXAMPLE 3

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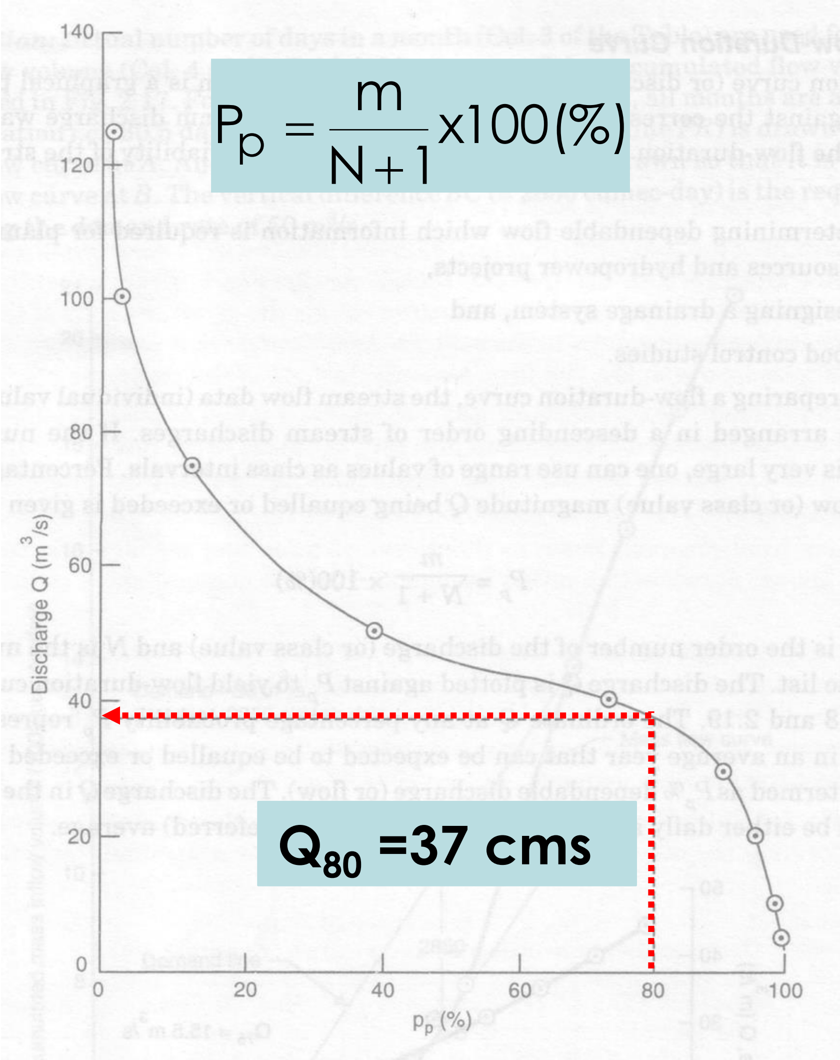
Column 1 of the table below gives the class interval of daily mean discharges of a streamflow data. Column 2, 3, 4, and 5 give the number of days for which the flow in the stream belonged to that class in 4 consecutive years. Estimate 80% dependable flow for the stream.

Daily Mean Discharge (cms)	Number of days the flow in the stream belonged to the class interval				Total of Col. 2, 3, 4, 5	Cumulative Total (m)	Pp (%) $P_p = \frac{m}{N+1} x100(\%)$
	1995	1996	1997	1998			
1/	2/	3/	4/	5/	6/	7/	8/
125-150	0	1	4	2	**7**	**7**	**0.48**
100-124.9	2	5	8	4	**19**	**26**	**1.78**
75-99.9	20	52	40	48	**160**	**186**	**12.72**
50-74.9	95	90	100	98	**383**	**569**	**38.92**
40-49.9	140	125	117	124	**506**	**1,075**	**73.53**
30-39.9	71	75	65	50	**261**	**1,336**	**91.38**
20-29.9	15	10	20	21	**66**	**1,402**	**95.90**
10-19.9	15	8	10	18	**51**	**1,453**	**99.38**
5-9.9	7	0	1	0	**8**	**1,461**	**99.93**
Total	365	366	366	365	N=1,461		

FLOW-DURATION CURVE: EXAMPLE 3



SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

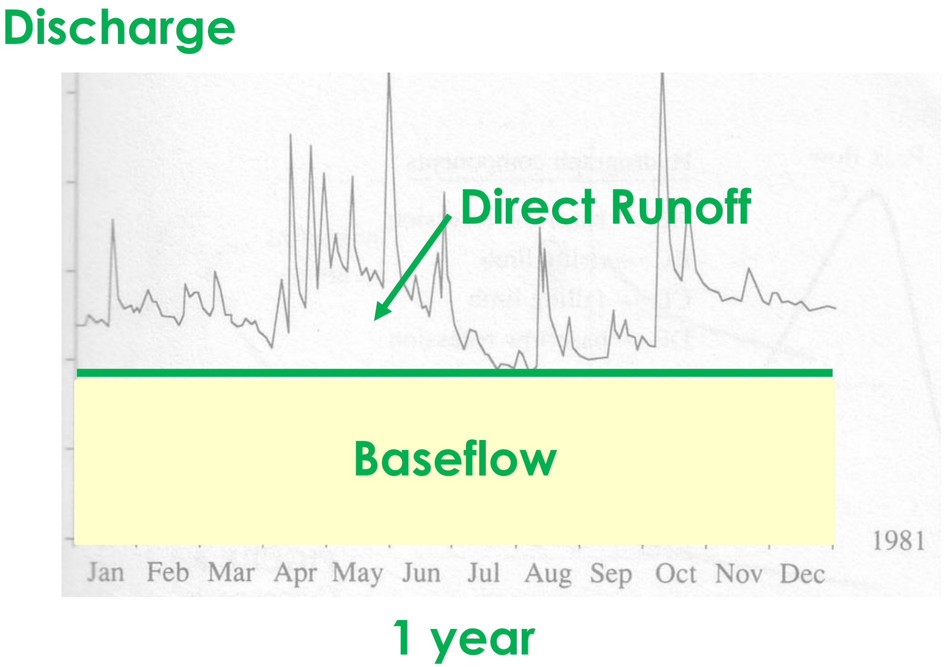
A Streamflow Hydrograph (Discharge Hydrograph)
Streamflow hydrograph is a graph or table showing the flow rate as a function of time at a given location on the stream.

Two type of hydrographs are particularly important
- Annual Hydrograph
- Storm Hydrograph

SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

Annual Hydrograph

The annual hydrograph is a plot of streamflow vs. time over a year showing the long term balance of precipitation, evaporation, and streamflow in a watershed.



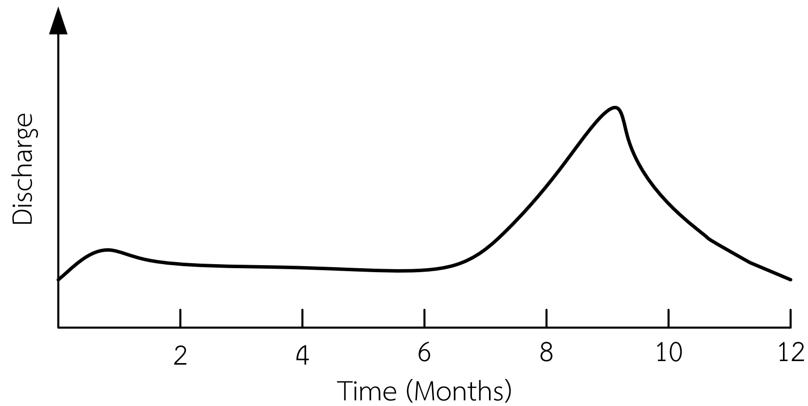
Direct Runoff (Quickflow):
Direct runoff is the spike caused by rain storms.

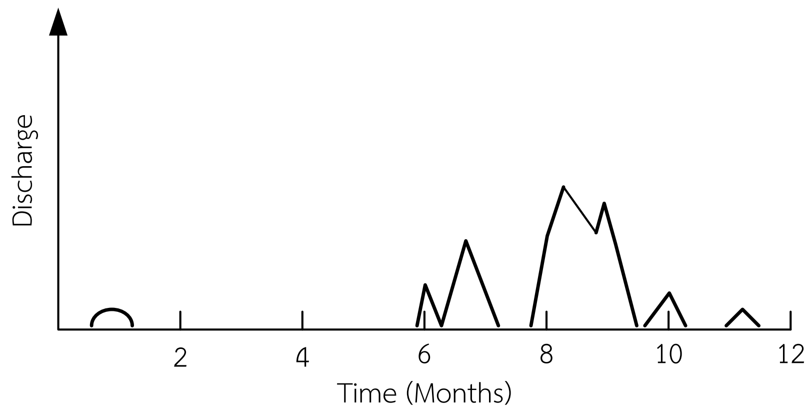
Baseflow:
Baseflow is the slow flow in rainless period.

Basin Yield:
The total volume of flow under the annual hydrograph

SURFACE WATER: GRAPHICAL REPRESENTATION OF STREAMFLOW

Annual Hydrograph





Perennial Streams

Most of the basin yield of perennial stream usually comes from **baseflow**, indicating that a large proportion of the rainfall is infiltrated into the basin and reaches the stream as subsurface flow.

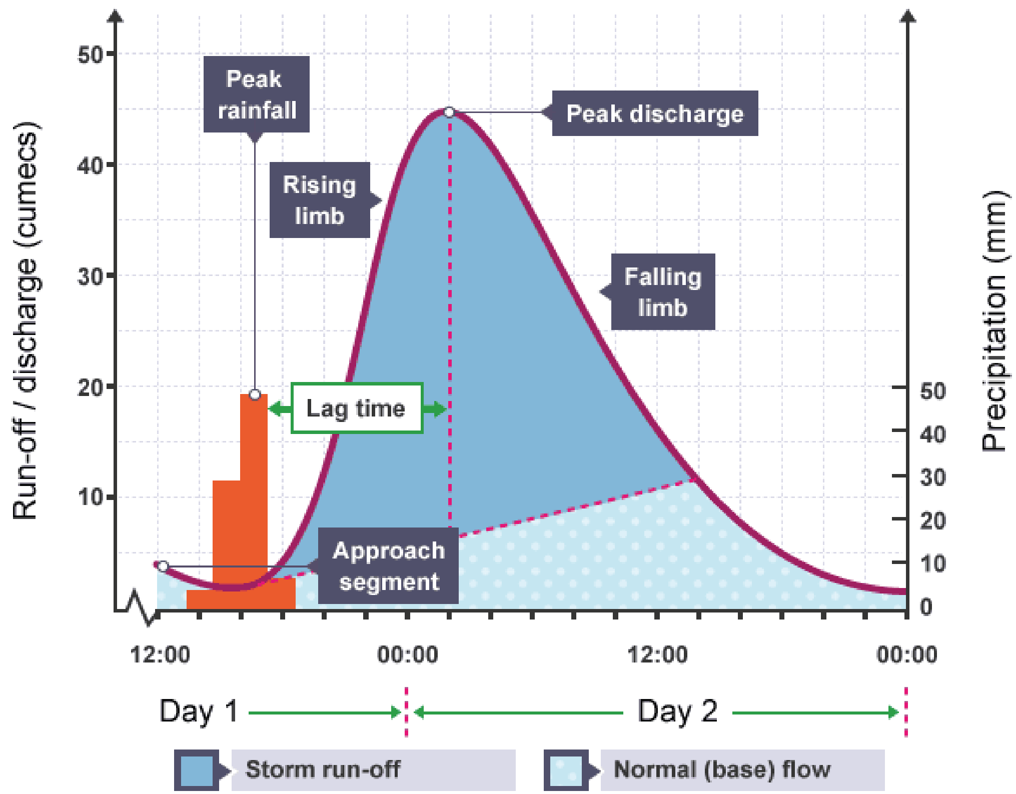
Ephemeral Stream

Most storm rainfall becomes **direct runoff** and little infiltration occurs. Basin yield from the watershed is the result of direct runoff from large storms.

SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

Storm Hydrograph

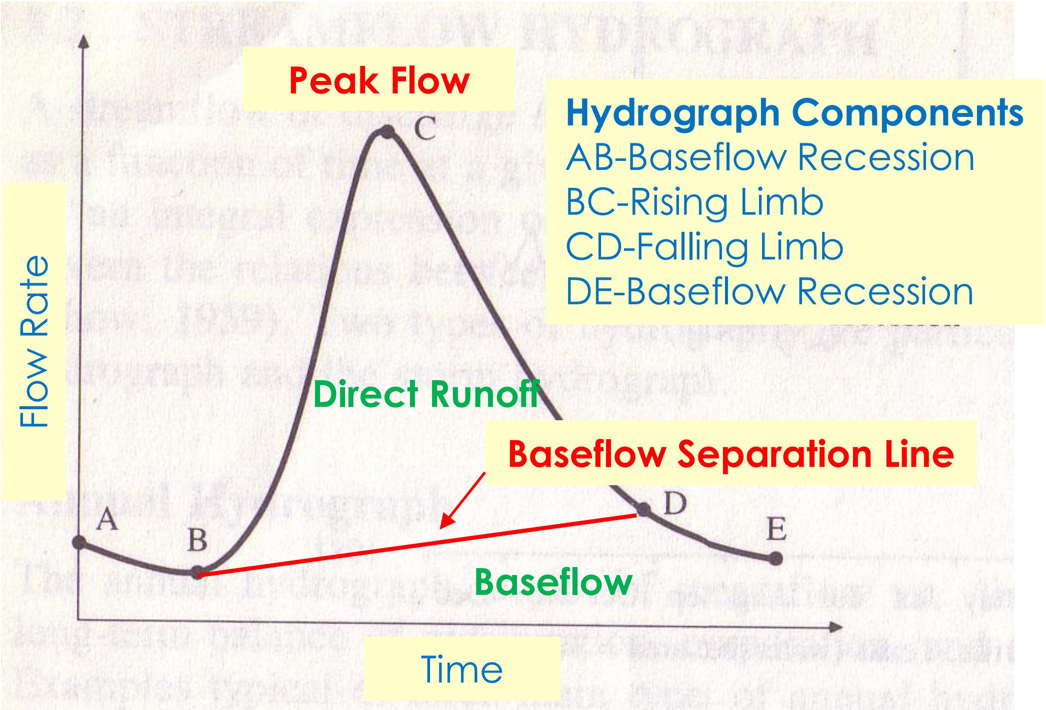
- The study of hydrograph during a storm is called "**Storm Hydrograph**".
- A storm hydrograph shows the response of a river drainage basin to a period of rainfall. They show the volume of water passing a certain point on a river, measured in cumecs in relation to volume of rainfall.



Source: Wordpress (2016)

SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

Components of Storm Hydrograph



The figure shows the components of streamflow hydrograph during a storm.

Prior to the time of intense rainfall, baseflow is gradually diminishing (segment AB).

Direct runoff begins at B, peaks at C and ends at D.

Segment DE follows as normal baseflow recession begins again.

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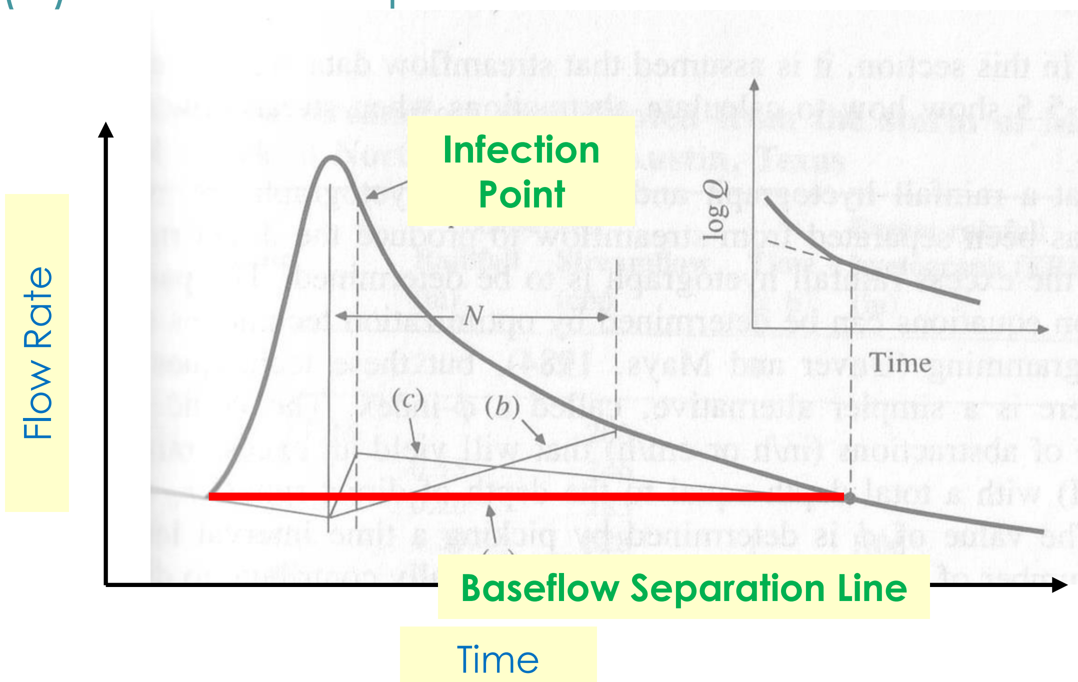
Methods of Baseflow Separation

Methods of baseflow separation are;

(a) Straight line method
(b) Fixed base method
(c) Variable slope method



Straight line method involves drawing a horizontal line from the point at which surface runoff begins to the intersection with the recession limb. This is applicable to ephemeral streams.

SURFACE WATER:
GRAPHICAL REPRESENTATION OF STREAMFLOW

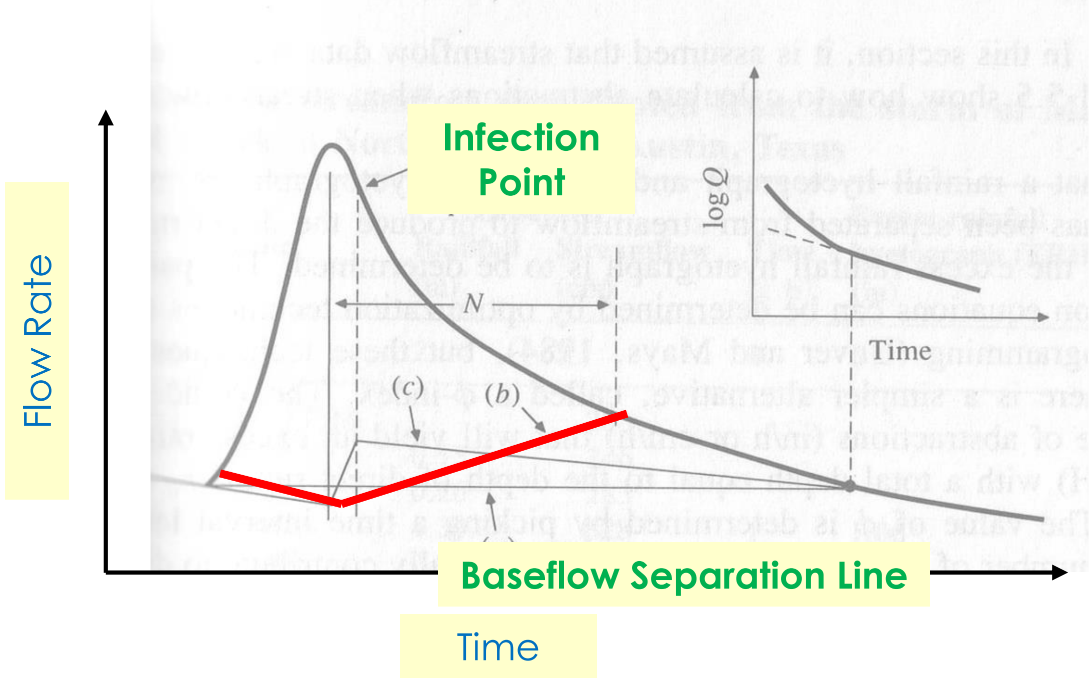
Methods of Baseflow Separation

Methods of baseflow separation are;

(a) Straight line method

(b) Fixed base method

(c) Variable slope method



Fixed base method, the surface runoff is assumed to end a fixed time N after the hydrograph peak. The baseflow before the surface runoff began is projected ahead to the time of the peak. A straight line is used to connect this projection at the peak to the point on the recession limb at time N after the peak.

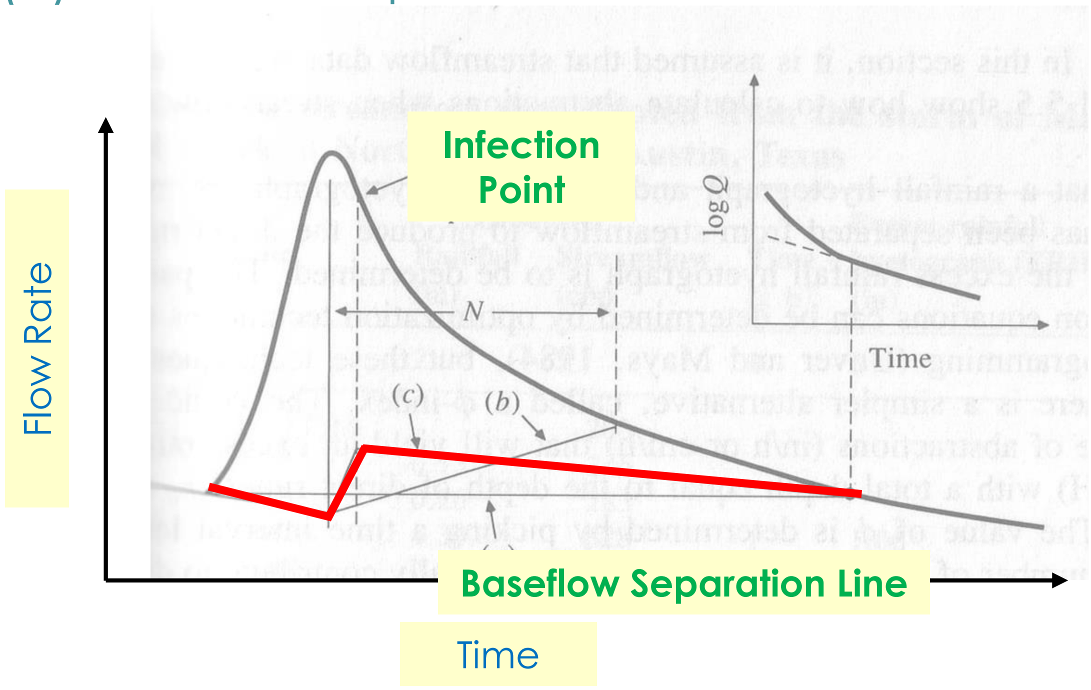
Methods of Baseflow Separation

Methods of baseflow separation are;

(a) Straight line method

(b) Fixed base method

(c) Variable slope method



Variable slope method, the baseflow curve before the surface runoff began is extrapolated forward to the time of peak discharge, and the baseflow curve after the surface runoff ceases is extrapolated backward to the time of point of inflection on the recession limb. A straight line is used to connect the endpoints of the extrapolated curves.

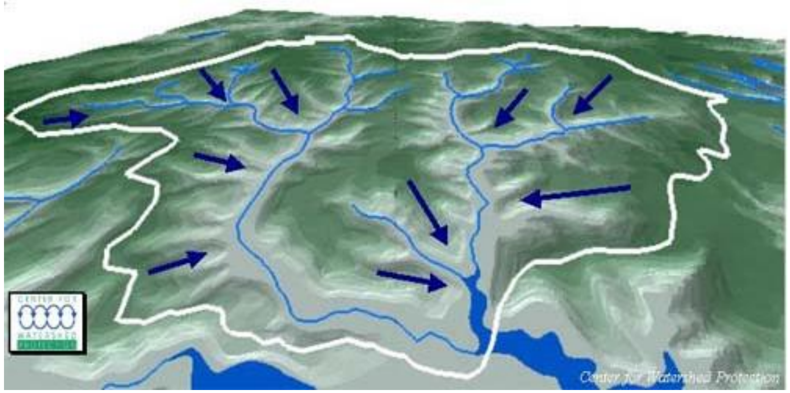
TRAVEL TIME OF FLOW

Watershed

Watershed is an area of land that drains all the streams and rainfall to a common outlet such as the outflow of a reservoir, mouth of a bay, or any point along a stream channel.

The word watershed is sometimes used interchangeably with "**Drainage Basin**" or "**Catchment**".

Ridges and hills that separate two watersheds are called the "**Drainage Divide**".



"A watershed is a precipitation collector"

Function of Watershed

The main function of watershed is to receive the incoming precipitation and then dispose it off. This is the essence of soil and water conservation.

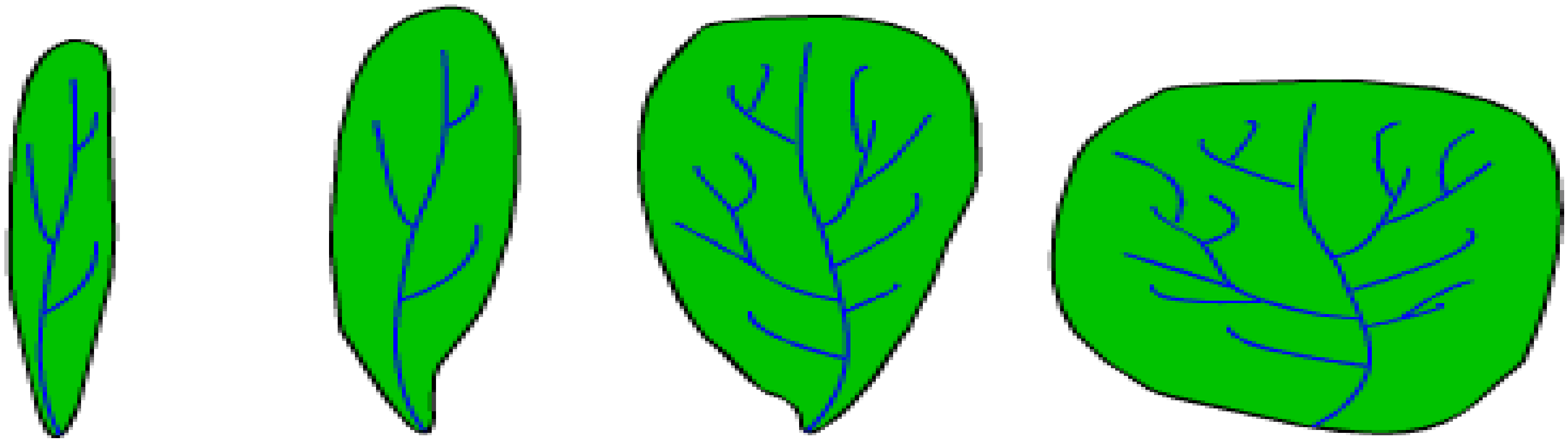
Types of Watershed

Type of Watershed	Area Covered
Micro Watershed	0-10 ha
Small Watershed	10-40 ha
Small Watershed)	40-200 ha
Sub Watershed	200-400 ha
Macro Watershed	400-1,000 ha
River Basin)	>>1000 ha

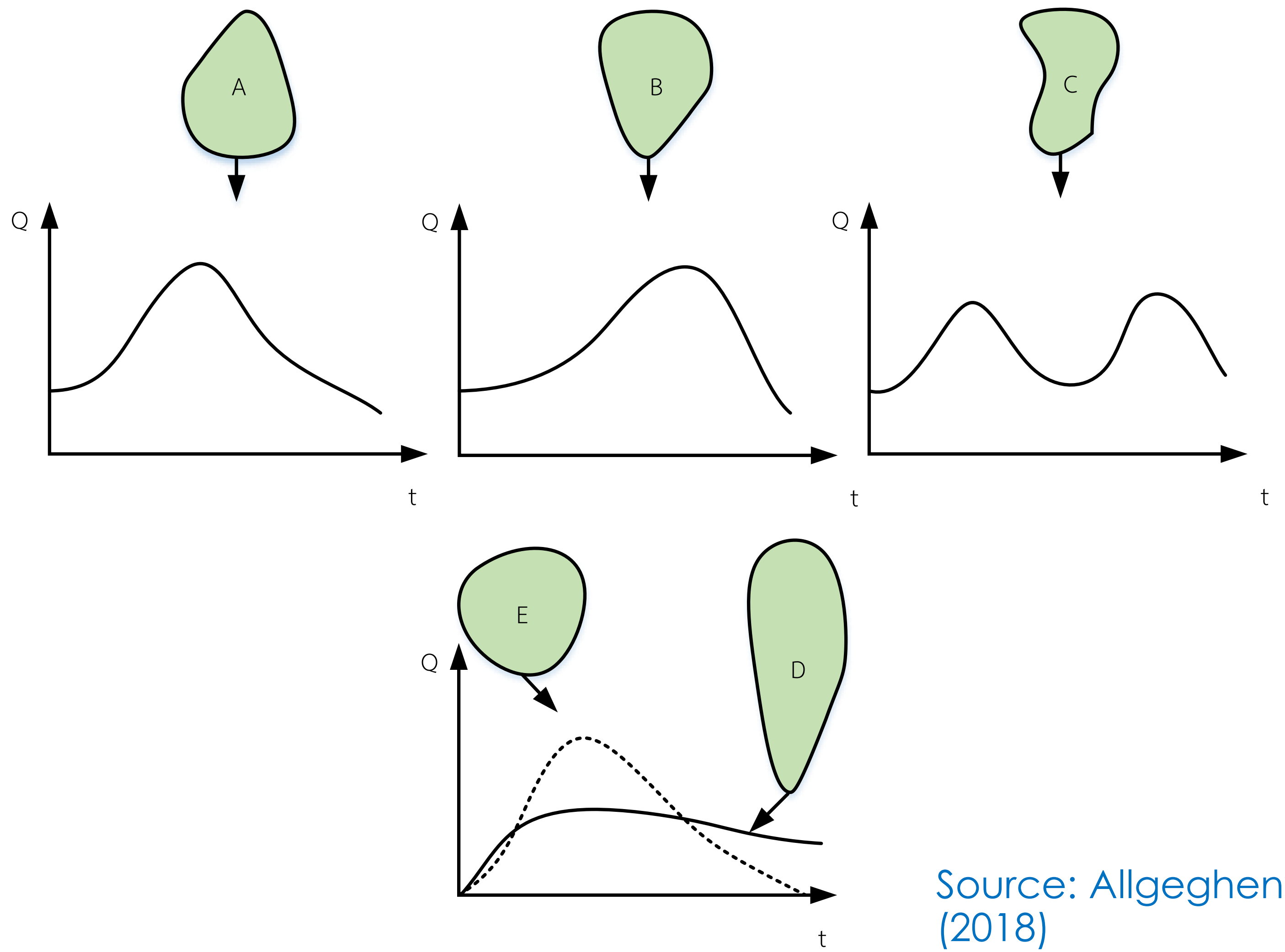
TRAVEL TIME OF FLOW

Types of Watershed

- Square Watershed
- Triangular Watershed
- Rectangular Watershed
- Oval Watershed
- Fern Leaf Shaped Watershed
- Palm Shaped Watershed
- Polygon Shaped Watershed
- Circular Watershed



Types of Watershed vs Hydrograph



Source: Allgeghenygeoquest (2018)

Travel Time of Flow

The travel time of flow from one point on a watershed to another can be deduced from the flow distance and velocity.

If two points on a stream are a distance L apart and the velocity along the path connecting them is v(l), where l is the distance along the path, then the travel time t is given by;

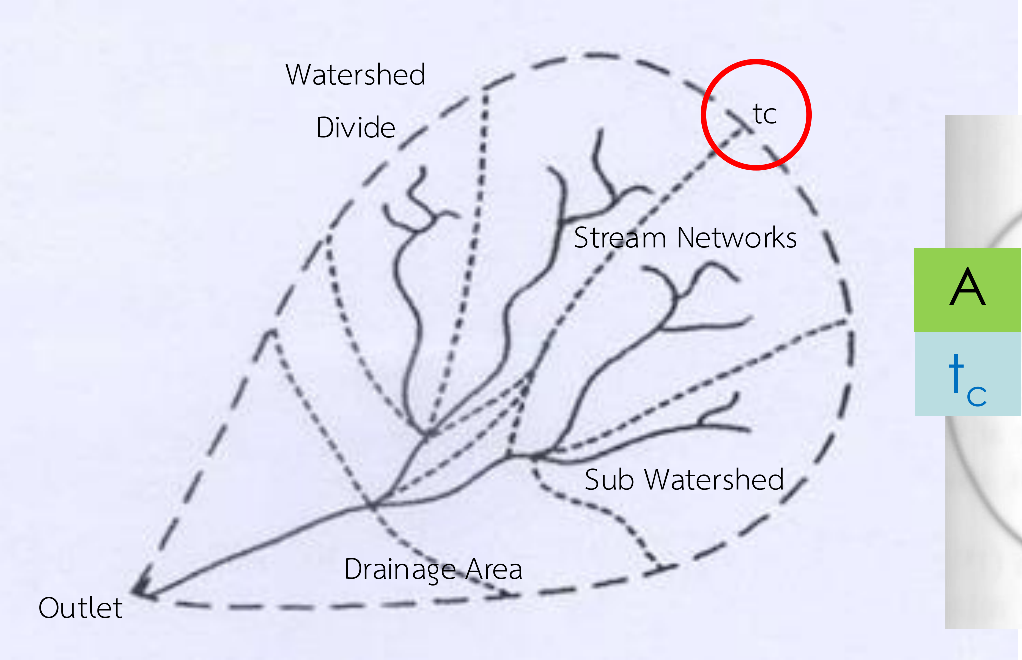
$$dl = v(l)dt \rightarrow \int_0^1 dt = \int_0^L \frac{dl}{v(l)} \rightarrow t = \int_0^L \frac{dl}{v(l)}$$

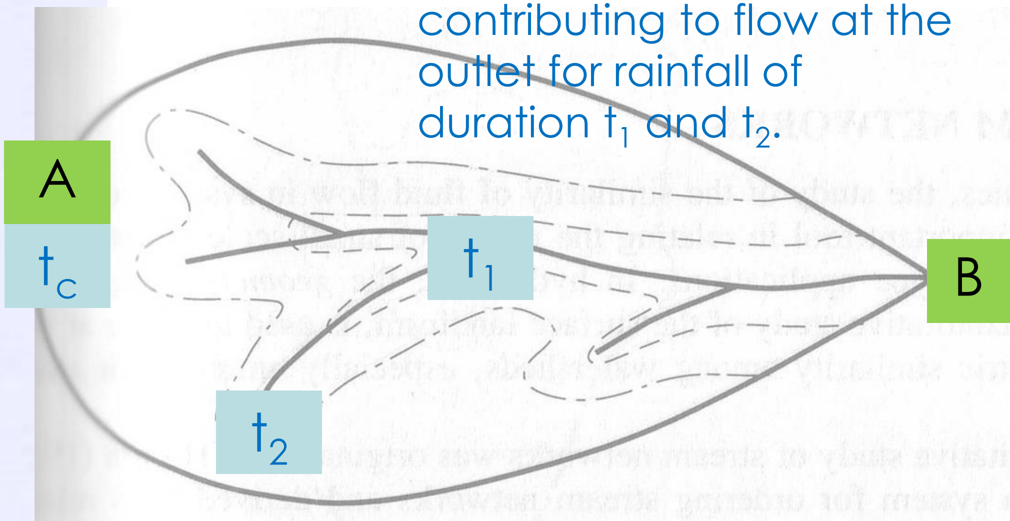
If the velocity can be assumed constant at v_i in an increment of length Δl_i, i=1, 2, 3, …., I then;

$$t = \sum_{i=1}^{I} \frac{\Delta l_i}{v_i}$$

TRAVEL TIME OF FLOW

Watershed vs Travel Time of Flow





Isochrone at t_1 and t_2 define the area contributing to flow at the outlet for rainfall of duration t_1 and t_2.

Time of concentration, tc is the time of flow from the farthest point in the watershed (A) to the outlet (B).

TRAVEL TIME OF FLOW

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Average Velocities in ft/s

Description of water course	Slope in Percent			
	0-3	4-7	8-11	12-
Unconcentrated*				
-Woodlands	0-1.5	1.5-2.5	2.5-3.25	3.25-
-Pastures	0-2.5	2.5-3.5	3.5-4.25	4.25-
-Cultivated	0-3.0	3.0-4.5	4.5-5.5	5.5-
-Pavements	0-8.5	8.5-13.5	13.5-17	17-
Concentrated**				
-Outlet channel-determine velocity by Manning's formula				
-Natural channel not well defined	0-2	2-4	4-7	7-

* This condition usually occurs in the upper extremities of a watershed prior to the overland flows accumulating in a channel.

** These values vary with the channel size and other conditions. Where possible, more accurate determinations should be made for particular conditions by the Manning channel formula for velocity.

TRAVEL TIME OF FLOW: EXAMPLE 4

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Calculate the time of concentration of a watershed in which the longest flow path covers 100 feet of pasture at a 5% slope, then enters a 1000 foot-long rectangular channel having width 2 ft, roughness n=0.015, and slope 2.5 percent, and receiving a lateral flow of 0.00926 cfs/ft.

Solution

Travel Time in a Channel

Distance along channel, l (ft)	0		200		400		600		800		1000
Δl		200		200		200		200		200	
Calculated velocity, v (ft/s)	0		4.63		5.97		6.86		7.56		8.02
Average velocity, v (ft/s)		2.32		5.30		6.42		7.21		7.79	
Travel time, Δt=Δl/v (s)		86.2		37.7		31.2		27.7		25.7	
											Σ Δt=208.5 sec

Travel Time on Land Surface = 100/3.0 = 33.33 sec and Tc = 208.5+33.33 = 241.83 sec

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Stream Networks

The quantitative study of stream networks was originated by Horton (1945). He developed a system for ordering stream networks and derived laws relating the number and length of streams of different order.

Horton's Stream Ordering System, as slightly modified by Strahler (1964) is as follows:

- The smallest recognizable channels are designated order 1; these channels normally flow only during wet weather.
- Where two channels of order 1 join, A channel of order 2 results downstream, where two channels of order I join, a channel of order i+1 results.

The order of the drainage basin is designated as the order of the stream draining its outlet, the highest stream order in the basin I.

Horton's Law of Stream Number

Horton (1945) found empirically that the "**Bifurcation Ratio, R_B**" or ratio of the number N_i, of channels of order i to the number N_{i+1} is relatively constant from one order to another.

$$\frac{N_i}{N_{i+1}} = R_B \qquad i = 1,2,\ldots,I-1$$

The theoretical minimum value of the $R_B = 2$. Values typically lie in the range 3-5.

Source: Chow et al. (1988)

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Horton's Law of Stream Lengths

By measuring the length of each stream, the average length of streams of each order, Li, can be found. Horton proposed a Law of Stream Lengths in which the average lengths of streams of successive orders are related by a length ratio, RL.

$$\frac{L_{i+1}}{L_i} = R_L$$

Law of Stream Area

Schumm (1956) proposed a Law of Stream Areas to relate the average areas, Ai drained by streams of successive order.

$$\frac{A_{i+1}}{A_i} = R_A$$

Source: Chow et al. (1988)

STREAM NETWORKS

Length of Overland flow, L

If the streams are fed by Hortonian overland flow from all of their contributing area, then the average length of overland flow, L_o is given approximately by

$$L_0 = \frac{1}{2D}$$

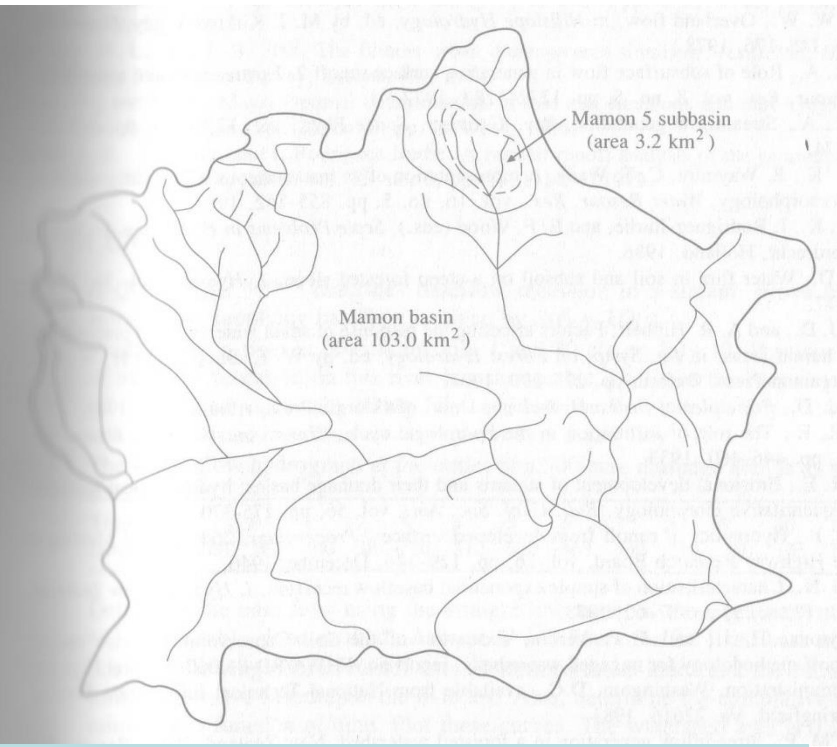
Drainage Density, D

Drainage density is the ratio of the total length of stream channels in a watershed to its area.

$$D = \frac{\sum_{i=1}^{I} \sum_{j=1}^{N_i} L_{ij}}{A_I}$$

Source: Chow et al. (1988)

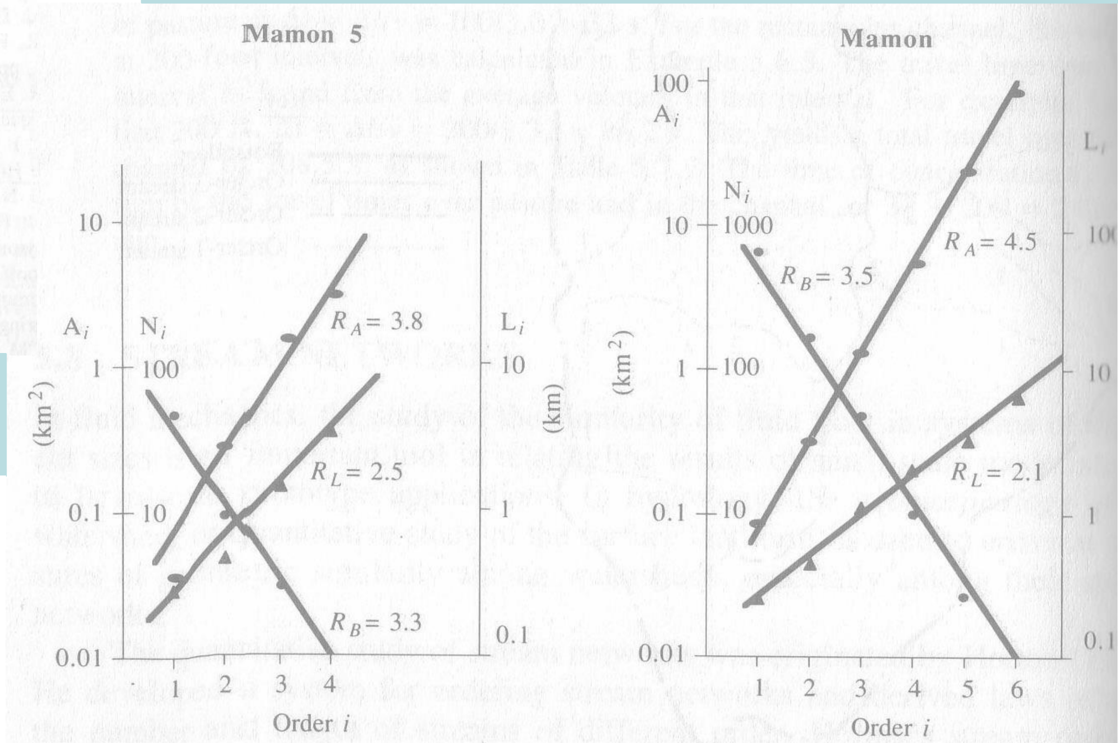
STREAM NETWORKS



Drainage basin of the Mamon watershed in Venezuala

Source: Chow et al. (1988)

Geomorphological parameters for the Mamon basins.



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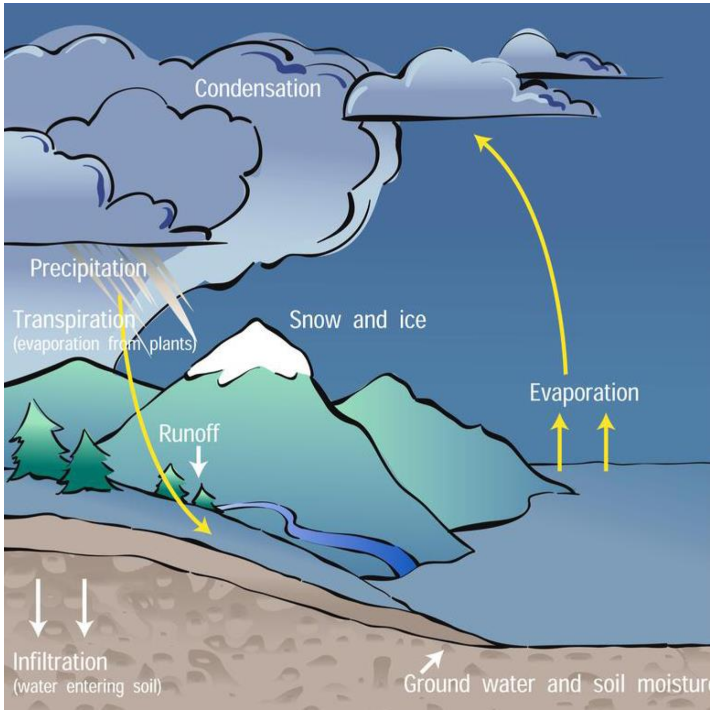

LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

Hydrologic Measurement

- Hydrologic Data Measurement Sequence
- Measurement of Atmospheric Water
- Measurement of Surface Water
- Measurement of Subsurface Water
- Hydrologic Measurement Systems

Hydrologic Measurement

Hydrologic measurement are made to obtain data on hydrologic process.

Hydrologic data is used **to better understand the hydrologic processes** as a direct input into hydrologic simulation models for design, analysis, and decision making.

Hydrologic processes vary in space and time and are random (probability) in character.

The **uncertainties** create requirement for hydrologic measurement to provide observed data at/near the location of interest.

HYDROLOGIC MEASUREMENT

Hydrologic Measurement
The hydrologic processes are measured as

(1) Point Sample
Measurements made through time at a fixed location in space. The resulting data forms a "**Time Series**".

(2) Distributed Samples
Measurement made over a line or area in space at a specific point in time. The resulting data forms a "**Space Series**".

HYDROLOGIC MEASUREMENT SEQUENCES

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Hydrologic Phenomenon

↓

Sensing	Transform the intensity of the phenomenon into an observable signal
Recording	Make an electronic or paper record of the signal
Transmission	Move the record to a central processing site
Translation	Convert the record into a computerized data sequence
Editing	Check the data and eliminate errors and redundant info
Storage	Archive the data on a computer tape or disk
Retrieval	Recover the data in the form required

↓

User of Data

Hydrologic Measurement

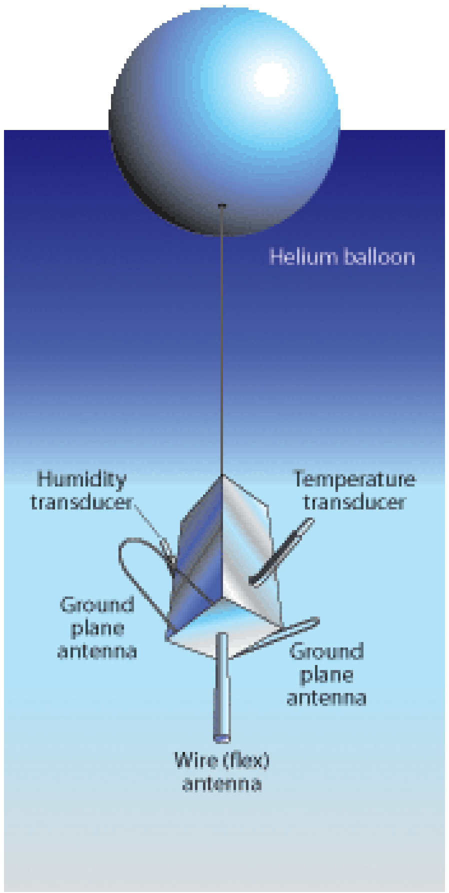
Data	Instrument
1.Atmospheric Moisture	Radiosonde
2.Temperature	Thermometer
3.Humidity	Hygrometer
4.Radiation	Radiometer
5.Rainfall	1) Nonrecording gage; standard gage, storage gage 2) Recording gage; weighting type, float type, tipping bucket type

Hydrologic Measurement

Data	Instrument
6.Interception	Water balane; -comparing the precipitation in gage beneath the tree with that recorded nearby under the open sky
7.Evaporation	Evaporation pan; -US class A pan -USSR GGI-3000 pan
8.Evapotranspiration	Lysimeter

MEASUREMENT OF ATMOSPHERIC WATER: CLIMATE DATA

Radiosonde



A radiosonde is an instrument package carried by a balloon that ascends to altitudes of 20 to 30 kilometers.

It measures temperature, humidity, and pressure in the atmosphere and broadcasts the information back to a ground station.

The Global Positioning System is used to record the trajectory during ascent to determine wind speed and direction.

MEASUREMENT OF ATMOSPHERIC WATER: RADIATION & TEMPERATURE

Radiometer

Ground Radiometers on Stand for Upwelling Radiation.

Thermometer

MEASUREMENT OF ATMOSPHERIC WATER: HUMIDITY

Hygrometer

Psychometer

MEASUREMENT OF ATMOSPHERIC WATER: RAINFALL

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Rain Guage

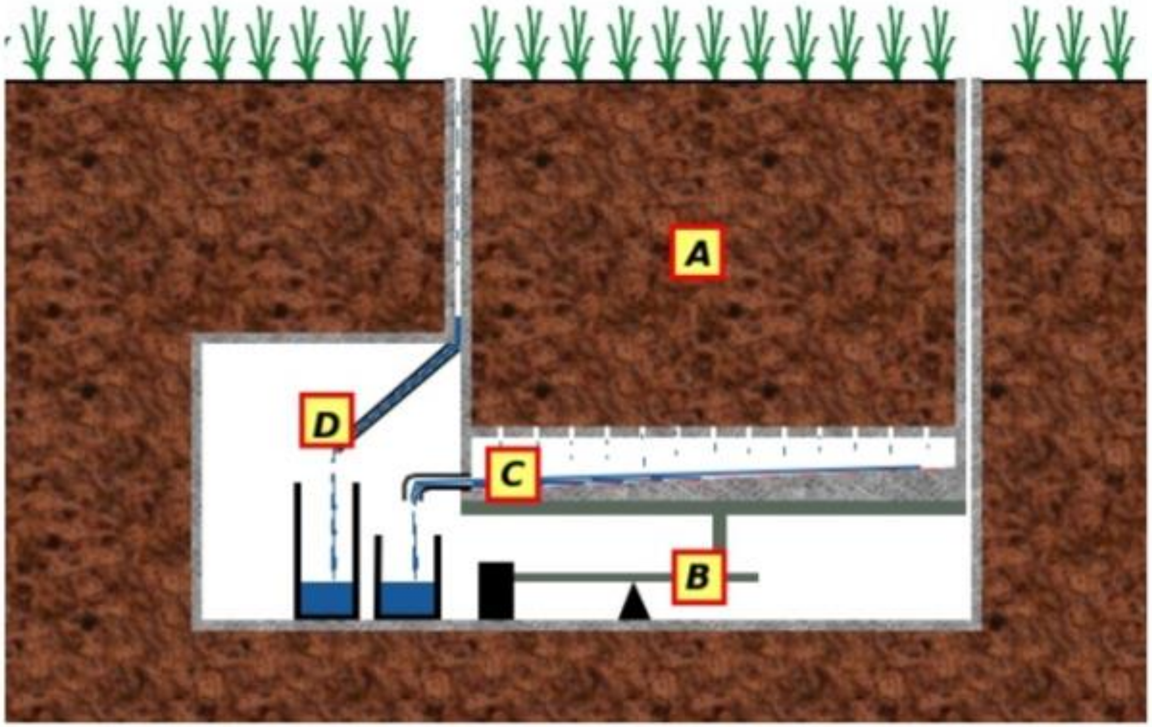
Tipping Bucket Rain Guage

MEASUREMENT OF ATMOSPHERIC WATER: EVAPORATION

Evaporation Pan

MEASUREMENT OF ATMOSPHERIC WATER: EVAPOTRANSPIRATION

Lysimeter



Hydrologic Measurement

Data	Instrument
1.Water Surface Elevation	Staff gage
2.Flow Velocity	1) Current meter 2) Electromagnetic sensing (VMFM)
3.Streamflow Rate	Rating Curve
4.Discharge Computation	Continuous equation
5.Rating Curve	Plotting discharge vs water level

MEASUREMENT OF SURFACE WATER: WATER ELEVATION

Vertical Staff Guage

Inclined Staff Guage

MEASUREMENT OF SURFACE WATER: FLOW VELOCITY

Current Meter: Cup Type

Current Meter: Propeller Type

MEASUREMENT OF SURFACE WATER: STREAMFLOW RATE

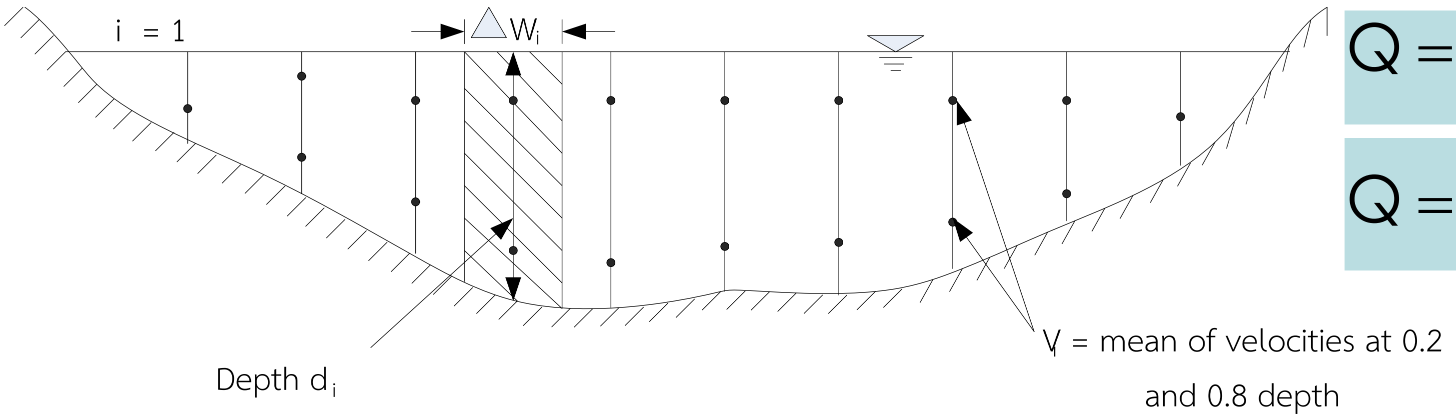
Streamflow

Streamflow is not directly recorded, even though this variable is perhaps the most important in hydrologic studies.

Instead, water level is recorded and streamflow is deducted by means of a "**Rating Curve**".

MEASUREMENT OF SURFACE WATER: DISCHARGE COMPUTATION

At known distances from an initial point on the stream bank, the measured depth and velocity of a stream are shown in the table. Calculate the corresponding discharge at this location.



$$Q = \iint_A V dA$$

$$Q = \sum_{i=1}^{n} V_i d_i \Delta w_i$$

MEASUREMENT OF SURFACE WATER: DISCHARGE COMPUTATION

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Measurement No, i	Distance from Initial Point, (ft)	Width, Δw (ft)	Depth, d (ft)	Mean Velocity, V (ft/s)	Area, dΔw (ft²)	Discharge, VdΔw (cfs)
1	0	6.0	0.0	0.00	4.7	0.0
2	12	16.0	3.1	0.37	49.6	18.4
3	32	20.0	4.4	0.87	88.0	76.6
4	52	20.0	4.6	1.09	92.0	100.3
5	72	20.0	5.7	1.34	114.0	152.8
6	92	20.0	4.5	0.71	90.0	63.9
7	112	20.0	4.4	0.87	88.0	76.6
8	132	20.0	5.4	1.42	108.0	153.4
9	152	17.5	6.1	2.03	106.8	216.7
10	167	15.0	5.8	2.22	87.0	193.1
11	182	15.0	5.7	2.51	85.5	214.6
12	197	15.0	5.1	3.06	76.5	234.1
13	212	15.0	6.0	3.12	90.0	280.8
14	227	15.0	6.5	2.96	97.5	288.6
15	242	15.0	7.2	2.62	108.0	283.0
16	257	15.0	7.2	2.04	108.0	220.3
17	272	15.0	8.2	1.56	123.0	191.9
18	287	15.0	5.5	2.04	82.5	168.3
19	302	15.0	3.6	1.57	54.0	84.8
20	317	11.5	3.2	1.18	36.8	43.4
21	325	4.0	0.0	0.00	3.2	0.0
Total		325.0			1,693.0	3,061.4

Width:

$\Delta w_2 = [(32-12)/2+(12-0)/2]$
$= 16$ ft

The corresponding area:

$d_2\Delta w_2 = 3.1\text{x}16.0 = 49.6 \text{ ft}^2$

The resulting discharge increment:

$V_2 d_2 \Delta w = 0.37\text{x}49.6$
$= 18.4 \text{ ft}^3/\text{s}$

Total discharge:

Q = 3,061 ft3/s

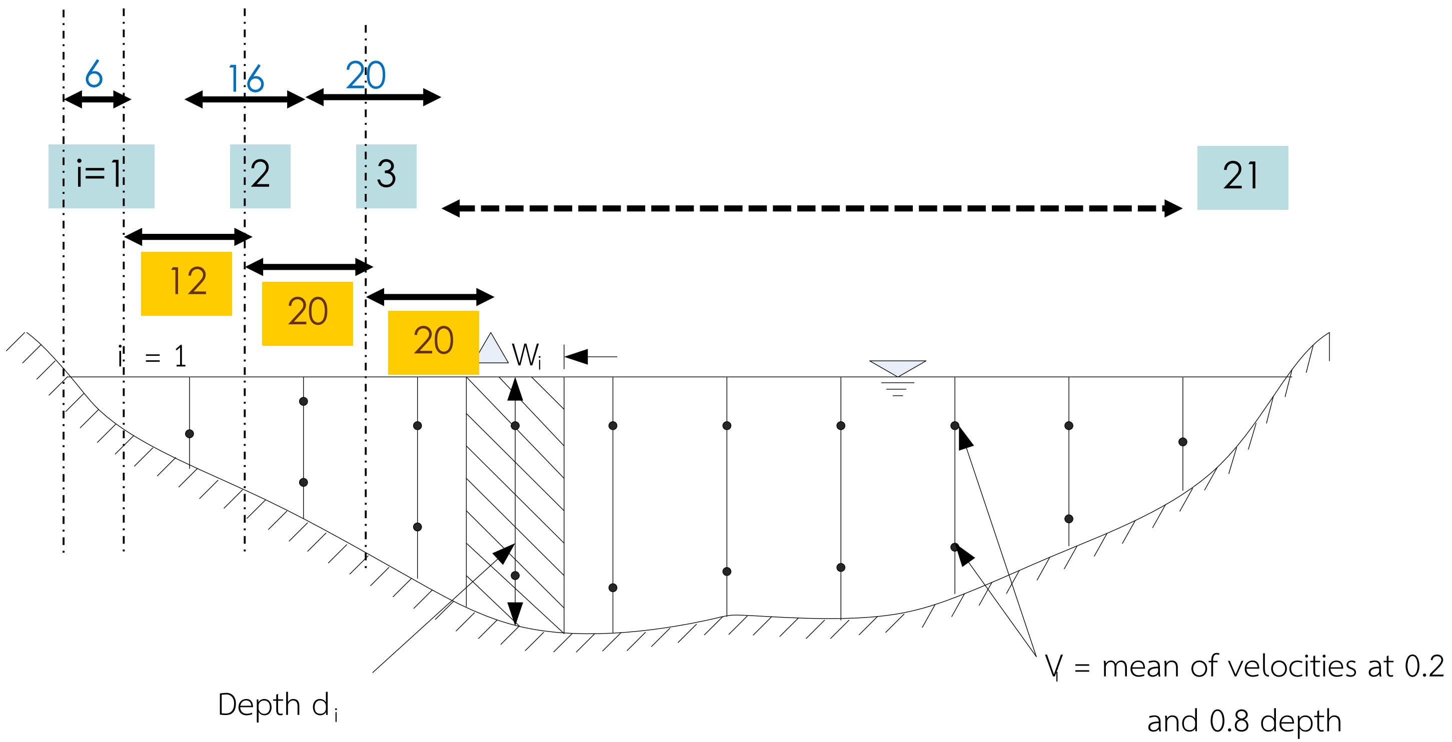
Total crossectional area:

A = 1,693 ft2

The average velocity at this cross section:

V = Q/A = 3,061/1,693
= 1.81 ft/s

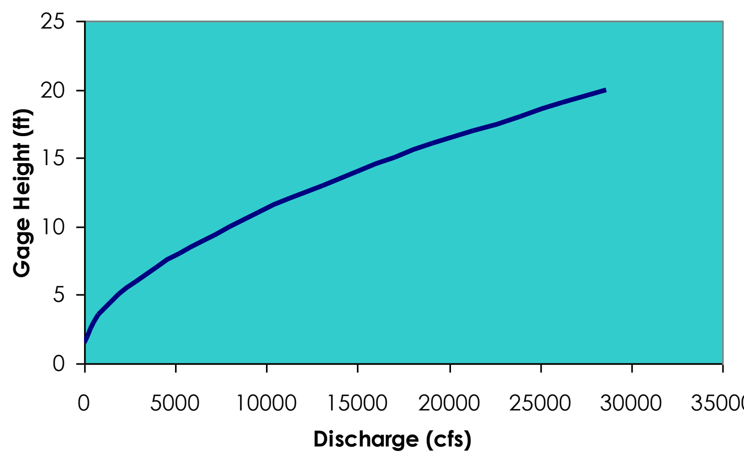
MEASUREMENT OF SURFACE WATER: DISCHARGE COMPUTATION



Rating Curve/Table

A rating curve is a relationship between stage and discharge at a cross section of a river. In most cases, data from stream gages are collected as stage data

Gage Height (ft)	Discharge (cfs)	Gage Height (ft)	Discharge (cfs)
1.5	20	10.0	8,000
2.0	131	11.0	9,588
2.5	307	12.0	11,300
3.0	530	13.0	13,100
3.5	808	14.0	15,000
4.0	1,130	15.0	17,010
4.5	1,498	16.0	19,110
5.0	1,912	17.0	21,340
6.0	2,856	18.0	23,920
7.0	3,961	19.0	26,230
8.0	5,212	20.0	28,610
9.0	6,561		



A Rating Curve/Table for the Colorado River at Austin, Texas, as applicable from Octorber 1974-July 1982.

Source: Chow et al. (1988)

Rating Curve/Table

- The rating curve is developed using a set of measurements of discharge and gage height in the stream, these measurements being made over a period of months or years so as to obtain an accurate relationship between the stream flow rate, or discharge and the gage height at the gaging site.
- Rating curve is used to convert records of water level into flow rate.
- The rating curve must be checked periodically to ensure that the relationship between the discharge and gage height has remained constant.
- Scouring of the stream bed or deposition of sediment in the stream can cause the rating curve to change so that the same recorded gage height produces a different discharge.

MEASUREMENT OF SUBSURFACE WATER

Hydrologic Measurement

Data	Instrument
1.Soil Moisture	1) Water content 2) Gypsum block & Neutron probes
2.Infiltration	Ring infiltrometer
3.Groundwater	Observation wells

MEASUREMENT OF SUBSURFACE WATER: MOISTURE CONTENT

Tensiometer

A **tensiometer** is a device used to determine matric water potential Ψm (Soil Moisture Tension) in the vadose zone.

The amount of moisture in the soil can be found by taking a sample of soil and oven drying. By comparing the weight of the sample before and after the drying and measuring the volume of the sample, the moisture content of the soil can be determined.

Neutron Probe

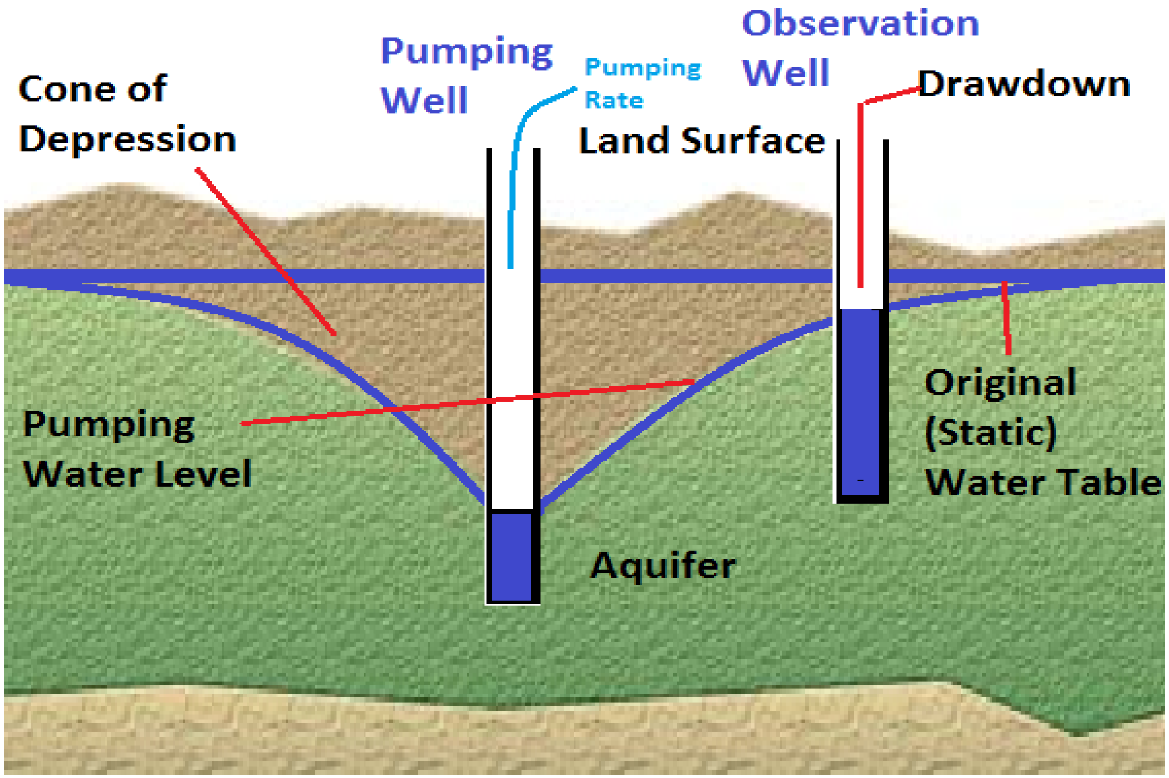
MEASUREMENT OF SUBSURFACE WATER: INFILTRATION

Single Ring Infiltrometer

Double Ring Infiltrometer

MEASUREMENT OF SUBSURFACE WATER: GROUNDWATER LEVEL

Observation Well & Pumping Well



Source: Allwelldrilling (2018)

Observation Well:
A well that is used to observe changes in groundwater levels over a period, or more specifically during a pumping test.

Pumping Well:
Groundwater is accessed for use either by pumping from wells drilled into the aquifer in the subsurface.

MEASUREMENT OF SUBSURFACE WATER: GROUNDWATER HEAD

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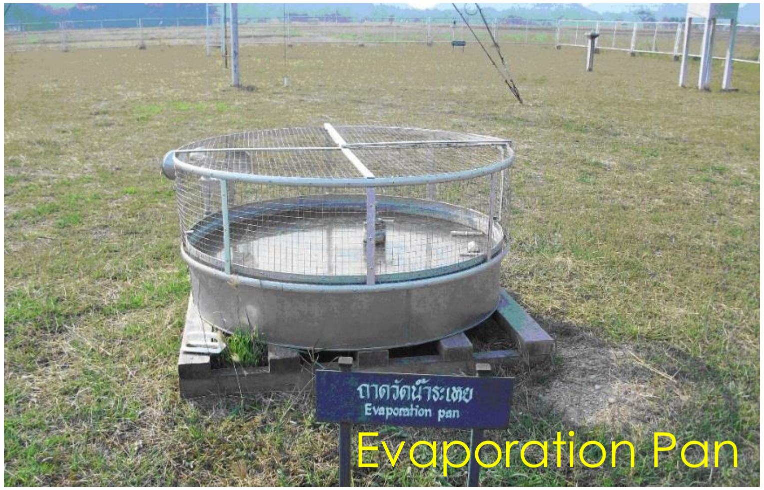
Observation Wells

Observation Well Network:
The primary purpose of the observation well network is to collect, analyze and interpret ground water hydrographs and ground water quality data from various developed aquifers.

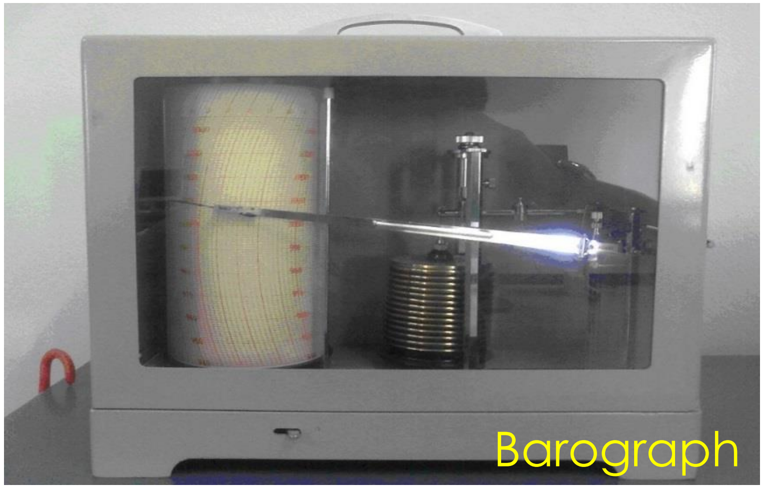
HYDROLOGICAL INSTRUMENTS IN THAILAND

Ordinary Rain Guage



Evaporation Pan

Soil Thermometer


Barograph

HYDROLOGICAL INSTRUMENTS IN THAILAND


Solarimeter

Sunshine Recorder

Thermometer Screen


Climate Station, MET

HYDROLOGICAL INSTRUMENTS IN THAILAND

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Telemetering System, MET

Runoff Station, RID

Online Monitoring Station, WMA

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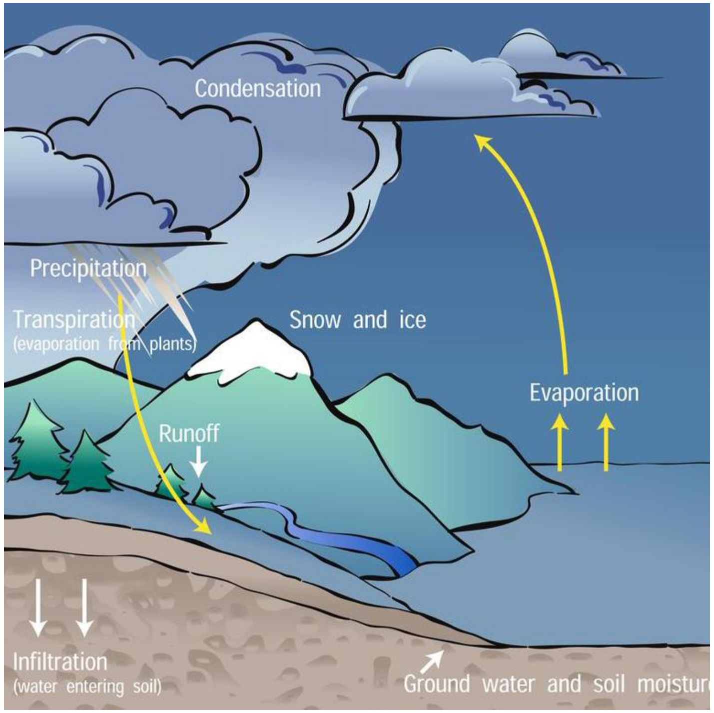
Allwelldrilling. (2018). *Pump testing*. Retrieved July 12, 2018 from http://allwelldrilling.com.au/service/pump-testing/.

Chow, V.T., Maidment, D.R., & Mays, L.W. (1988). *Applied hydrology*. New York: McGraw-Hill Book Company.



LECTURE NOTES
EGCE 323 HYDROLOGY

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Analysis of Unit Hydrograph

- The Unit Hydrograph
- Unit Hydrograph Derivation
- Unit Hydrograph Application
- Synthetic Unit Hydrograph

UNIT HYDROGRAPH: DEFINITION

Unit Hydrograph

- The unit hydrograph is the unit pulse response function of a linear hydrologic system.
- First proposed by Sherman (1932), **the unit hydrograph (originally named unit-graph) of a watershed is defined as a direct runoff hydrograph (DRH) resulting from 1 in (usually taken as 1 cm in SI units) of excess rainfall generated uniformly over the drainage area at a constant rate for an effective duration.**
- Sherman originally used the word "unit" to denote a unit of time. But since that time it has often been interpreted as a unit depth of excess rainfall.
- Sherman classified runoff into surface runoff and groundwater runoff and defined the unit hydrograph for use only with surface runoff.

UNIT HYDROGRAPH: ASSUMPTIONS

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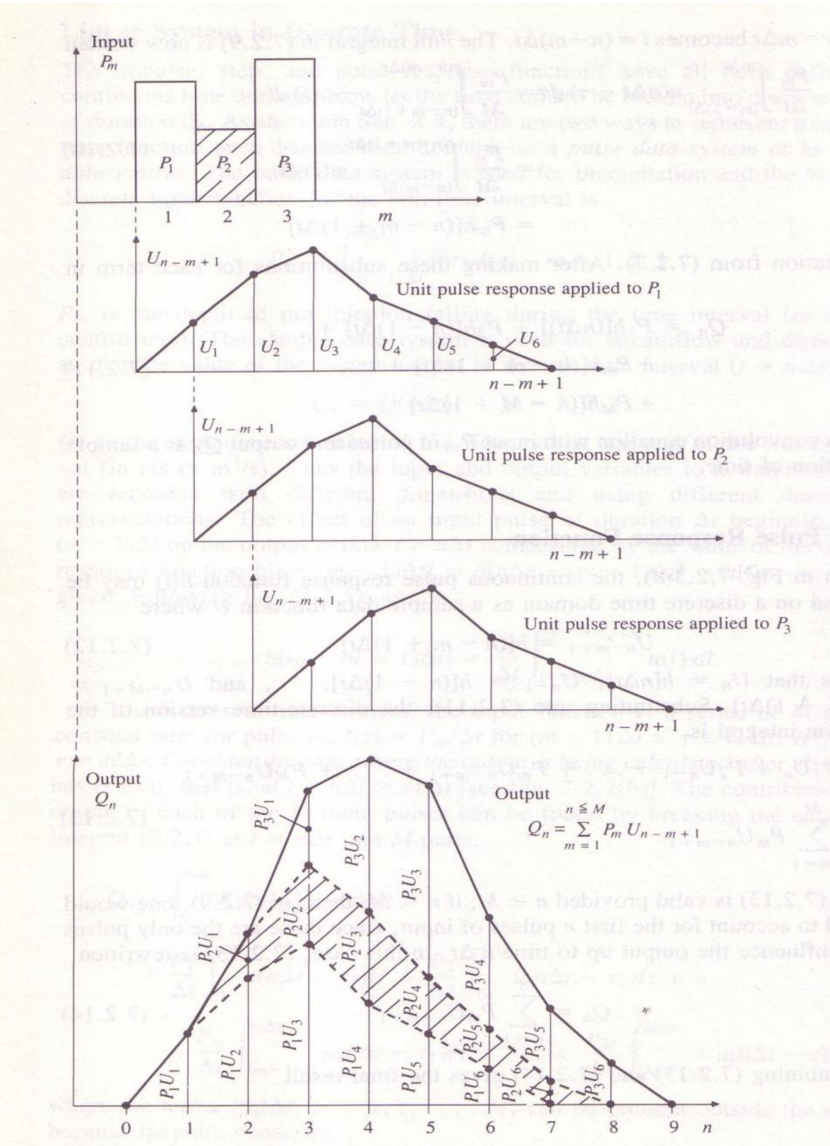
Unit Hydrograph

- The unit hydrograph is a simple linear model that can be used to derive the hydrograph resulting from any amount of excess rainfall. The following basic assumptions are inherent in this model;
    - The excess rainfall has a constant intensity within the effective duration.
    - The excess rainfall is uniformly distributed throughout the whole drainage area.
    - The base time of the DRH (the duration of direct runoff) resulting from an excess rainfall of given duration is constant.
    - The ordinates of all DRH's of a common base time are directly proportional to the total amount of direct runoff represented by each hydrograph.
    - For a given watershed, the hydrograph resulting from a given excess rainfall reflects the unchanging characteristics of the watershed.

UNIT HYDROGRAPH DERIVATION: DISCRETE CONVOLUTION EQUATION



$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

When Q_n = Direct runoff

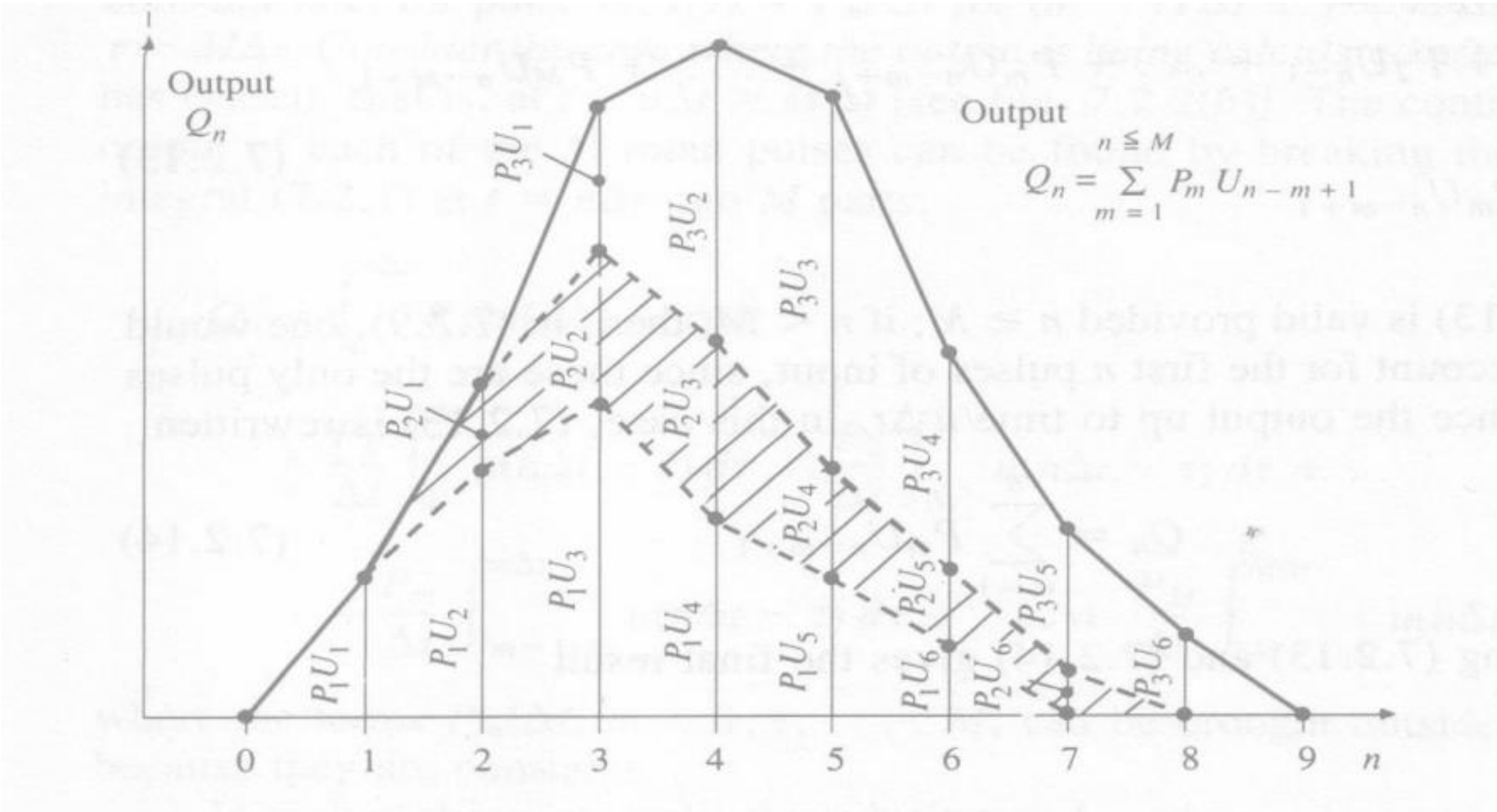
P_m = Excess rainfall

U_{n-m+1} = Unit hydrograph

Suppose that there are M pulses of excess rainfall N pulses of direct runoff in the storm considered, then N equations can be written for Q_n = 1, 2, …,N in terms of N-M+1unknown values of unit hydrograph.

Source: Chow et al. (1988)

UNIT HYDROGRAPH DERIVATION: DISCRETE CONVOLUTION EQUATION



UNIT HYDROGRAPH DERIVATION: DISCRETE CONVOLUTION EQUATION

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The set of equations for discrete time convolution

$$Q_1 = P_1U_1$$

$$Q_2 = P_2U_1 + P_1U_2$$

$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3$$

$$\vdots$$

$$Q_M = P_MU_1 + P_{M-1}U_2 + \ldots.. + P_1U_M$$

$$Q_{M+1} = 0 + P_MU_2 + \ldots.. + P_2U_M + P_1U_{M+1}$$

$$\vdots$$

$$Q_{N-1} = 0 + 0 + \ldots.. + 0 + 0 + \ldots.. + P_MU_{N-M} + P_{M-1}U_{N-M+1}$$

$$Q_{N-1} = 0 + 0 + \ldots.. + 0 + 0 + \ldots.. + 0 + P_{M-1}U_{N-M+1}$$

$$Q_n = \sum_{m=1}^{n \le M} P_mU_{n-m+1}$$

$n = 1, 2, \ldots, N$

DISCRETE CONVOLUTION EQUATION: EXAMPLE 1

Find the half-hour unit hydrograph using the excess rainfall hyetograph and direct runoff hydrograph given in the table.

Solution

The ERH and DRH in table have M=3 and N=11 pulses respectively.

Hence, the number of pulses in the unit hydrograph is N-M+1=11-3+1=9.

Substituting the ordinates of the ERH and DRH into the equations in table yields a set of 11 simultaneous equations.

Time (1/2hr)	Excess Rainfall (in)	Direct Runoff (cfs)
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

DISCRETE CONVOLUTION EQUATION: EXAMPLE 1

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

$$U_1 = \frac{Q_2 - P_2U_1}{P_1} = \frac{1{,}928 - 1.93\text{x}404}{1.06} = 1{,}079 \text{ cfs/in}$$

$$U_3 = \frac{Q_3 - P_3U_1 - P_2U_2}{P_1} = \frac{5{,}297 - 1.81\text{x}404 - 1.93\text{x}1{,}079}{1.06} = 2{,}343 \text{ cfs/in}$$

and similarly for the remain ordinates

$$U_4 = \frac{9{,}131 - 1.81\text{x}1{,}079 - 1.93\text{x}2{,}343}{1.06} = 2{,}506 \text{ cfs/in}$$

$$U_5 = \frac{10{,}625 - 1.81\text{x}2{,}343 - 1.93\text{x}2{,}506}{1.06} = 1{,}460 \text{ cfs/in}$$

$$U_6 = \frac{7{,}834 - 1.81\text{x}2{,}506 - 1.93\text{x}1{,}460}{1.06} = 453 \text{ cfs/in}$$

DISCRETE CONVOLUTION EQUATION: EXAMPLE 1

$$U_7 = \frac{3,921 - 1.81x1,460 - 1.93x453}{1.06} = 381\ cfs/in$$

$$U_8 = \frac{1,846 - 1.81x453 - 1.93x381}{1.06} = 274\ cfs/in$$

$$U_9 = \frac{1,402 - 1.81x381 - 1.93x274}{1.06} = 173\ cfs/in$$

Unit hydrograph

n	1	2	3	4	5	6	7	8	9
U_n (cfs/in)	404	1,079	2,343	2,506	1,460	453	381	274	173

UNIT HYDROGRAPH APPLICATION

Application of Unit Hydrograph

Once the unit hydrograph has been determined, it may be applied to direct runoff and streamflow hydrograph.

Procedures

- A rainfall hyetograph is selected.
- The abstractions are estimated.
- The excess rainfall is calculated.
- The time interval used in defining the excess rainfall hyetograph ordinates must be the same as that for which the unit hydrograph was specified.
- The discrete convolution equation may then be used to yield the direct runoff hydrograph.
- By adding an estimated baseflow to the direct runoff hydrograph, the streamflow hydrograph is obtained.

DISCRETE CONVOLUTION EQUATION: EXAMPLE 2

Calculate the streamflow hydrograph for a storm of 6 in excess rainfall, with 2 in the first half-hour, 3 in in the second half-hour and 1 in in the third half-hour. Use the half-hour unit hydrograph computed in example 1 and assume the baseflow is constant at 500 cfs throughput the flood. Check that the total depth of direct runoff is equal to the total excess precipitation. (Watershed area = 7.03 mi^2)

Solution

$$Q_1 = P_1U_1 = 2.00x404 = 808\,cfs$$

$$Q_2 = P_2U_1 + P_1U_2 = 3.00x404 + 2.00x1{,}079 = 1{,}212 + 2{,}158 = 3{,}370\,cfs$$

$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3 = 1.00x404 + 3.00x1{,}079 + 2.00x2{,}343$$
$$= 404 + 3{,}237 + 4{,}686 = 8{,}327\,cfs$$

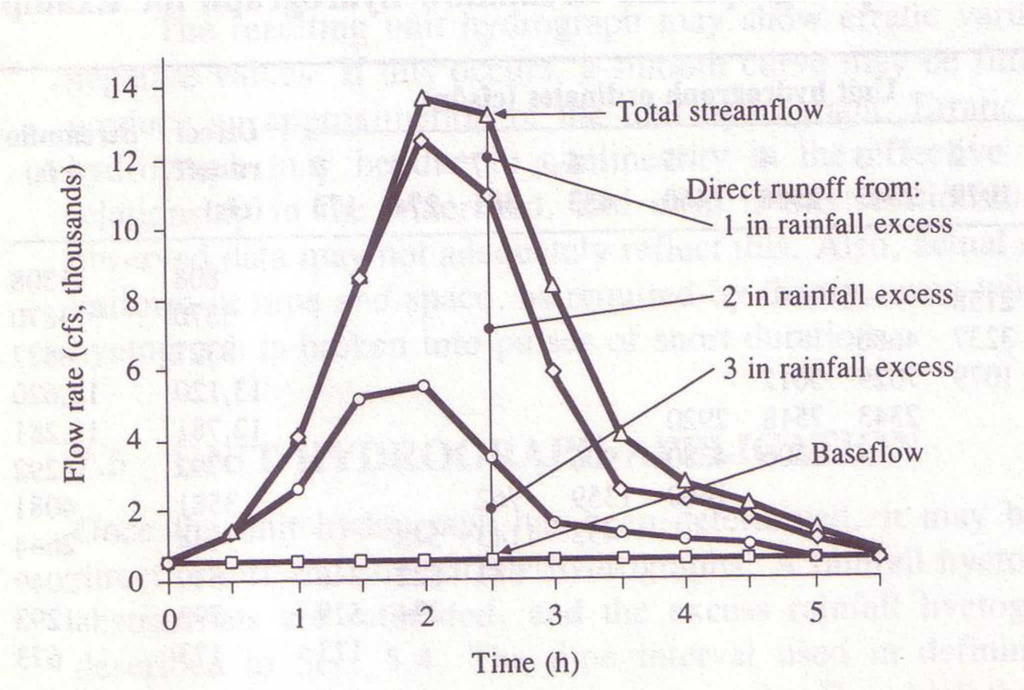
DISCRETE CONVOLUTION EQUATION: EXAMPLE 2

Calculation of the direct runoff hydrograph and streamflow hydrograph

Time (1/2 hr)	Excess Precipitation (in)	Unit Hydrograph Ordinates (cfs/in)									Direct Runoff (cfs)	Stream flow (cfs)
		1	2	3	4	5	6	7	8	9		
		404	1079	2343	2506	1460	453	381	274	173		
N=1	2.00	808									808	1308
2	3.00	1212	2158								3370	3870
3	1.00	404	3237	4686							8327	8827
4			1079	7029	5012						13120	13620
5				2343	7518	2920					12781	13281
6					2506	4380	906				7792	8292
7						1460	1359	762			3581	4081
8							453	1143	548		2144	2644
9								381	822	346	1549	2049
10									274	519	793	1293
11										173	173	673

Baseflow = 500 cfs

DISCRETE CONVOLUTION EQUATION: EXAMPLE 2



The total direct runoff volume =

$$V_d = \sum_{n=1}^{N} Q_n \Delta t$$

$$= 54,438\text{x}0.5 \text{ cfs.h}$$

$$= 54,438\text{x}0.5 \ \frac{ft^3.h}{s} \text{x} \frac{3,600\,s}{1s}$$

$$= 9.80\text{x}10^7 \ ft^3$$

The corresponding depth of direct runoff is found by dividing by the watershed area A=7.03 mi2=7.03x5280 ft^2=1.96x108 ft^2

$$r_d = \frac{V_d}{A} = \frac{9.80\text{x}10^7}{1.96\text{x}10^8} \text{ ft} = 0.50 \text{ ft} = 6.00 \text{ in}$$

UNIT HYDROGRAPH VS SYNTHETIC UNIT HYDROGRAPH

Unit Hydrograph (UH)
developed from rainfall and streamflow data on a watershed applies only for that watershed and for the point on the stream where the streamflow data were measured.

Synthetic Unit Hydrograph (SUH)
Synthetic unit hydrograph procedures are used to develop unit hydrographs for other locations on the stream in the same watershed or for nearby watersheds of a similar character.

SYNTHETIC UNIT HYDROGRAPH

Synthetic Unit Hydrograph (SUH)

There are three types of synthetic unit hydrograph:

- Those relating hydrograph characteristics (peak flow rate, base time, etc.) to watershed characteristics (Snyder, 1938).
- Those based on a dimensionless unit hydrograph (Soil Conservation Service, 1972).
- Those based on models of watershed storage (Clark, 1943).

SYNTHETIC UNIT HYDROGRAPH: SNYDER'S UH

Snyder's UH

Snyder defined a standard unit hydrograph as one whose rainfall duration t_r is related to the basin lag t_p by

$$t_p = 5.5 t_r$$

For a standard unit hydrograph Snyder found that

$$t_p = C_1 C_t (L L_c)^{0.3}$$

The basin lag is

tp = the basin lag (hr)
L = Length of the main stream from the outlet to the upstream (km)
Lc = The distance from the outlet to a point on the stream nearest the centroid of watershed area.
C1 = 0.75
Ct = Coefficient derived from gaged watersheds in the same region.

SYNTHETIC UNIT HYDROGRAPH: SNYDER'S UH

The peak discharge per unit drainage area in $m^3/s.km^2$ of the standard unit hydrograph is

$$q_p = \frac{C_2 C_p}{t_p}$$

→ C_2 = 2.75 and C_p = Coefficient derived from gaged watersheds in the same region.

If $t_{pR}=5.5t_R$ → $t_R=t_r$, $t_{pR}=t_p$, $q_{pR}=q_p$

c_t, c_p are computed from $t_p = C_1 C_t (LL_c)^{0.3}$

If $t_{pR} \neq 5.5t_R$ → $$t_p = t_{pR} + \frac{t_r - t_R}{4}$$

$t_{pR}=t_p$, $q_{pR}=q_p$

c_t, c_p are computed from $t_p = C_1 C_t (LL_c)^{0.3}$

SYNTHETIC UNIT HYDROGRAPH: SNYDER'S UH

The relationship between q_p and the peak discharge per unit drainage area q_{pR} of the required unit hydrograph is

$$q_{pR} = \frac{q_p t_p}{t_{pR}}$$

The base time t_b in hours of the unit hydrograph can be determined using the fact that the area under the unit hydrograph is equivalent to a direct runoff of 1 cm. Assuming a triangular shape for the unit hydrograph, the base time may be estimated by

$$t_b = \frac{C_3}{q_{pR}}$$

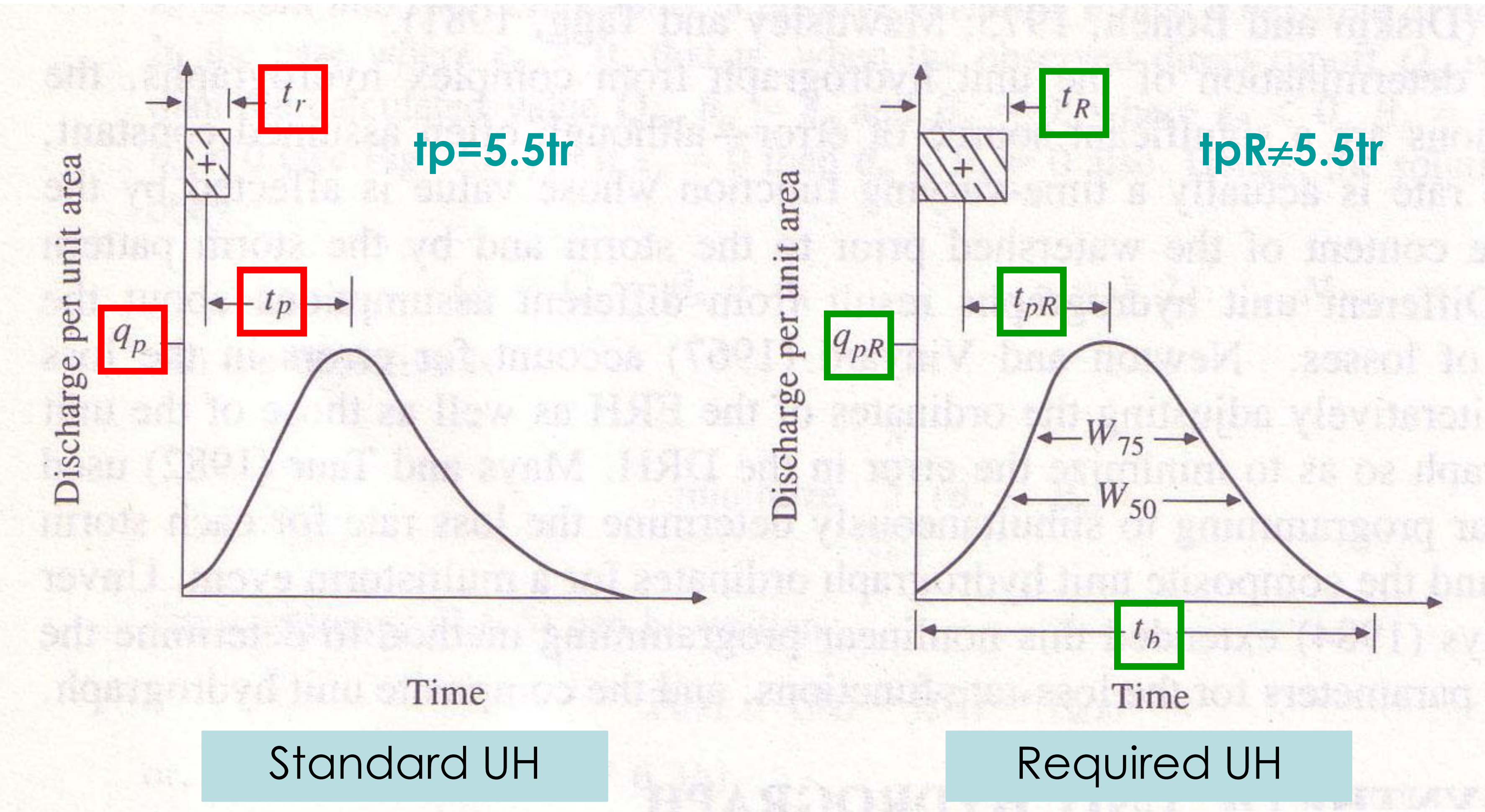
$$C_3 = 5.56$$

The width in hours of a unit hydrograph at a discharge equal to a certain percent of the peak discharge q_{pR} is given by

$$W = C_w q_{pR}^{-1.08}$$

C_w = 1.22 for the 75 percent width and 2.14 for the 50 percent width.

SYNTHETIC UNIT HYDROGRAPH: SNYDER'S UH



SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 3

From the basin map of a given watershed, the following quantities are measured: L = 150 km, L_c = 75 km, and drainage area = 3,500 km^2. From the unit hydrograph derived for the watershed, the following are determined: t_R = 12 hr, t_{pR} = 34 hr, and peak discharge = 157.5 m^3/s.cm. Determine the coefficients C_t and C_p for the synthetic unit hydrograph of the watershed.

Solution From the given data, $5.5t_R$ = 66 hr, which is quite different from t_{pR}. Thus

$$t_p = t_{pR} + \frac{t_r - t_R}{4} = 34 + \frac{t_r - 12}{4}$$ **

Solving * and ** simultaneously gives t_p = 32.5 hr and t_r = 5.9 hr.
To calculate C_t, use

$$t_p = C_1 C_t (L L_c)^{0.3}$$

$$32.5 = 0.75 C_t (150 \times 75)^{0.3}$$ $$C_t = 2.65$$

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 3

The peak discharge per unit area is

$$q_{pR} = 157.5/3500$$
$$= 0.045 m^3/s \cdot km^2 \cdot cm$$

The coefficient is calculated by

$$q_p, q_{pR} \text{ and } t_p = t_{pR}$$

$$q_{pR} = \frac{C_2 C_p}{t_{pR}}$$

$$0.045 = \frac{2.75 C_p}{34.0} \qquad C_p = 0.56$$

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 4

Compute the six-hour synthetic unit hydrograph of a watershed having a drainage area of 2,500 km^2 with L = 100 km and L_c = 50 km. This watershed is a sub-drainage area of the watershed in Example 3.

Solution

Standard UH

$C_t = 2.64$ and $C_p = 0.56$

$t_p = 0{,}75 \times 2.64 \times (100 \times 50)^{0.3} = 25.5$ hr ➔ $t_p = C_1 C_t (L L_c)^{0.3}$

$t_r = 25.5/5.5 = 4.64$ hr ➔ $t_p = 5.5 t_r$

$q_p = 2.75 \times 0.56/25.5 = 0.0604$ m^3/s.km^2.cm ➔ $q_p = \dfrac{C_2 C_p}{t_p}$

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 4

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Required UH (6 hr UH)

t_R = 6 hr

t_{pR} = 25.5-(4.64-6)/4 = 25.8 hr ➔ $t_p = t_{pR} + \frac{t_r - t_R}{4}$

q_{pR} = 0.0604x25.5/25.8 = 0.0597 m³/s.km².cm ➔ $q_{pR} = \frac{q_p t_p}{t_{pR}}$

Peak discharge = 0.0597x2,500 = 149.2 m³/s.cm

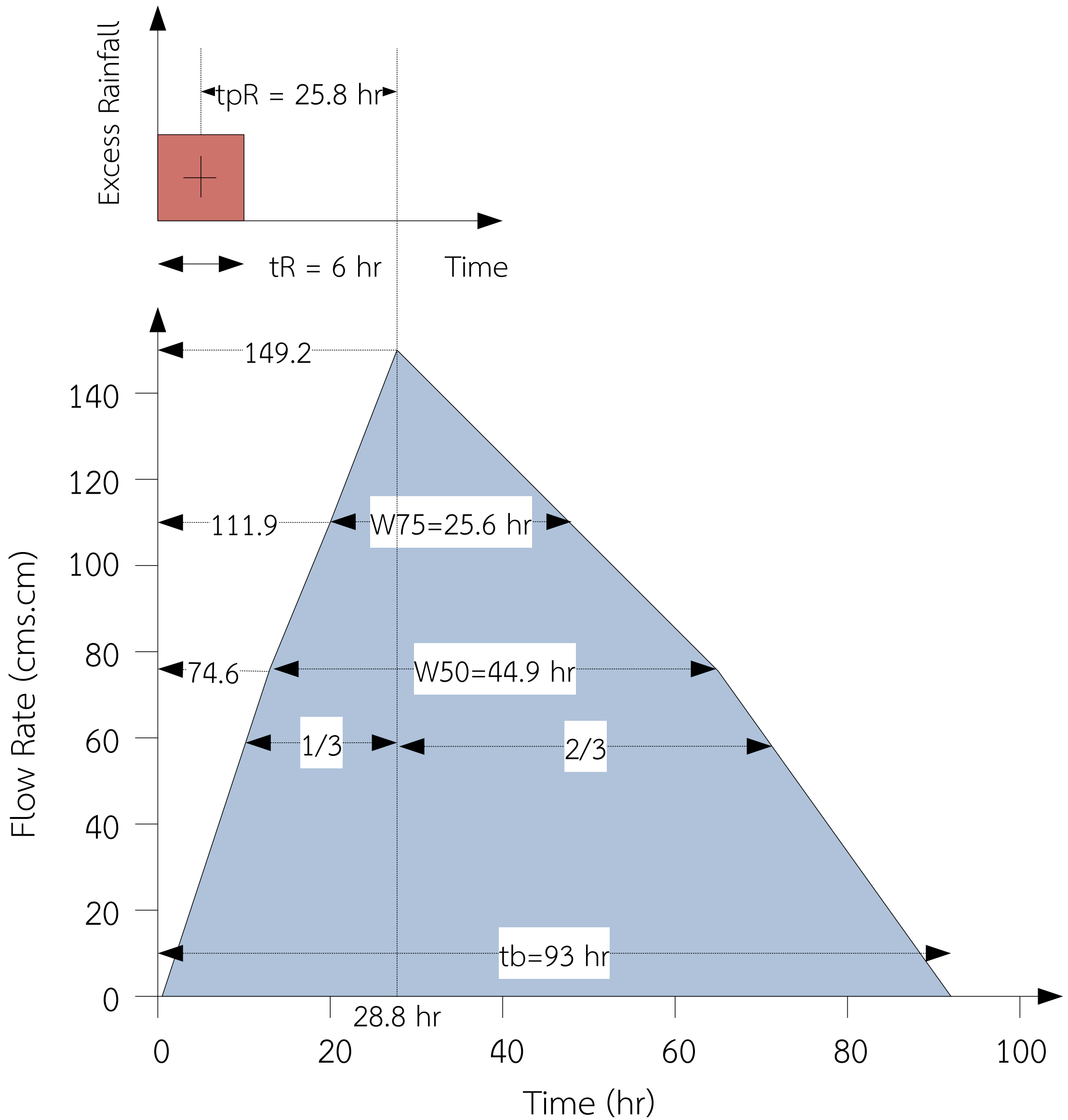
Width of UH

At 75% of q_{pR}, $W_{75\%}$ = 1.22x0.0597$^{-1.08}$ = 25.6 hr ➔ $W = C_w q_{pR}^{-1.08}$

At 50% of q_{pR}, $W_{50\%}$ = 2.14x0.0597$^{-1.08}$ = 44.9 hr

t_b = 5.56/0.0597 = 93 hr ➔ $t_b = \frac{C_3}{q_{pR}}$

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 4



SYNTHETIC UNIT HYDROGRAPH: SCS DIMENSIONLESS UH

SCS Diminsionless UH

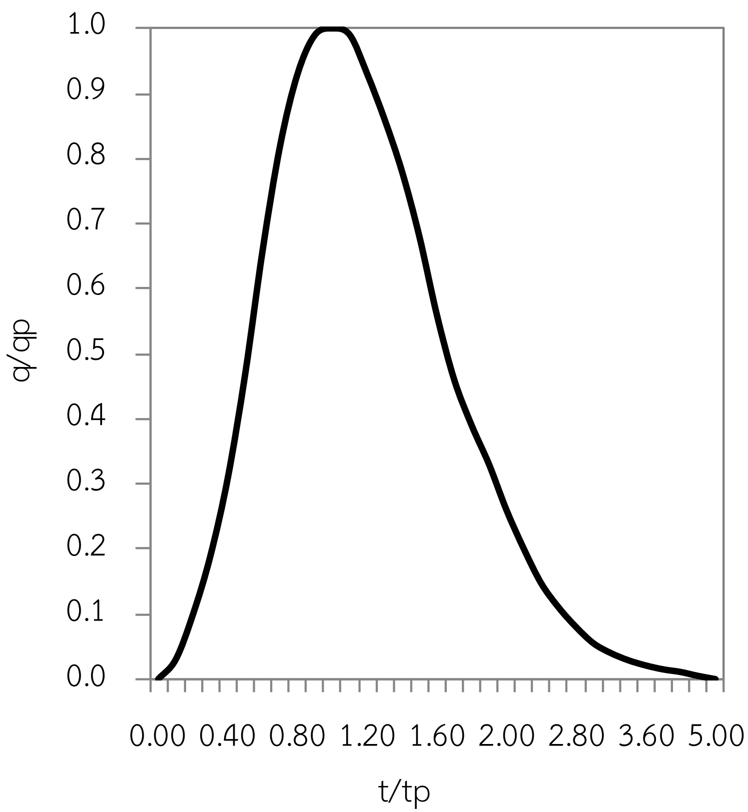
The SCS-UH is a synthetic unit hydrograph in which the discharge is expressed by

- The ratio of discharge: discharge (q) to peak discharge (q_p)
- The time: the ratio of the time (t) to the time of rise of the unit hydrograph (T_p)

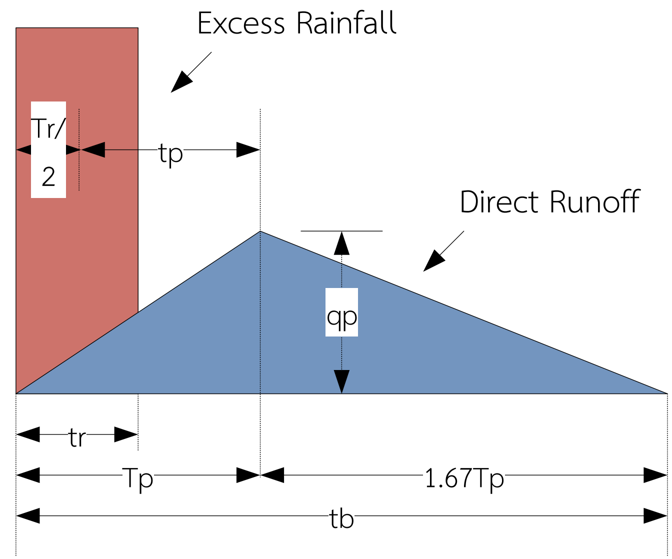
Given the peak discharge and lag time for the duration of excess rainfall, the UH can be estimated from the synthetic dimensionless hydrograph for the given basin.

SYNTHETIC UNIT HYDROGRAPH: SCS DIMENSIONLESS UH

Diminsionless UH



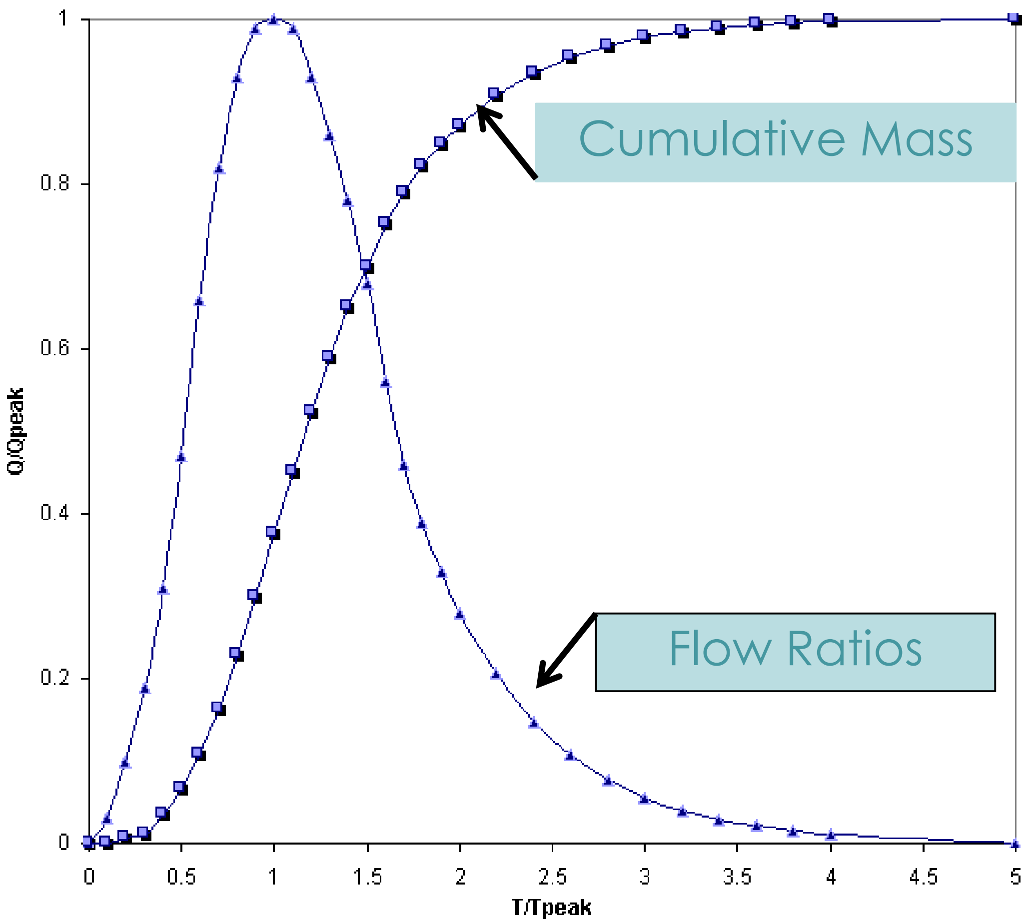
Triangular UH



The figure shows a dimensionless hydrograph, prepared from the unit hydrographs of variety of watersheds. The values of q_p and T_p may be estimated using a simplified model of triangular unit hydrograph.

SYNTHETIC UNIT HYDROGRAPH: SCS DIMENSIONLESS UH

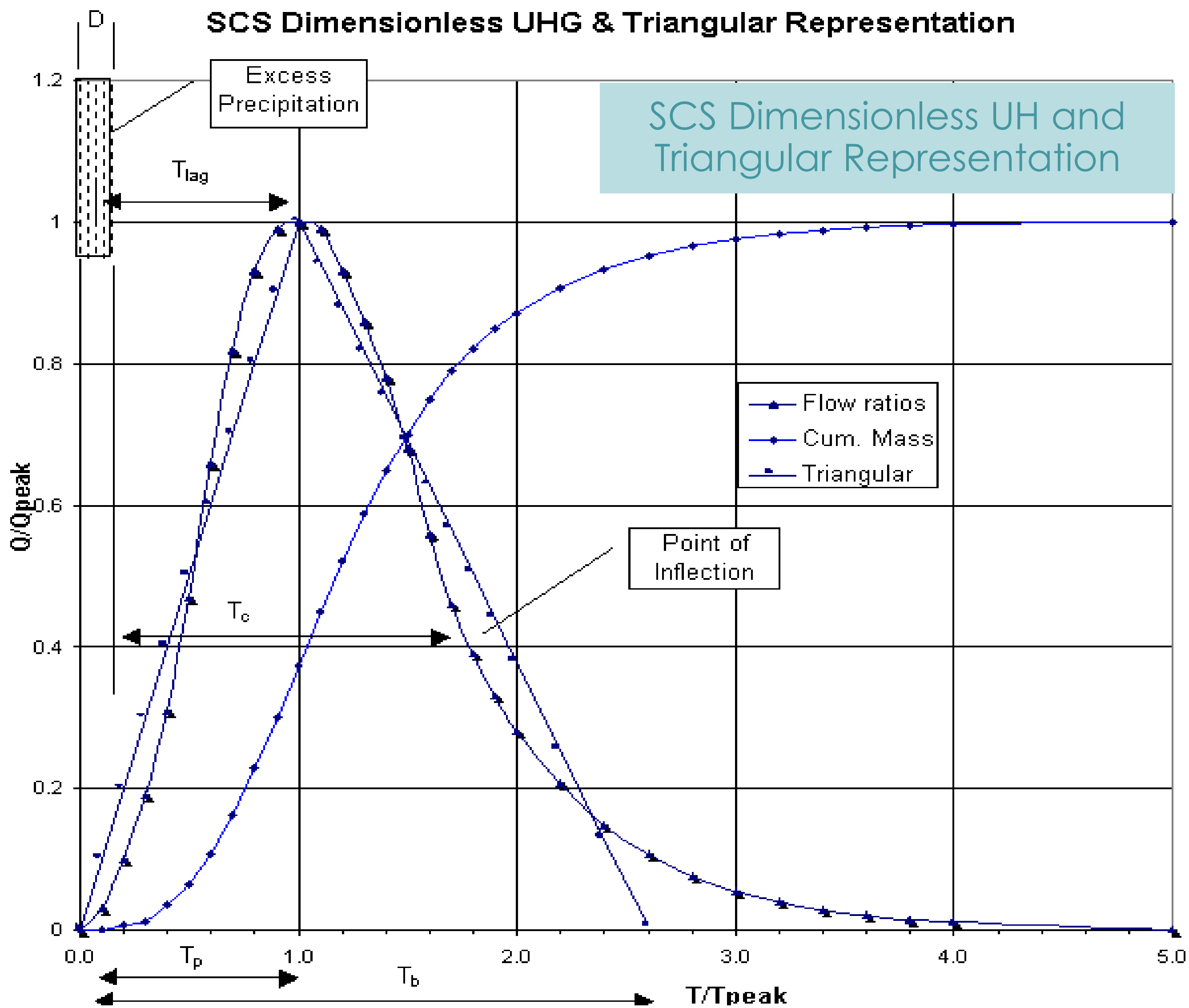
SCS Diminsionless UH



Ratios for dimensionless unit hydrograph and mass curve.

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)	Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0.0	0.000	0.000	1.6	0.560	0.751
0.1	0.030	0.001	1.7	0.460	0.790
0.2	0.100	0.006	1.8	0.390	0.822
0.3	0.190	0.012	1.9	0.330	0.849
0.4	0.310	0.035	2.0	0.280	0.871
0.5	0.470	0.065	2.2	0.207	0.908
0.6	0.660	0.107	2.4	0.147	0.934
0.7	0.820	0.163	2.6	0.107	0.953
0.8	0.930	0.228	2.8	0.077	0.967
0.9	0.990	0.300	3.0	0.055	0.977
1.0	1.000	0.375	3.2	0.040	0.984
1.1	0.990	0.450	3.4	0.029	0.989
1.2	0.930	0.522	3.6	0.021	0.993
1.3	0.860	0.589	3.8	0.015	0.995
1.4	0.780	0.650	4.0	0.011	0.997
1.5	0.680	0.700	4.5	0.005	0.999
			5.0	0.000	1.000

SYNTHETIC UNIT HYDROGRAPH: SCS DIMENSIONLESS UH



SCS Dimensionless UH and Triangular Representation

SCS Diminsionless UH

- SCS suggests the time of recession may be approximated as $1.67T_p$.
- As the area under the unit hydrograph should be equal to a direct runoff of 1 cm.

$$q_p = \frac{CA}{T_p}$$

T_p = peak time, hr
q_p = peak discharge, cms.m
c = 2.08
A = the drainage area, sq.km.

- A study of unit hydrographs of many large and small rural watersheds indicates that the basin lag

$$t_p \;;\; 0.6t_c$$

t_c = time of concentration of watersheds

- Time to rise, T_p can be expressed in terms of lag time, t_p and the duration of effective rainfall, t_r

$$T_p = \frac{t_r}{2} + t_p$$

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 5

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Construct a 10-minute SCS-UH for a basin of area 3.0 km2 and time of concentration 1.25 hr.

Solution The duration t_r = 10 min = 0.166 hr

Lag time t_p = 0.6Tc = 0.6x1.25 = 0.75 hr

Rise time T_p = tr/2 + tp = 0.166/2 + 0.75 = 0.833 hr

q_p = 2.08x3.0/0.833 = 7.49 m3/s.cm → $q_p = \frac{CA}{T_p}$

The dimensionless hydrograph in the figure may be converted to the required dimensions by multiplying the values on the horizontal axis by T_p and those on the vertical axis by qp. Alternatively, the triangular unit hydrograph can be drawn with $t_b = 2.67T_p$ = 2.22 hr. The depth of direct runoff is checked to equal 1 cm.

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 5

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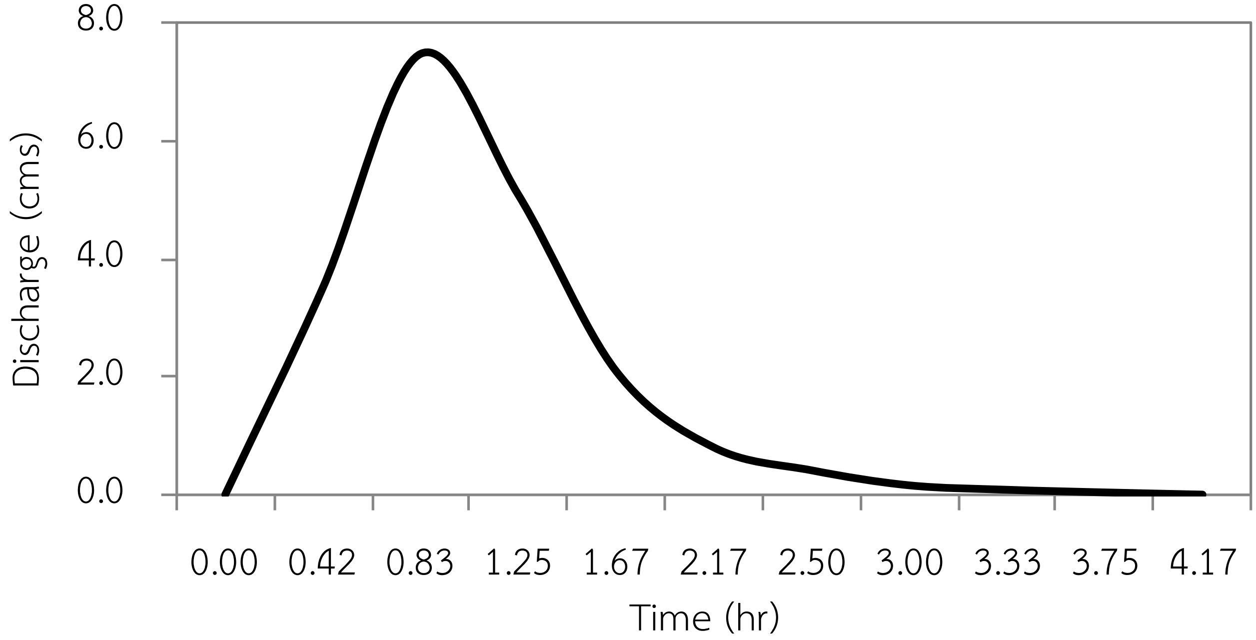
Unit Hydrograph

t/Tp	T (hr)	Q/Qp	Q (cms)
0	0	0	0
0.5	0.417	0.470	3.520
1	0.833	1.000	7.49
1.5	1.250	0.680	5.093
2	1.666	0.280	2.097
2.6	2.166	0.107	0.801
3	2.499	0.055	0.412
3.6	2.999	0.021	0.157
4	3.332	0.011	0.082
4.5	3.749	0.005	0.037
5	4.165	0	0

SYNTHETIC UNIT HYDROGRAPH: EXAMPLE 5



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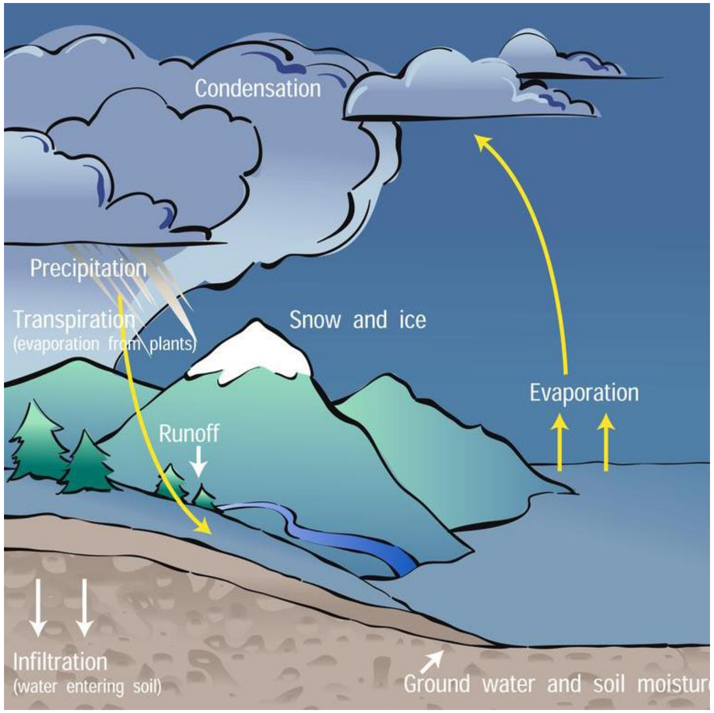

LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

LECTURE OUTLINE

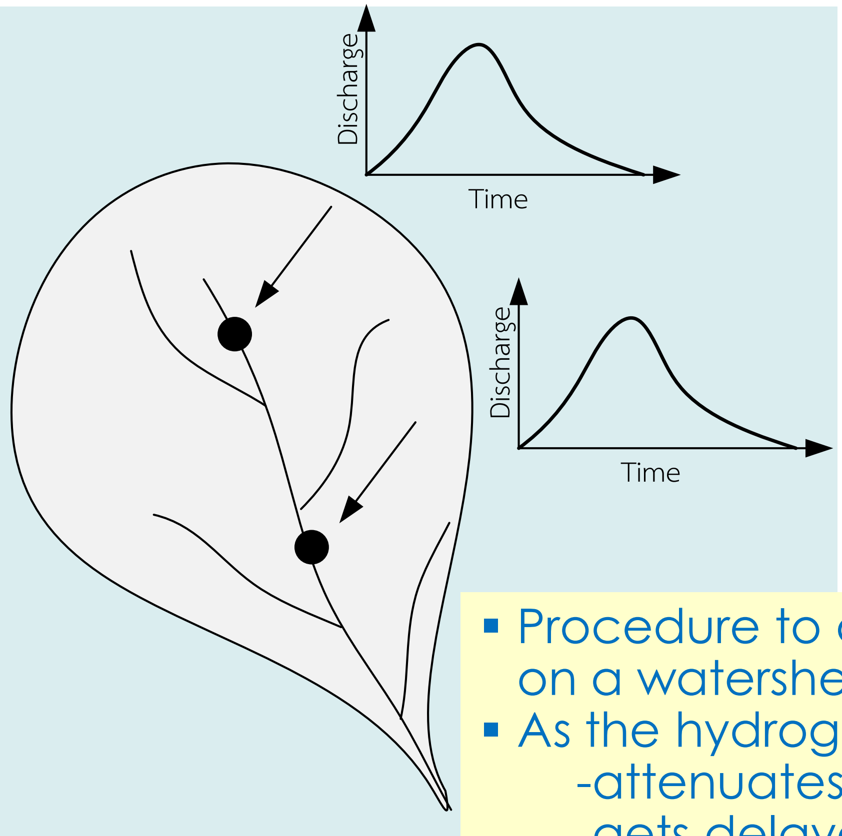
Flow/Flood Routing Analysis

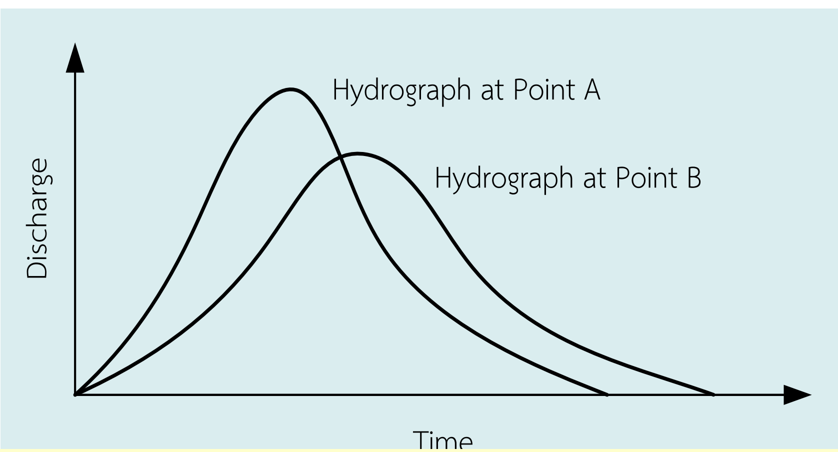
- Channel Routing
- Reservoir Routing

FLOW ROUTING

Flow Routing

Flow routing is the process of predicting temporal and spatial variation of a flood wave as it travels through a river (or channel reach or reservoir).





- Procedure to determine the flow hydrograph at a point on a watershed from a known hydrograph upstream.
- As the hydrograph travels, it
  - -attenuates
  - -gets delayed

Source: Chow et al. (1988)

FLOW ROUTING

Types of Flow Routing

Two types of routing can be performed:

Hydrologic Routing (Lumped Routing)

- Flow is calculated as a function of time alone at a particular location.
- Governed by continuity equation and flow/storage relationship.

Hydraulic Routing (Distributed Routing)

- Flow is calculated as a function of space and time throughout the system.
- Governed by continuity and momentum equations.

HYDROLOGIC ROUTING

Hydrologic Routing
In hydrologic routing techniques, the equation of continuity and some linear or curvilinear relation between storage and discharge within the river or reservoir is used.

Applications of routing techniques:

- Flood predictions
- Evaluation of flood control measurements
- Assessment of effects of urbanization
- Flood warning
- Spillway design for dams

Hydrologic Routing

Continuity Equation:

$$I - O = \frac{\Delta S}{\Delta t}$$

Where

I = Inflow

O = Outflow

$\Delta S/\Delta t$ = Rate of change of storage

Problem:

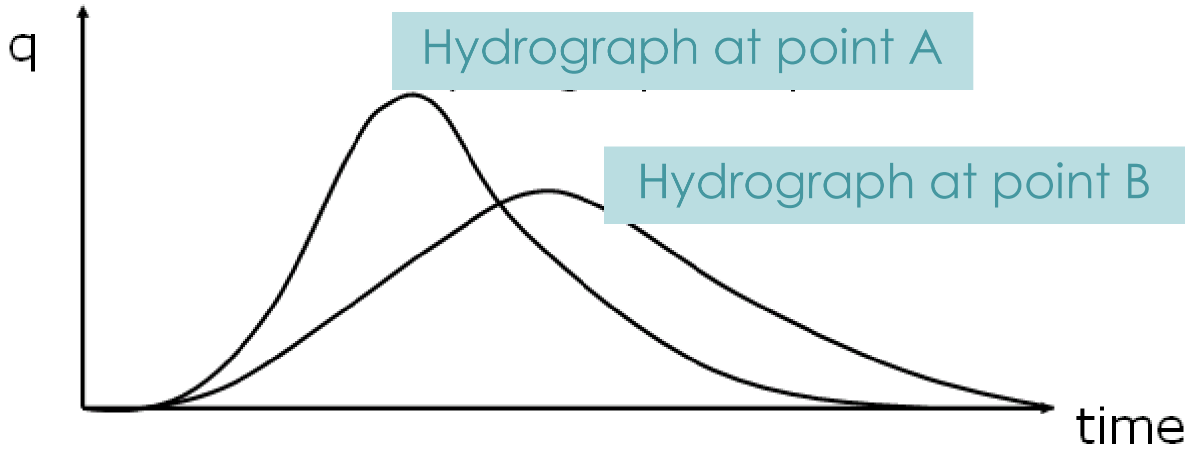
You have a hydrograph at one location (I)

You have river characteristics (S=f(I,O))

Need:

A hydrograph at different location (O)

Hydrologic Routing



The hydrograph at B is attenuated due to storage characteristics of the stream reach.
Assumption: no seepage, leakage, evaporation, or inflow from the sides.

HYDROLOGIC ROUTING

Lumped Flow Routing

Three famous types of flow routing technique;

(1) Level Pool Method (Modified Puls)

Procedure for calculating outflow hydrograph Q(t) from a reservoir with horizontal water surface, given its inflow hydrograph I(t) and storage-outflow relationship. Storage is nonlinear function of Q.

(2) Muskingum Method

Muskingum method was developed for hydrologic river routing. Storage is linear function of I and Q.

(3) Series of Reservoir Models

Storage is linear function of Q and its time derivatives.

HYDROLOGIC ROUTING: CHANNEL ROUTING

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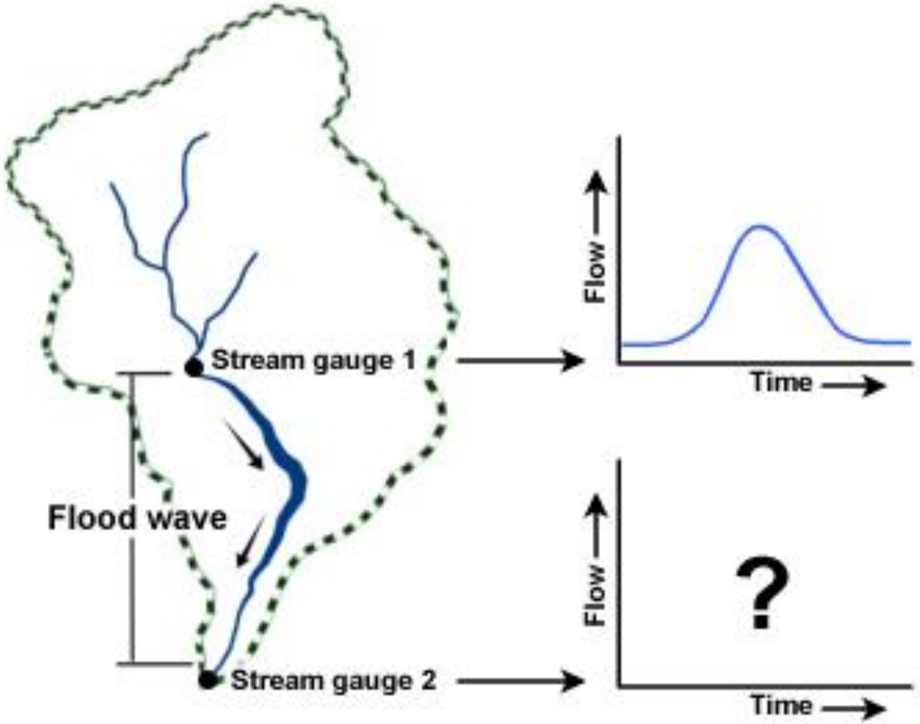
Chanel Routing

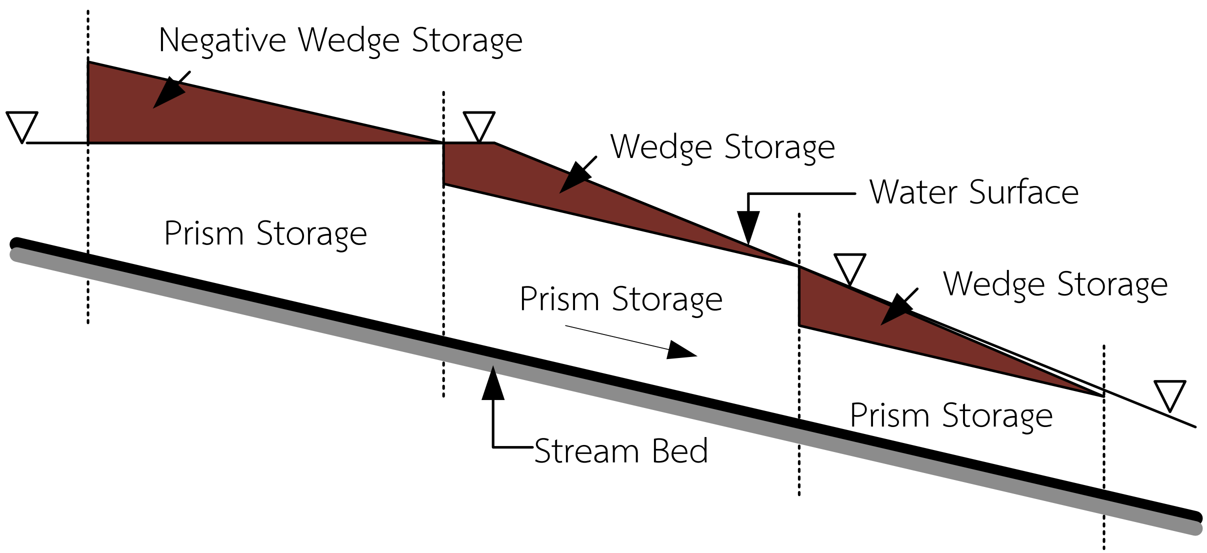
- Channel routing simulates the movement of water through a channel.
- It is used to predict the magnitudes, volumes, and temporal patterns of the flow (often a flood wave) as it translates down a channel.

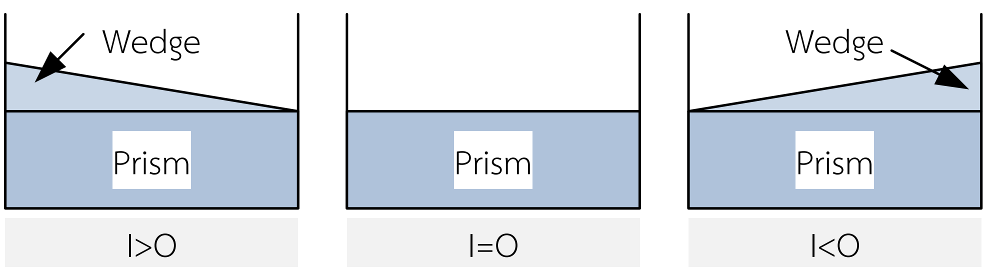
Channel Routing Methods

- Muskingum Method
- Muskingum-Cunge Method



Muskingum Method: Flow in a Channel





Storage in wedge: $KX(I-O)$

Storage in prism: KO

So, Storage: $S=KX(I-O)+KO$

HYDROLOGIC ROUTING: CHANNEL ROUTING

Muskingum Method

Storage $S=KO+KX(I-O)$ rewritten as

$$S=K[XI+(1-X)O]$$

Where

S = Storage in the river reach

K = Storage time constant (T)

X = A weighting factor that varies between 0 and 0.5 (defines relative importance of inflow and outflow on storage)

If X=0.5 pure translation, if X=0 max attenuation

HYDROLOGIC ROUTING: CHANNEL ROUTING

Muskingum Method

How it works:

Write continuity equation as

$$\bar{I} - \bar{O} = \frac{\Delta S}{\Delta t}$$

Where

$\bar{I}$ = Average inflow during Δt

$\bar{O}$ = Average outflow during Δt

or

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

$$S = k[XI - (1 - X)O]$$

Combine and rearrange

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{K}{\Delta t}[X(I_2 - I_1) + (1 - X)(O_2 - O_1)]$$

Simplified into the routing equation:

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 I_0$$

Subscript 1 refers to t_1 and 2 to t_2 = $(t + \Delta t)$

Muskingum Method

$$C_0 = \frac{-KX + 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_1 = \frac{KX + 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_2 = \frac{K - KX - 0.5\Delta t}{K - KX + 0.5\Delta t}$$

$$C_0 + C_1 + C_2 = 1$$

Need K and Δt in the same units

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Estimation of K, X and Δt

K is estimated to be the travel time through the reach. This may pose somewhat of a difficulty, as the travel time will obviously change with flow. The travel time may be estimated using the kinematic travel time or a travel time based on Manning's equation.

$K=0.6L/v_{avg}$

Where

- L = Length of river reach
- V_{avg} = Average velocity in reach

Constraint K<tp/5 (divide reach up if needed)

- X = 0.2 for most cases
- X = 0.4 for steep channels with narrow flood plains
- X = 0.1 for mild channels with broad flood plains

$2KX<\Delta t<2K(1-X)$ and ideally $\Delta t<tp/5$.

Choose Δt in numbers that divide into 24 (daily data)

HYDROLOGIC ROUTING: EXAMPLE 1

Tp = 4 hr, L = 2 mi, v_{avg} = 2.5 ft/s, wide flat floodplain. Estimate K, X and Δt.

Solution

$K = 0.6L/v_{avg} = 0.6(2x5280)/2.5=2,534$ sec = 0.7 hr

X = 0.1

Δt:

2KX = 2(0.7)0.1 = 0.14

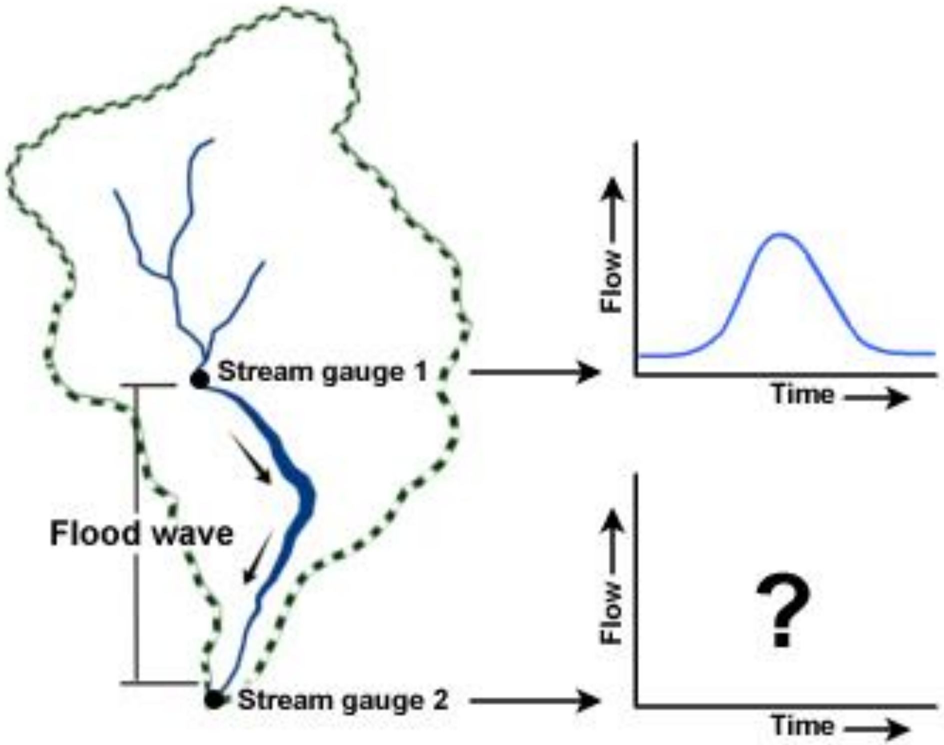
2K(1-X) = 2(0.7)0.9 = 1.26

0.14<Δt<1.26 and Δt<tp/5 or Δt<0.8 hr,

so Δt = 0.5 hr is most accurate.

CHANNEL ROUTING: EXAMPLE 2

Channel Routing in Excel Spreadsheet



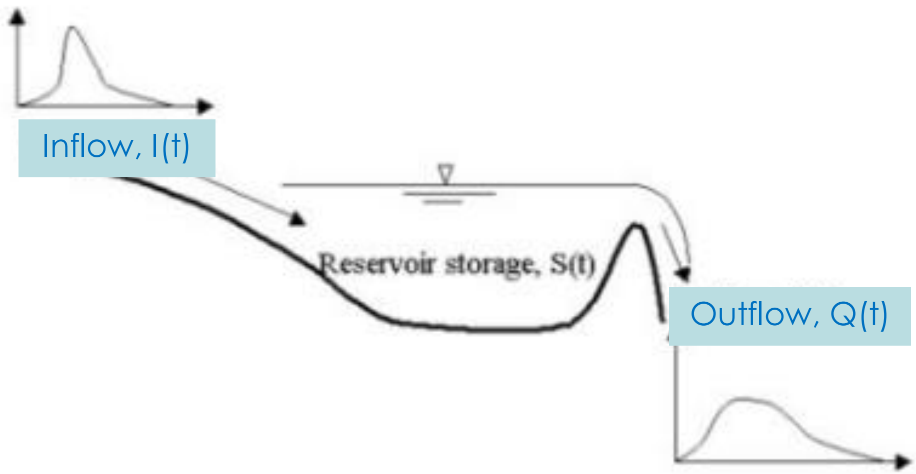
HYDROLOGIC ROUTING: RESERVOIR ROUTING

Reservoir Routing

- Reservoir routing is used to determine the peak flow attenuation that a hydrograph undergoes as it enters a reservoir.
- Reservoir acts to store water and release through control structure later.

Reservoir Routing Methods

- Inflow-Storage-Discharge Curve Method (Puls Method)
- Storage-Indication Method (Modified Puls Method)



HYDROLOGIC ROUTING: RESERVOIR ROUTING

Storage-Indication Method

- Apply the storage-indication method for reservoirs that have a spillway.
- Assume that storage (S)=0 when no overflow occurs (surcharge storage).
- Apply this to an ungated spillway like a weir, outlet discharge pipe, or gated spillway with fixed position.

Storage-Indication Method

Use a relationship between outflow (O) and elevation head (H).

For uncontrolled weir outflow:

$Q_w = CLH^{3/2}$

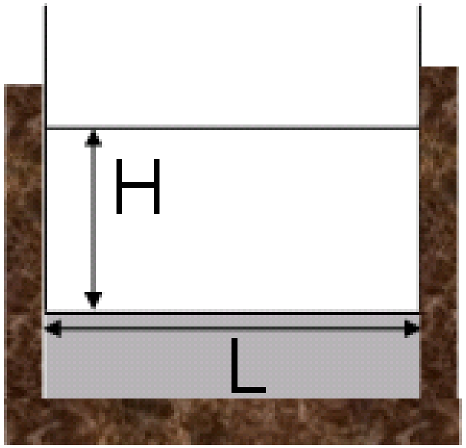
Where

Q_w = O = Discharge at the outlet (cfs)

C = Discharge coefficient of weir (cfs)

L = Length of crest (ft)

H = Depth above spillway (ft)



HYDROLOGIC ROUTING: RESERVOIR ROUTING

Storage-Indication Method
For controlled orifice outflow:

$$Q_o = C A_o \sqrt{2gH}$$

Where

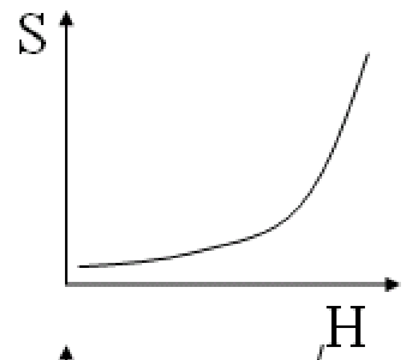
Q_o = O = Orifice discharge rate (cms)
C = Orifice coefficient (cms)
A_o = Orifice area (cms)
g = gravitational constant (9.81 m/s^2)
H = depth of water above the centre line of orifice (m)

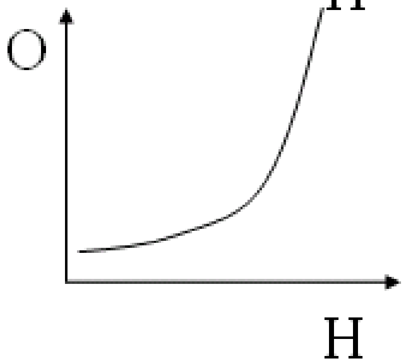
HYDROLOGIC ROUTING: RESERVOIR ROUTING

Storage-Indication Method

Two relationships specific for reservoir:

- Storage-Head Relationship
- Outflow-Head Relationship





Need:

- An inflow hydrograph
- A starting elevation above spillway

Use the continuity equation as:

$$\bar{I} - \bar{O} = \frac{\Delta S}{\Delta t}$$

Where

I = Average inflow during Δt

O = Average outflow during Δt

Or

$$\frac{I_i + I_{i+1}}{2} - \frac{O_i + O_{i+1}}{2} = \frac{S_{i+1} - S_i}{\Delta t}$$

Where subscripts denote the time interva

HYDROLOGIC ROUTING: RESERVOIR ROUTING

Storage-Indication Method

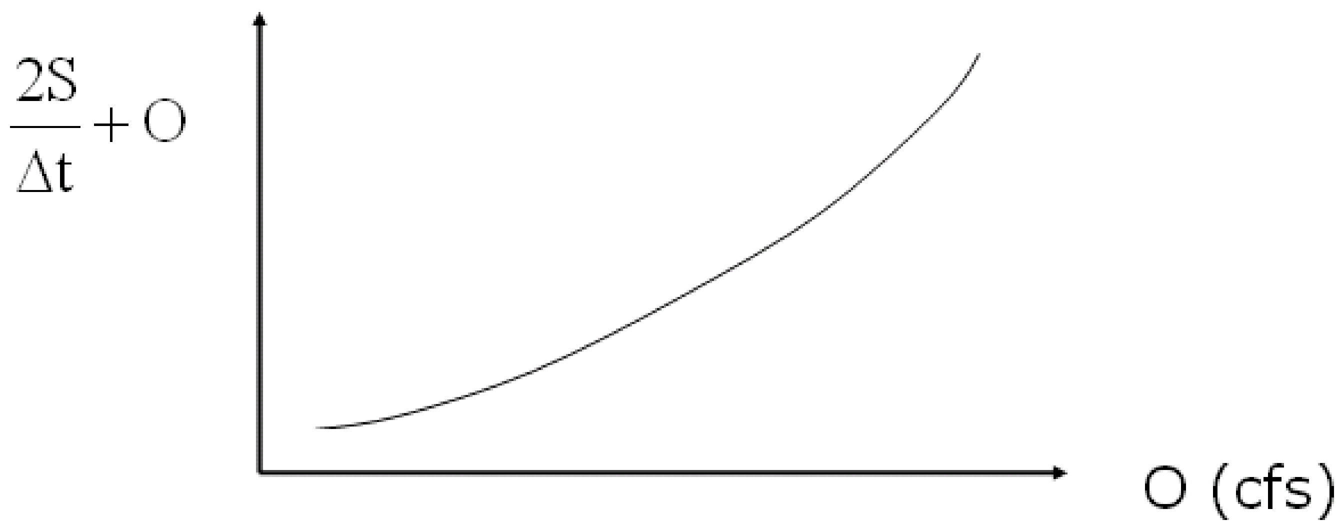
$$\frac{I_i + I_{i+1}}{2} - \frac{O_i + O_{i+1}}{2} = \frac{S_{i+1} - S_i}{\Delta t}$$

For i=1, we know Ii and Ii+1 (Initially) and Si (Initially)
We do not know Oi+1 and Si+1
So, we rewrite "Knowns = Unknowns"

$$I_i + I_{i+1} + \frac{2S_i}{\Delta t} - O_i = \frac{2S_{i+1}}{\Delta t} + O_{i+1}$$

Storage-Indication Method

We can find Oi+1, if we have a relationship between term on RHS and O. This is possible using the so-called “**Storage-Indication Curve**”.

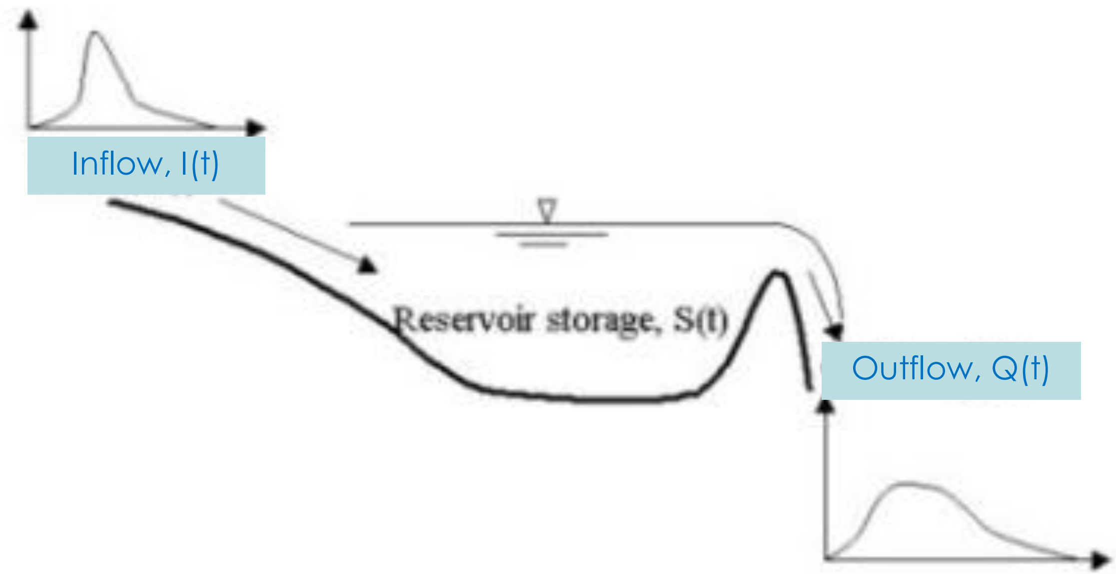


Storage-Indication Method: Routing Steps

- Set i=1, obtain initial head and inflow hydrograph.
- Find initial outflow O_1 corresponding to initial head above spillway.
- Find $2S/\Delta t$ for S(H) relationship.
- From the continuity equation, calculate $\frac{2S_2}{\Delta t} + O_2$
- Enter storage-indication curve to find O_2.
- Calculate $\frac{2S_2}{\Delta t} - O_2 = [\frac{2S_2}{\Delta t} + O_2] - 2O_2$
- Change i=2
- From continuity equation, calculate $\frac{2S_3}{\Delta t} + O_3$
- Repeat steps 4-7, and so on…..

RESERVOIR ROUTING: EXAMPLE 3

Reservoir Routing in Excel Spreadsheet



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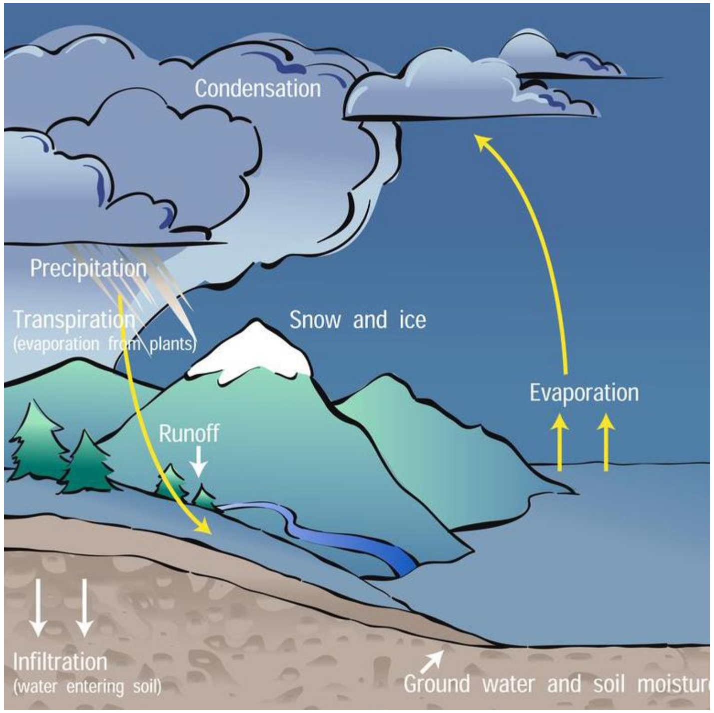

LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

Peak Flow Analysis

- Methods for Peak Flow Estimation
- Empirical Method
- Rational Method

Estimation of Peak Flow

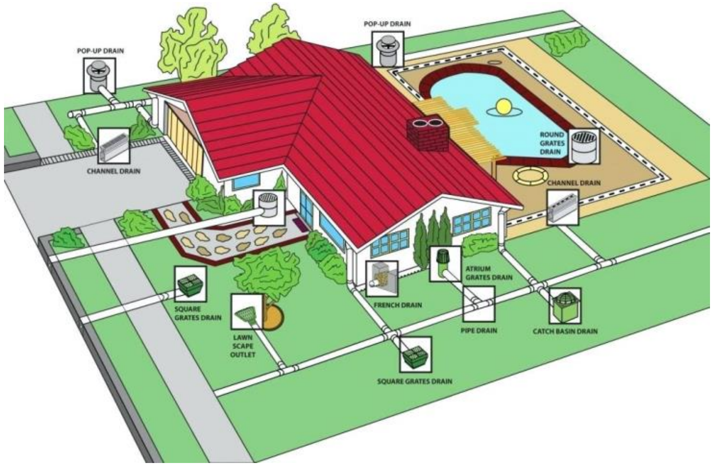
- For the purpose of designing any hydraulic structure, one needs to know the **magnitude of the peak flow/flood** that can be expected with an assigned frequency during the life of the structure.
- Estimation of peak flow rates from small and mid-size watersheds is a common application of engineering hydrology.
- Simpler approaches are justified when designing small hydraulic structures such as culverts or storm drainage systems.
- For these design problems, peak flows usually provide information to determine the appropriate pipe size.

PEAK FLOW ANALYSIS

Estimation of Peak Flow

The following alternative methods are used for estimation of the peak flow:

- Empirical Method
- Rational Method
- Unit Hydrograph Method
- Flood Frequency Method



The choice of a method for estimation of the peak flow primarily depends upon the importance of the work and available data.

PEAK FLOW ANALYSIS: EMPIRICAL METHOD

Empirical Method

The empirical relations are based on statistical correlation between the observed peak flow, Qp (cms) and the area of the catchment, A (km^2) in a given region.

The following empirical relations are often used:

- Dicken's formula
- Ryves formula
- Inglis formula
- Envelope curve technique
- Fanning formula
- Myers formula

PEAK FLOW ANALYSIS: EMPIRICAL METHOD

Empirical Method

- Fanning Formula

$Q = CA^{5/6}$

Where Q = Peak flow (cfs)

A = Area (sq.mi.)

C = Constant value (equal to 200 for Q=cfs)

- Myers Formula

$Q = 100pA^{1/2}$

Where Q = Max flow (cfs)

p = Myers rating

A = Area (sq.mi.)

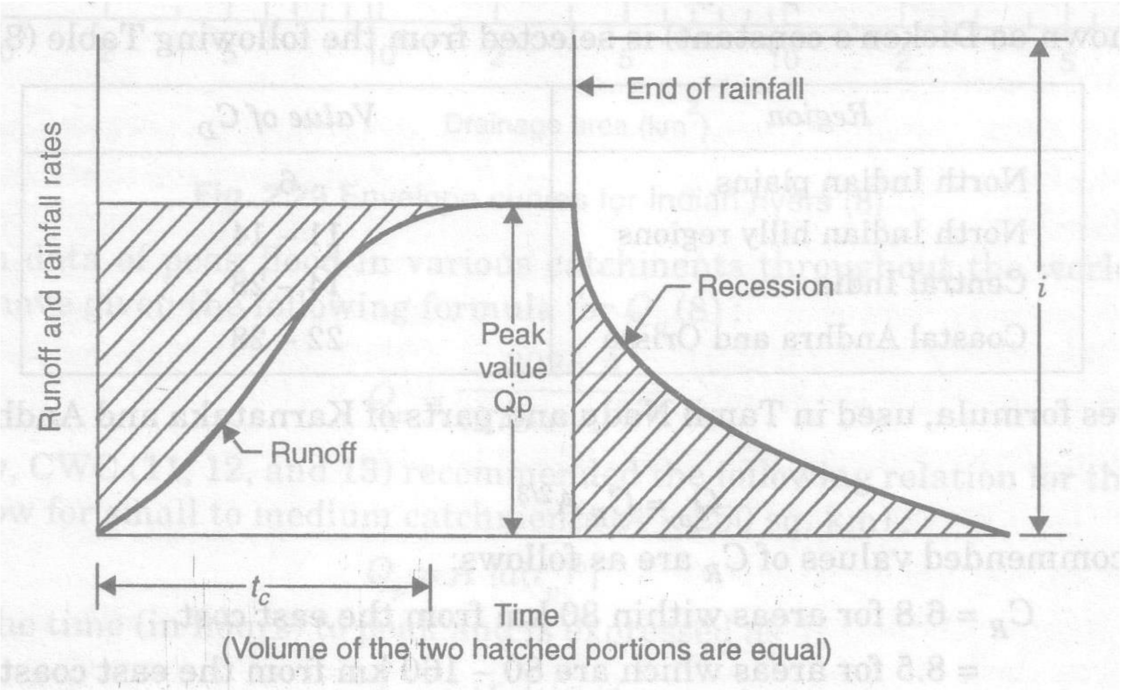
PEAK FLOW ANALYSIS: EMPIRICAL METHOD

Empirical Method

- For the above formulas, there is no attempt to consider rainfall amounts or intensities as parameter, or to relate the value of q to any probability or return period.
- They simply provide an upper limit of Q that would represent an extremely conservative design flow value.
- Most designs are based on a return period (highway culverts: 50 year return period)
- A frequency analysis using peak flows from gaged stream flow would provide desired peak flow.
- Drawbacks: gaged data may not exist, watershed may have changed land use, gaged data may not be at the location of design.

Rational Method

The runoff rate during and after precipitation of uniform intensity and long duration typically varies as shown in the figure.



The runoff increases from zero to a constant peak value when the water from the farthest area of the catchment basin reaches the basin outlet.

If tc= time taken for water from the farthest part of catchment to reach the outlet and the rainfall continues beyond tc, the runoff will have attained constant peak value. When the rain stops the runoff start decreasing.

PEAK FLOW ANALYSIS: RATIONAL METHOD

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Rational Method

- Developed in 1800s in England as the first dimensionally correct equation.
- Used by 90% of engineers still today.
- Equation assumes that Q is a function of rainfall intensity applied uniformly over the watershed for a duration D.
- Equation also assumes that frequency of Q is equal to frequency of rainfall intensity.
- The proper rainfall duration is equal to the time of concentration.

PEAK FLOW ANALYSIS: RATIONAL METHOD

Rational Method: Metric Unit

The peak value of runoff, Qp (cms) is given as;

$$Q_P = \frac{1}{36} CiA$$

Where C = Coefficient of runoff depending upon the nature of the catchment surface and the rainfall intensity, i

i = The mean intensity of rainfall (mm/hr) for a duration equal to or exceeding tc and an exceedance probability, P

A = Catchment area (km^2)

PEAK FLOW ANALYSIS: RATIONAL METHOD

Rational Method: English Unit

The equation is;

$$Q = 1.008CIA$$

Where Q = Peak flow (cfs)

C = Dimensionless coefficient of runoff

I = Average rainfall intensity (in/hr)

A = Catchment area (acre)

1.008 =Unit conversion factor

The conversion factor is usually ignored.

PEAK FLOW ANALYSIS: RATIONAL METHOD

What is needed?

(1) Time of concentration

(2) A set of rainfall intensity-duration-frequency curve (IDF curve)

(3) Drainage area size

(4) An estimate of the coefficient C

PEAK FLOW ANALYSIS: RATIONAL METHOD

Time of Concentration

Time of concentration, tc can be obtained from Kirpich equation.

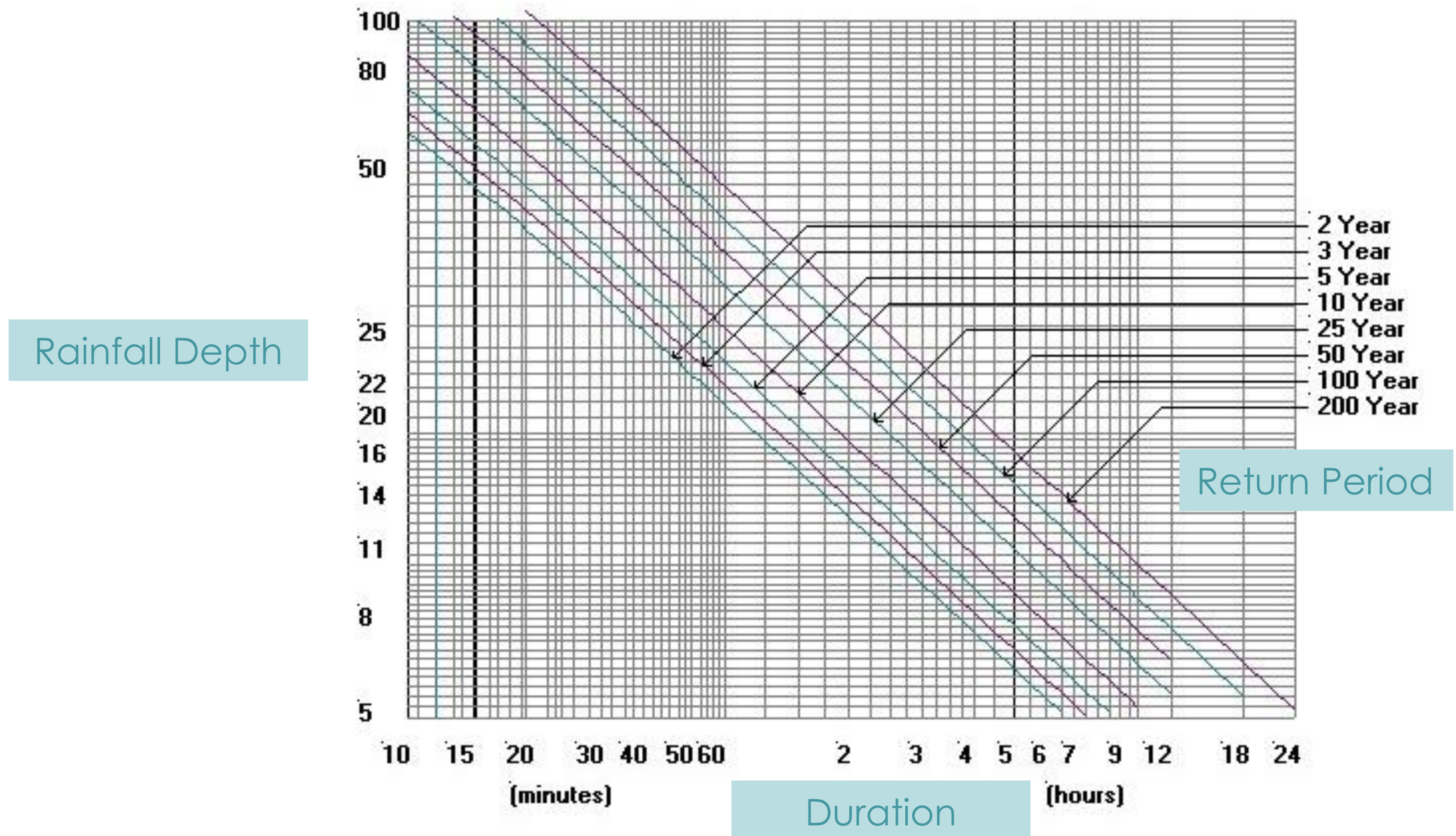
$$t_C = 0.01947 L^{0.77} S^{-0.386}$$

tc = Time of concentration in minutes
L = The maximum length of travel of water from the upstream end of the catchment basin to the basin
S = The slope of catchment which is equal to ΔH/L
ΔH = The difference of elevations of the upstream end of the catchment and the outlet.

Source: Chow et al. (1988)

PEAK FLOW ANALYSIS: RATIONAL METHOD

IDF Curve



PEAK FLOW ANALYSIS: RATIONAL METHOD

Coefficient of Runoff

Type of Surface	Value of C
Wooded area	0.01-0.20
Parks, open spaces lawns, meadows	0.05-0.30
Unpaved streets, vacant lands	0.10-0.30
Gravel roads and walks	0.15-0.30
Macadamized roads	0.25-0.60
Inferior block pavements with open joints	0.40-0.50
Stone, brick and wood-block pavements with open or uncemented joints	0.40-0.70
Stone, brick and wood-block pavements with tightly cemented joints	0.75-0.85
Asphalt pavements in good order	0.85-0.90
Watertight roof surfaces	0.75-0.90

PEAK FLOW ANALYSIS: RATIONAL METHOD

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Wisdom of the Land

Coefficient of Runoff

- C is know as runoff coefficient and can be found for the different land uses.
- If land use is mixed, you can calculate a composite C value as follows:

$$C = (C_A A_A + C_B A_B)/(A_A + A_B) \text{ or}$$

$$C = (\sum C_i A_i)/(A_i)$$

Where C_A, C_B = C values for land use A and B

A_A, A_B = Areas of land use A and B

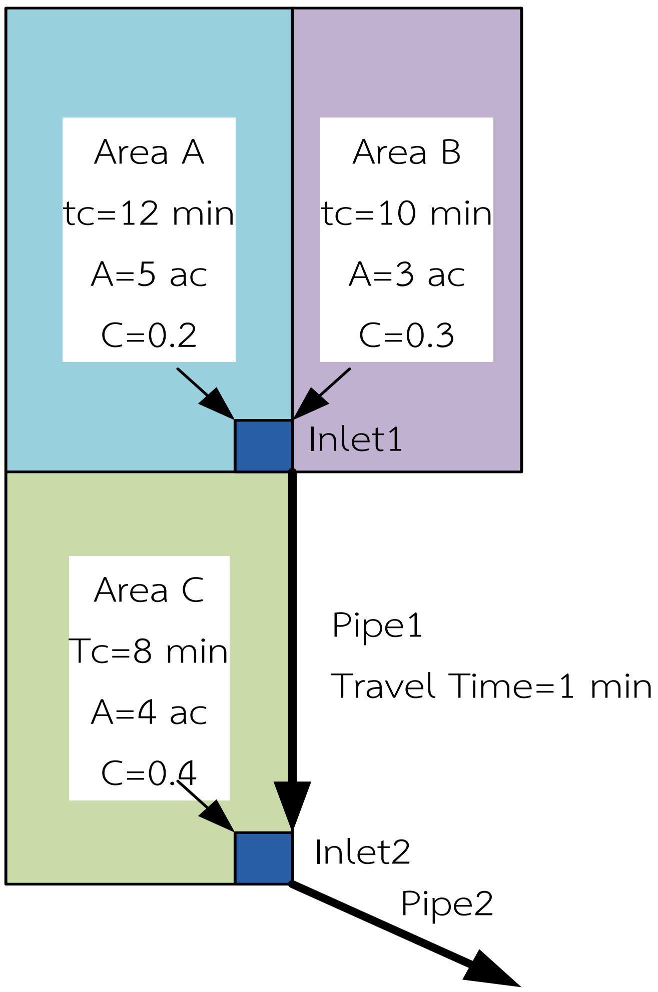
C_i, A_i = C and A for land use i

PEAK FLOW ANALYSIS: EXAMPLE 1

A storm drain system consisting of two inlets and pipe is to be designed using rational method. A schematic of the system is shown. Determine the peak flow rates to be used in sizing the two pipes and inlets.

Rainfall intensity (in/hr) as a function of t is:

$$I = \frac{30}{(t+5)^{0.70}}$$



PEAK FLOW ANALYSIS: EXAMPLE 1

Size Inlet 1 and pipe 1:

Area A and B contribute

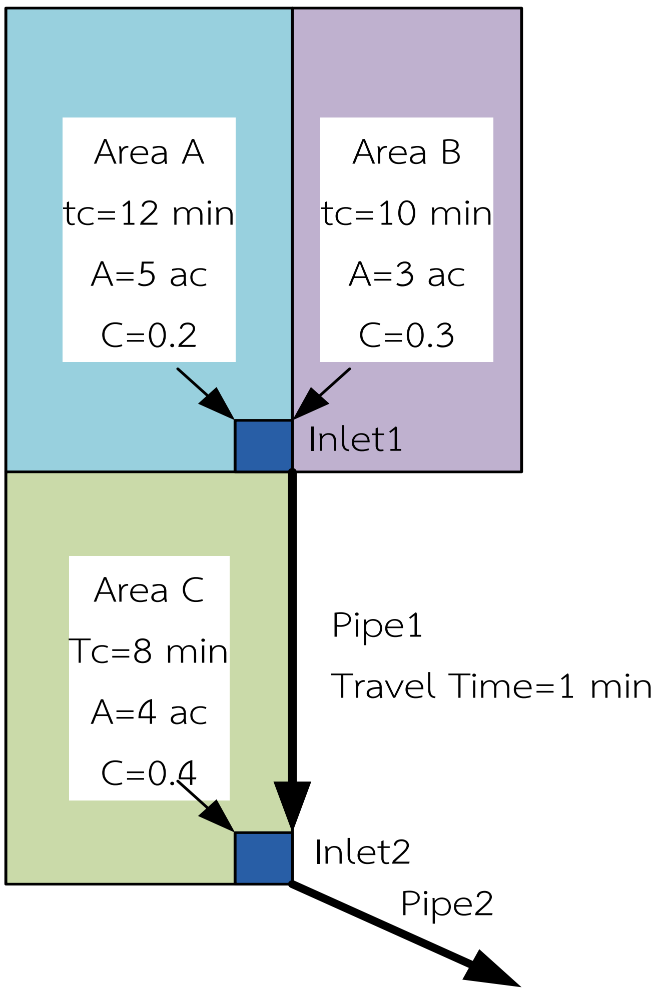
Take largest tc = 12 min

A = 5+3 = 8 acre

C = (5*0.2+3*0.3)/8 = 0.24

$I = 30/(12+5)^{0.7} = 4.13$ in/hr

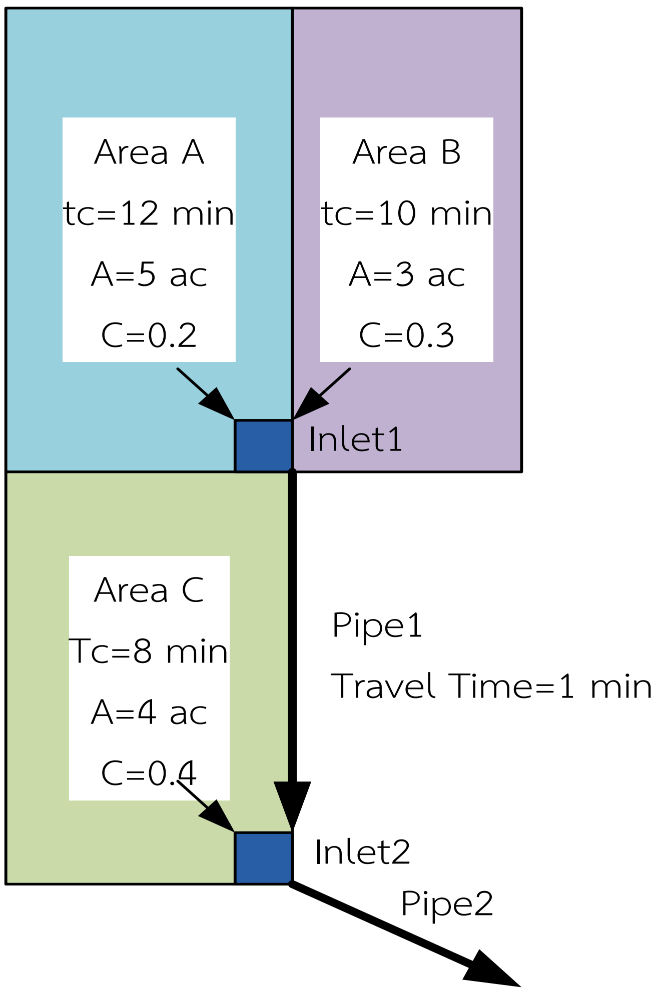
Q = CIA = 0.24*4.13*8 = 7.9 cfs



PEAK FLOW ANALYSIS: EXAMPLE 1

Size Inlet 2:
Flow from area C contributes
Take tc = 8 min

A = 4 acre
C = 0.4
$I = 30/(8+5)^{0.7} = 4.98$ in/hr
Q = CIA = 0.4*4.98*4 = 8.0 cfs

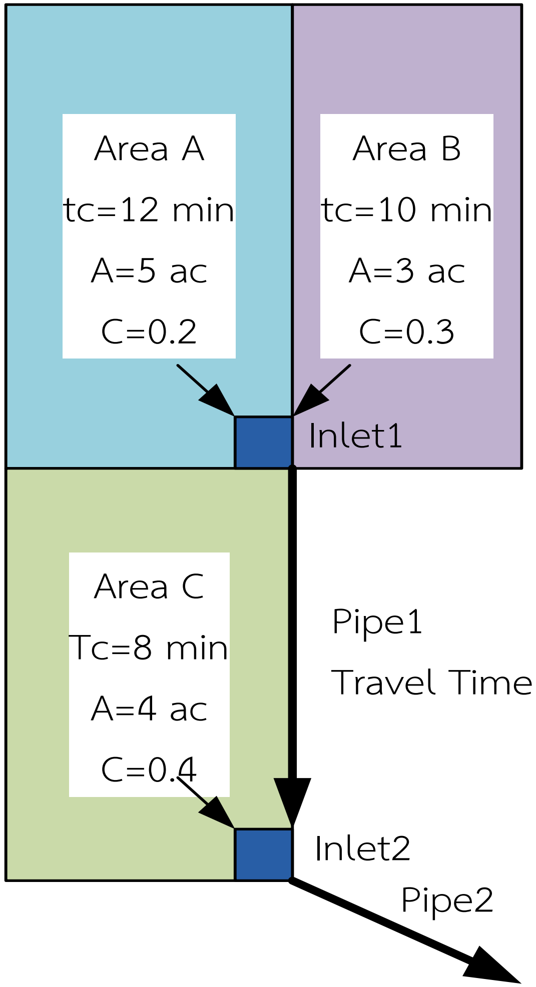


PEAK FLOW ANALYSIS: EXAMPLE 1

Size pipe 2:
Flow from all areas
Take tc = 12+1 = 13 min

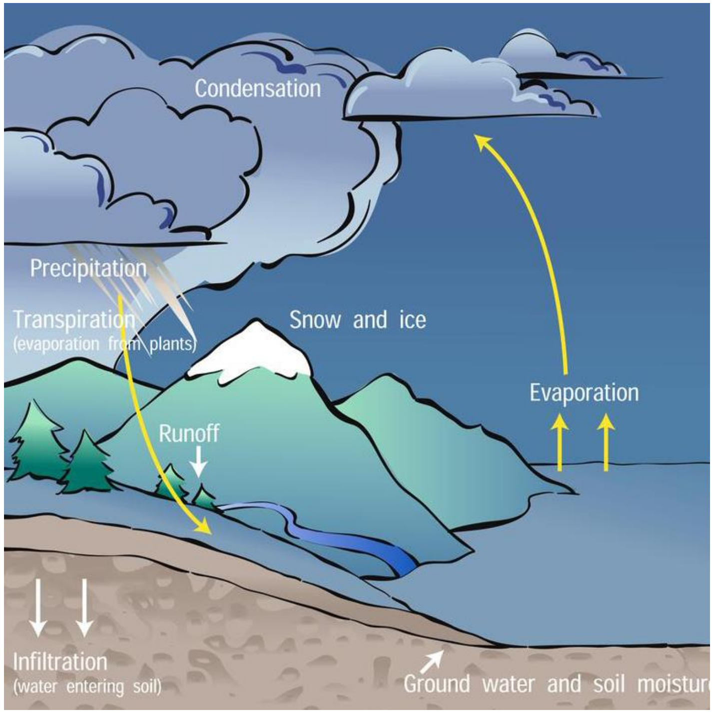
A = 5+4+3 = 12 acre
C = (5*0.2+4*0.4+3*0.3)/12
= 0.29
$I = 30/(13+5)^{0.7} = 3.97$ in/hr
Q = CIA = 0.29*3.97*12 = 13.8 cfs

Note: tc is taken as the largest value (12 min) plus travel trough pipe1.



REFERENCES

Chow, V.T., Maidment, D.R., & Mays, L.W. (1988). *Applied hydrology*. New York: McGraw-Hill Book Company.



LECTURE NOTES
EGCE 323 HYDROLOGY

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Revised in 2018

Hydrologic Statistics

- Hydrologic Statistics
- Frequency and Probability Functions
- Statistical Parameters
- Fitting a Probability Distribution
- Probability Distribution for Hydrologic Variables

Hydrological Data

The hydrologic processes are measured as;

Point Sample

- Measurements made through time at a fixed location in space.
- The resulting data forms a "**Time Series**".

Distributed Samples

- Measurement made over a line or area in space at a specific point in time.
- The resulting data forms a "**Space Series**".

The hydrologic processes evolve in space and time.

HYDROLOGIC STATISTICS

Hydrological Processes

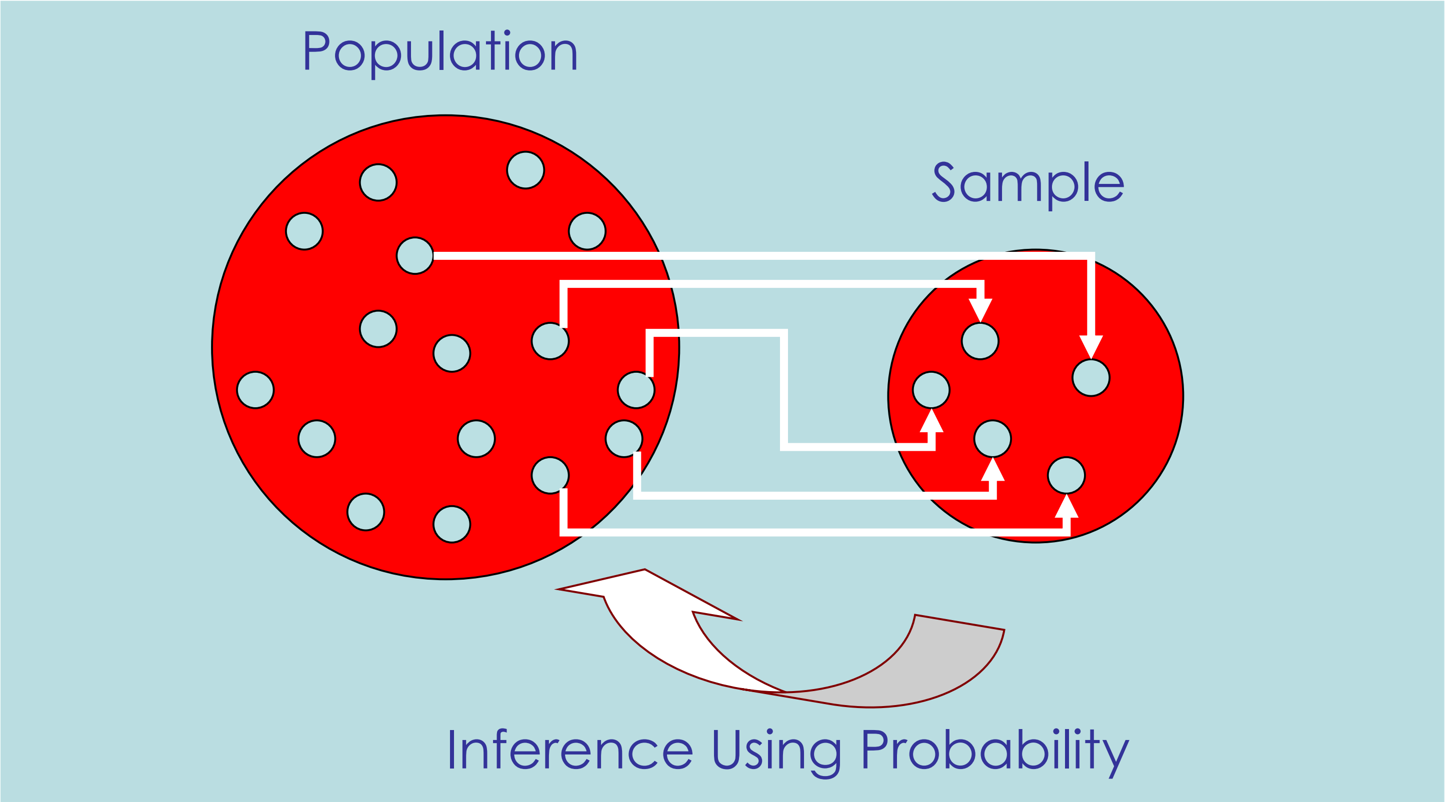
The hydrologic process is partly predictable or "**Deterministic Process**".

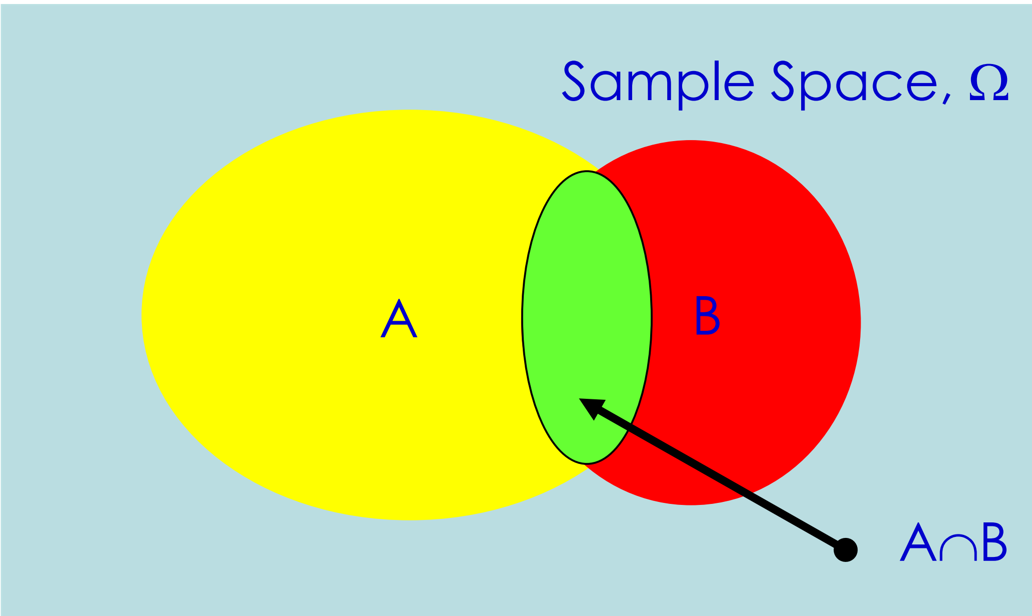
Some hydrologic process is partly unpredictable (random) or "**Stochastic Process**"

- X = random variable described by probability distribution.
- x = observation of the variable.
- x1, x2, x3,.., xn = Set of observation of random variable = "**Sample**".

Population vs Sample

It is assumed that samples are drawn from an infinite population possessing constant statistical properties.





- P(A) = Probability of Event A
- P(A) = lim(nA/n) ; n $\rightarrow \infty$
- nA/n = relative frequency

- **Sample Space**: the set of all possible sample that could be drawn from the population.
- **Event:** a subset of sample space.

HYDROLOGIC STATISTICS

Statistic Law

- **Total Probability**

  $P(A1)+P(A2)+.........+P(Am) = P(\Omega) = 1$

- **Complementarity**

  $P(A) = 1-P(A)$

- **Conditional Probability**

  -Dependent Events

  $P(A \cap B) = P(B/A)*P(A)$

  $P(B/A) = P(A \cap B)/P(A)$

  -Independent Events

  $P(B/A) = P(B)$

  $P(A \cap B) = P(B)*P(A)$

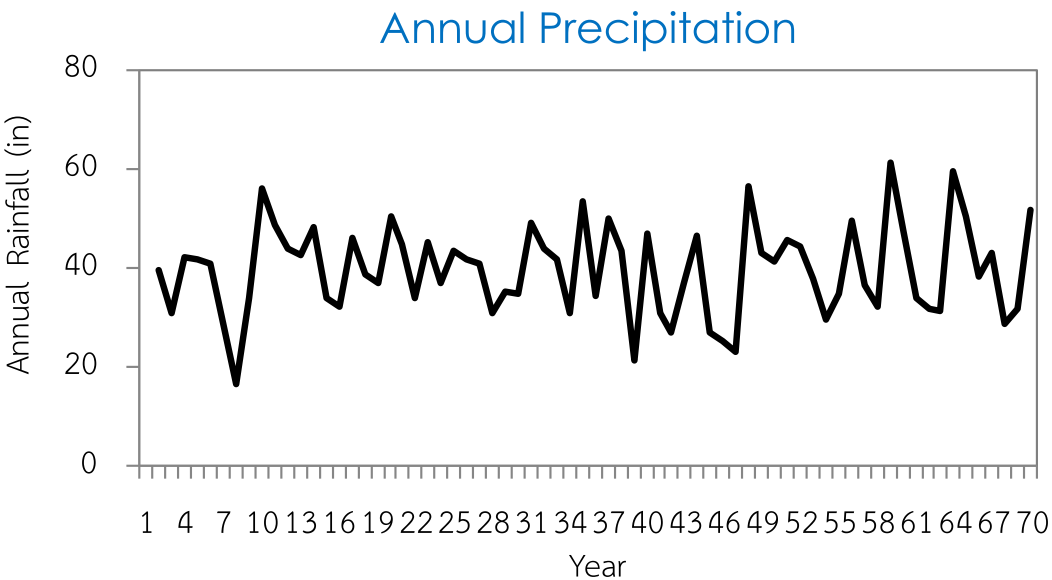
HYDROLOGIC STATISTICS: EXAMPLE 1

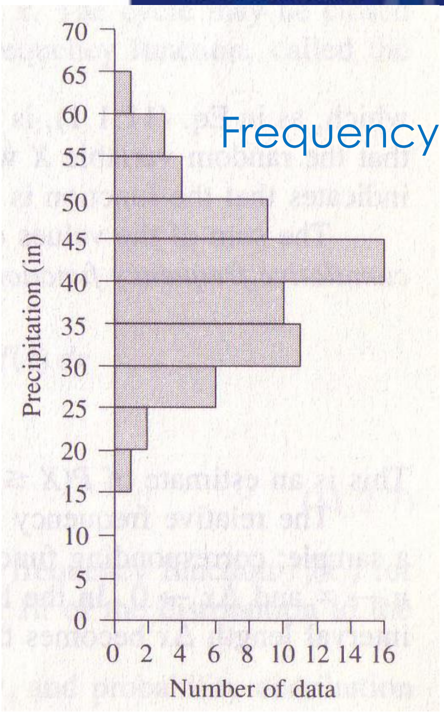
Mahidol University
Wisdom of the Land

The values of annual precipitation in College Station, Texas, from 1911 to 1979 are shown in table and plotted as a time series in the figure.
What is the probability that the annual precipitation R in any year will be less than 35 in? Between 35 and 45 in?

Year	1910	1920	1930	1940	1950	1960	1970
0		48.7	44.8	49.3	31.2	46.0	33.9
1	39.9	44.1	34.0	44.2	27.0	44.3	31.7
2	31.0	42.8	45.6	41.7	37.0	37.8	31.5
3	42.3	48.4	37.3	30.8	46.8	29.6	59.6
4	42.1	34.2	43.7	53.6	26.9	35.1	50.5
5	41.1	32.4	41.8	34.5	25.4	49.7	38.6
6	28.7	46.4	41.1	50.3	23.0	36.6	43.4
7	16.8	38.9	31.2	43.8	56.5	32.5	28.7
8	34.1	37.3	35.2	21.6	43.4	61.7	32.0
9	56.4	50.6	35.1	47.1	41.3	47.4	51.8

HYDROLOGIC STATISTICS: EXAMPLE 1





Solution n = 79-11+1 = 69 data.

Let A be the event $R < 35.0$ in.

B be the event $R > 45.0$ in.

The numbers of values falling in these ranges are $n_A = 23$ and $n_B = 19$

So, $P(A) \approx 23/69 = 0.333$

$P(B) \approx 19/69 = 0.275$

The probability that the annual precipitation is between 35 and 45 in can be calculated

$$P(35.0 \le R \le 45.0) = 1 - P(R<35.0) - P(R>35.0)$$
$$= 1 - 0.333 - 0.275$$
$$= 0.392$$

HYDROLOGIC STATISTICS: EXAMPLE 2

Assuming that annual precipitation in College Station is an independent process, calculate the probability that there will be two successive years of precipitation less than 35.0 in. Compare this estimated probability with the relative frequency of this event in the data set from 1911 to 1979.

Solution

Let C be the event that R < 35.0 in for two successive years.

From Example 1, $P(R < 35.0 \text{ in}) = 0.333$,

and assuming independent annual precipitation.

$$P(C) = [P(R<35.0)]^2$$
$$= (0.333)^2 = 0.111$$

From the data set, there are 9 pairs of successive years of precipitation less than 35.0 in out of 68 possible such pairs, so from a direct count it would be estimated that

$$P(C) = nc/n$$
$$= 9/68 = 0.132$$

FREQUENCY AND PROBABILITY FUNCTIONS

Frequency & Probability Functions

- If the observations in a sample are identically distributed, they can be arranged to form a frequency histogram.
- First, the feasible range of the random variable is divided into discrete intervals, then the number of observations falling into each intervals is counted, and finally the result is plotted as a bar graph.
- The width Δx of the interval used in setting up the frequency histogram is chosen to be as small as possible.

Frequency & Probability Functions

- If the number of observations n_i in interval i, covering the range $[x_i-\Delta x, x_i]$, is divided by the total number of observations n, the result is called "**Relative Frequency Function**, $f_s(x_i)$".

$$f_s(x_i) = \frac{n_i}{n}$$

- This is an estimate of $P(x_i-\Delta x \le X \le x_i)$, the probability that the random variable X will lie in the interval $[x_i-\Delta x, x_i]$.
- The subscript s indicates that the function is calculated from sample data.

FREQUENCY AND PROBABILITY FUNCTIONS

Frequency & Probability Functions

- The sum of the values of the relative frequencies up to a given point is the "**Cumulative Frequency Function**, $F_s(xi)$".

$$F_s(x_i) = \sum_{j=1}^{i} f_s(x_j)$$

- This is an estimate of $P(X \leq x_i)$, the cumulative probability of x_i.
- The relative frequency and cumulative frequency functions are defined for a sample.

FREQUENCY AND PROBABILITY FUNCTIONS

Frequency & Probability Functions

- Corresponding functions for the population are approached as limits as $n \to \infty$ and $\Delta x \to 0$. In the limit, the relative frequency function divided by the interval length Δx becomes the "**Probability Density Function, f(x)/PDF**".

$$f(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} \frac{f_s(x)}{\Delta x}$$

- The cumulative frequency function becomes the "**Probability Distribution Function/Cumulative Distribution Function (CDF)**".

$$F(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} F_s(x)$$

- Whose the derivative is the probability density function.

$$f(x) = \frac{dF(x)}{dx}$$

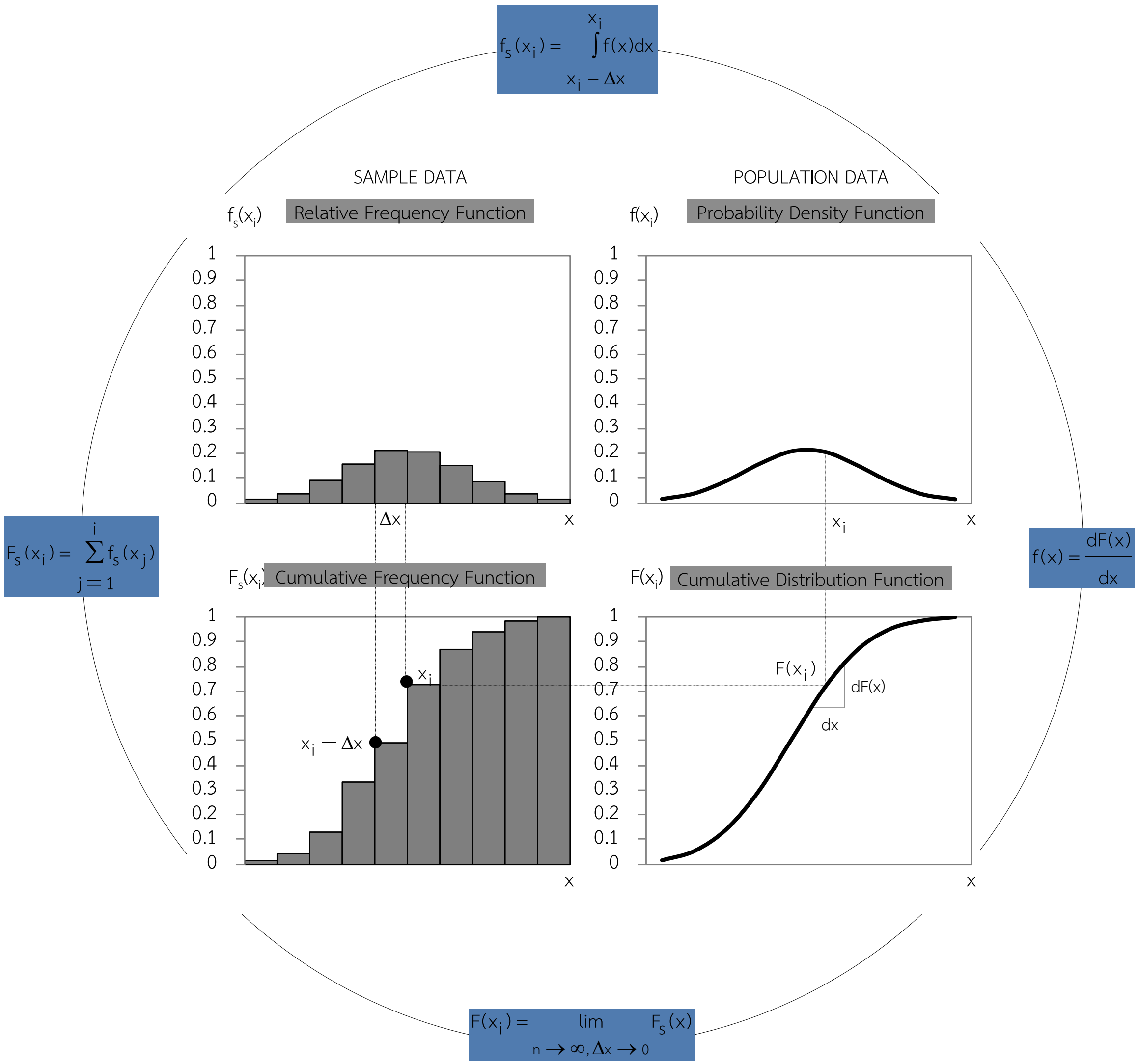
FREQUENCY AND PROBABILITY FUNCTIONS

Frequency & Probability Functions
From the point of view of fitting sample data to a theoretical distribution, the 4 functions

(1) Relative Frequency Function, fs(xi) or p(xi)
(2) Cumulative Frequency Function, Fs(xi)
(3) Probability Density Function, f(xi)
(4) Cumulative Distribution Function, F(xi)

May be arranged in a cycle as shown in the figure.

Frequency & Probability Functions



FREQUENCY AND PROBABILITY FUNCTIONS

Relative Frequency Function
The relative frequency function is computed from a sample data divided into intervals.

Cumulative Frequency Function
The relative frequency function is accumulated to form the cumulative frequency function shown at the lower left.

Probability Density Function, PDF
The probability distribution function, at the upper right, is the value of the slope of the distribution function for a specified value of x.

Cumulative Distribution Function, CDF
The cumulative distribution function, at the lower right, is the theoretical limit of the cumulative frequency function as the sample size becomes infinitely large and the data interval infinitely small.

FREQUENCY AND PROBABILITY FUNCTIONS

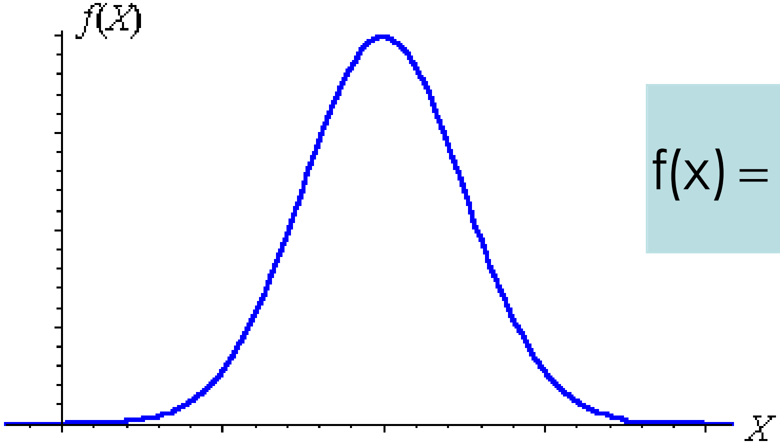
Frequency & Probability Functions

$$p(x_i) = P(x_i - \Delta x \leq X \leq x_i)$$
$$= F(x_i) - F(x_{i-1})$$

- The **Relative Frequency Function**, **Cumulative Frequency Function**, and **Cumulative Distribution Function** are all dimensionless functions varying over the range [0,1].
- The **Probability Density Function** $f(x) = dF(x)/dx$ has dimensions $[X]^{-1}$ and varies over the range $[0,\infty$

Normal Probability Distribution

One of the best-known probability density functions is that forming the familiar bell-shaped curve for the normal distribution.



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{(x-\mu)^2}{2\sigma^2}]$$

μ, σ = parameter

This function can be simplified by defining the "**Standard Normal Variable, z**".

PROBABILITY FUNCTIONS: NORMAL PROBABILITY DISTRIBUTION

Standard Normal Variable, z

$$z = \frac{x - \mu}{\sigma}$$

The corresponding standard normal distribution has PDF:

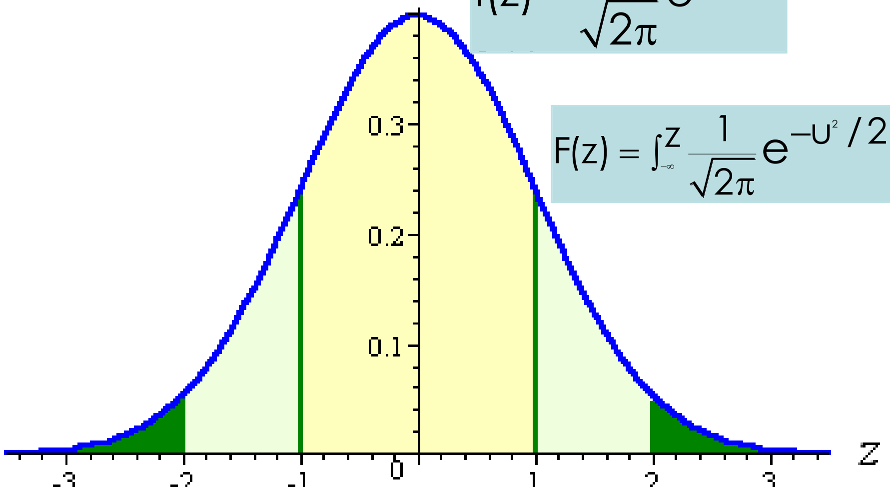
$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$-\infty \le z \le \infty$$

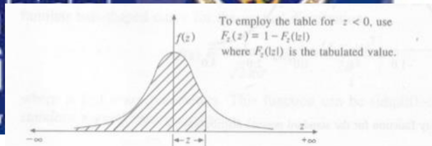
Which depends only on the value of x and is plotted in the figure.

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$



PROBABILITY FUNCTIONS: NORMAL PROBABILITY DISTRIBUTION-Z TABLE



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

PROBABILITY FUNCTIONS:
NORMAL PROBABILITY DISTRIBUTION

F(z) values are tabulated in the table and may be approximated by the following polynomial (Abramowitz and Stegun, 1965) :

$$B = \frac{1}{2}\left[\begin{array}{l} 1 + 0.196854|z| + 0.115194|z|^2 \\ +0.000344|z|^3 + 0.019527|z|^4 \end{array}\right]^{-4}$$

$$\begin{aligned} F(Z) &= B && \text{for } z < 0 \\ &= 1 - B && \text{for } z \geq 0 \end{aligned}$$

The error in F(z) as evaluated by this formula is less than 0.00025.

HYDROLOGIC STATISTICS: EXAMPLE 3

What is the probability that the standard normal random variable z will be less than -2? Less than 1?. What is P(-2<z<1).

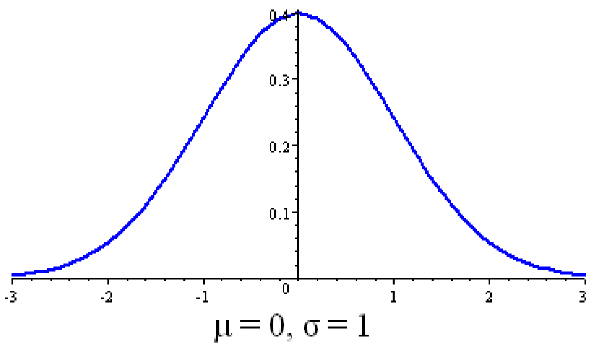
Solution $P(z \leq -2) = F(-2)$

$|z|=|-2|=2$

$B = ½[1+0.196854x2=0.115194x(2)^2+0.000344x(2)^3+0.019527x(2)^4]^{-4}$

$= 0.023$

Therefore; $F(-2) = B = 0.023$



HYDROLOGIC STATISTICS: EXAMPLE 3

$P(z \leq 1) = F(1)$

$|z| = |1| = 1$

$B = \frac{1}{2}[1 + 0.196854 \times 1 = 0.115194 \times (1)^2 + 0.000344 \times (1)^3 + 0.019527 \times (1)^4]^{-4}$

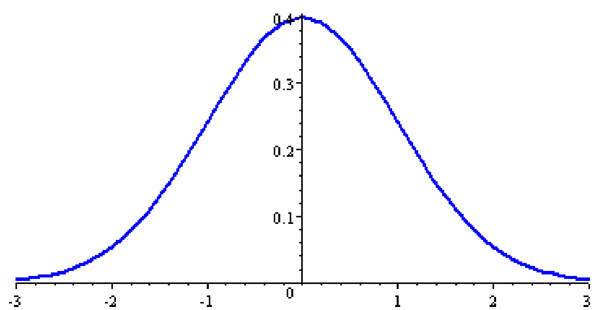
$= 0.159$

Therefore; $F(1) = 1 - B = 1 - 0.159 = 0.841$

Finally, $P(-2 < z < 1) = F(1) - F(-2)$

$= 0.841 - 0.023$

$= 0.818$



$\mu = 0, \sigma = 1$

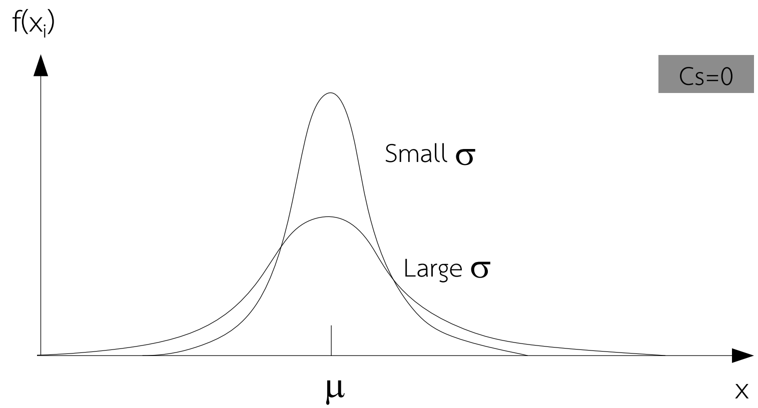
Statistical Parameters

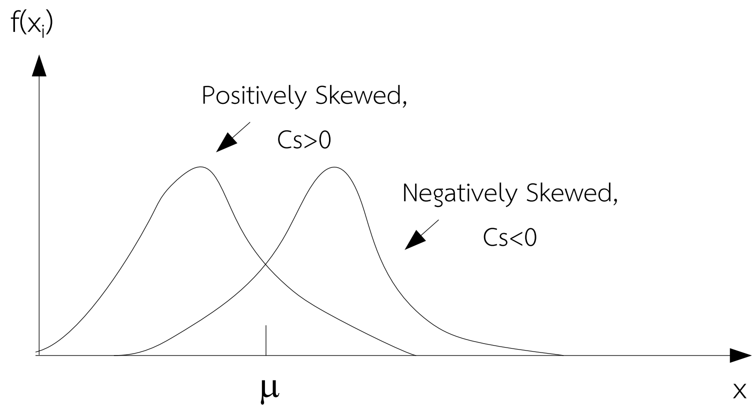
- The objective of statistics is to extract the essential information from a set of data, reducing a large set of numbers to a small set of numbers.
- Statistics are numbers calculated from a sample which summarize its important characteristics.
- Statistical parameters are characteristics of a population such as μ and σ.

STATISTICAL PARAMETERS

Population parameter	Sample statistic
1. *Midpoint*	
Arithmetic mean	
$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Median	
x such that $F(x) = 0.5$	50th-percentile value of data
Geometric mean	
antilog $[E(\log x)]$	$\left(\prod_{i=1}^{n} x_i\right)^{1/n}$
2. *Variability*	
Variance	
$\sigma^2 = E[(x-\mu)^2]$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
Standard deviation	
$\sigma = \{E[(x-\mu)^2]\}^{1/2}$	$s = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{1/2}$
Coefficient of variation	
$CV = \frac{\sigma}{\mu}$	$CV = \frac{s}{\bar{x}}$
3. *Symmetry*	
Coefficient of skewness	
$\gamma = \frac{E[(x-\mu)^3]}{\sigma^3}$	$C_s = \frac{n\sum_{i=1}^{n}(x_i - \bar{x})^3}{(n-1)(n-2)s^3}$

Population Parameters and Sample Statistics





The effect on the PDF of changes in the standard deviation and coefficient of skewness.

HYDROLOGIC STATISTICS: EXAMPLE 4

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Calculate the sample mean, sample standard deviation, and sample coefficient of skewness of the data for annual precipitation in College Station, Texas from 1970 to 1979. The data is given in the table.

Solution

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{401.7}{10} = 40.17\,in$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1{,}016.9}{9} = 113.0\,in^2$$

$$s = (113.0)^2 = 10.63\ in$$

$$C_s = \frac{n\sum_{i=1}^{n} (x_i - \bar{x})^3}{(n-1)(n-2)s^3} = \frac{10x6{,}480.3}{9x8x(10.63)^3} = 0.749$$

HYDROLOGIC STATISTICS: EXAMPLE 4

Year	Precipitation, x	$(x-x)^2$	$(x-x)^3$
1970	33.9	39.3	-246.5
1971	31.7	71.7	-607.6
1972	31.5	75.2	-651.7
1973	59.6	377.5	7,335.3
1974	50.5	106.7	1,102.3
1975	38.6	2.5	-3.9
1976	43.4	10.4	33.7
1977	28.7	131.6	-1,509.0
1978	32.0	66.7	-545.3
1979	51.8	135.3	1,573.0
Total	401.7	1,016.9	6,480.3

PARAMETER ESTIMATION

Parameter Estimation Methods

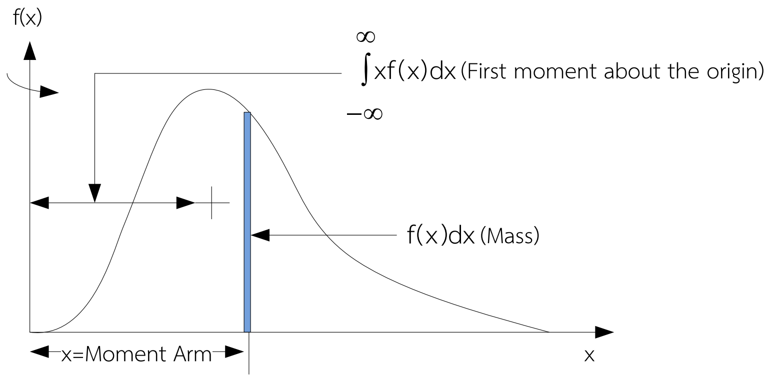
- Method of Moments
- Method of Maximum Likelihood

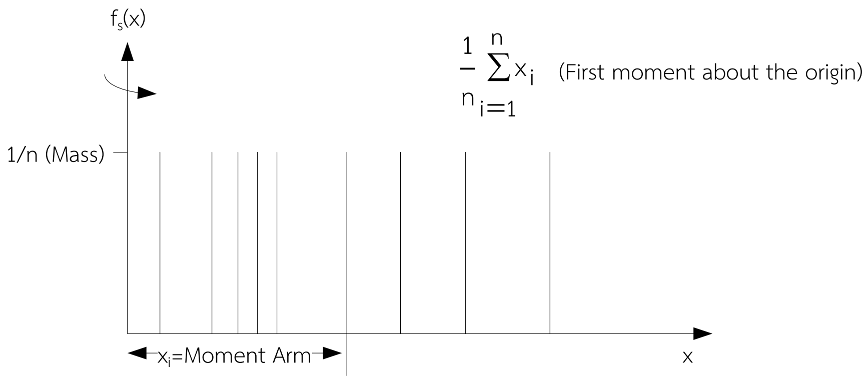
PARAMETER ESTIMATION METHODS: METHOD OF MOMENTS

Method of Moments

The method of moments was first developed by Karl Pearson in 1902. He considered that

"The good estimate of parameters of a probability distribution are those for which moments of the probability density function about the origin are equal to the corresponding moments of the sample data".





PARAMETER ESTIMATION METHODS: METHOD OF MAXIMUM LIKELIHOOD

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Method of Maximum Likelihood

The method of maximum likelihood was developed by R.A. Fisher in 1922. He reasoned that

"The best value of a parameter of a probability distribution should be that value which maximizes the likelihood or joint probability of occurrence of observed sample".

$$\ln L = \sum_{i=1}^{n} \ln\left[f(x_i)\right]$$

- The method of maximum likelihood is the most theoretically correct method of fitting probability distributions to data in the sense that it produces the most efficient parameter estimates.
- In general, the method of moments is easier to apply than the method of maximum likelihood and is more suitable for practical hydrologic analysis.

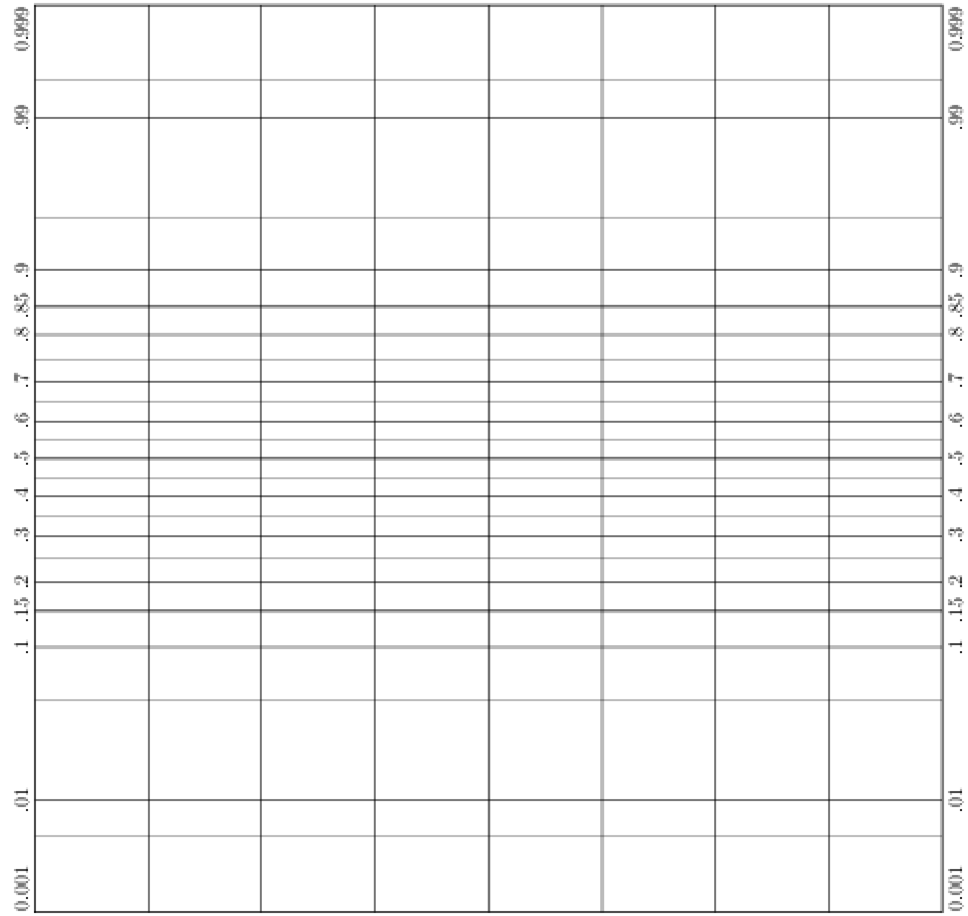
TESTING OF GOODNESS OF FIT: GRAPHICAL METHOD

Graphical Method

- To check the probability distribution fitting, the data may be plotted on specially "**Designed Probability Paper**" or using a plotting scale that linearizes the distribution function.
- The plotted data are then fitted with a straight line for interpolation and extrapolation purposes.

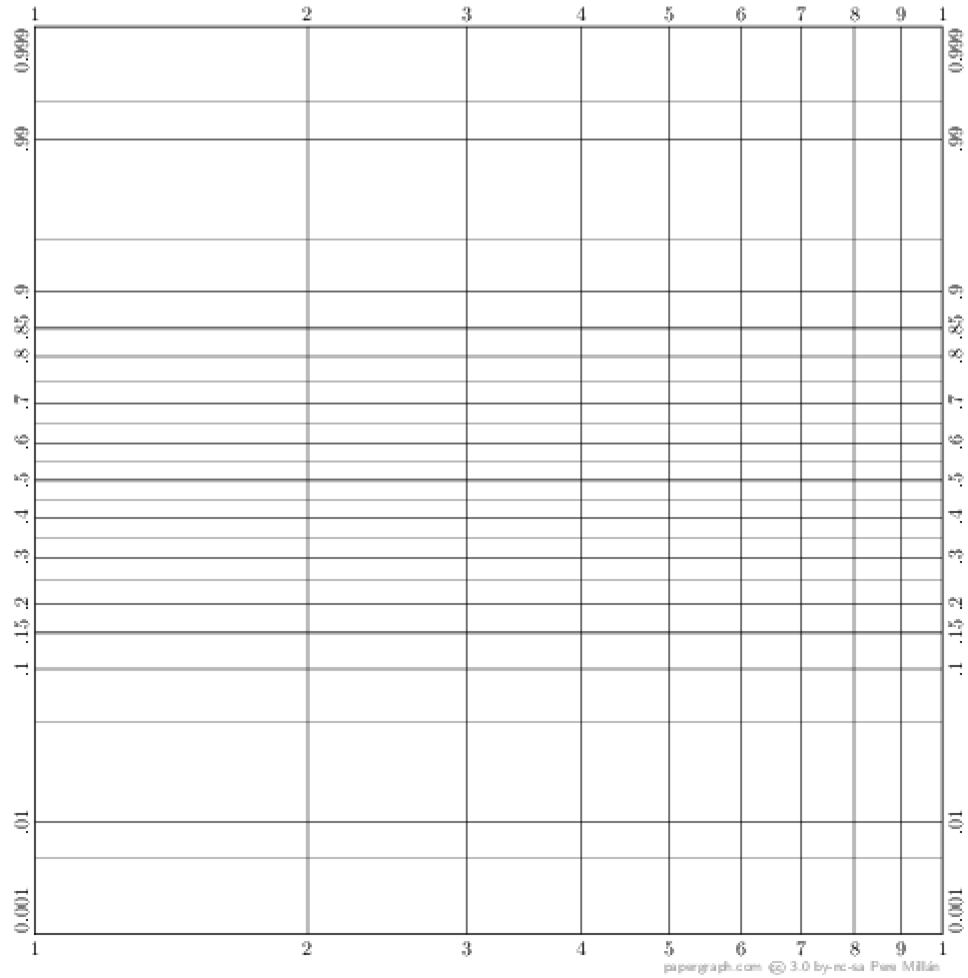
TESTING OF GOODNESS OF FIT: GRAPHICAL METHOD

Probability Paper: Normal



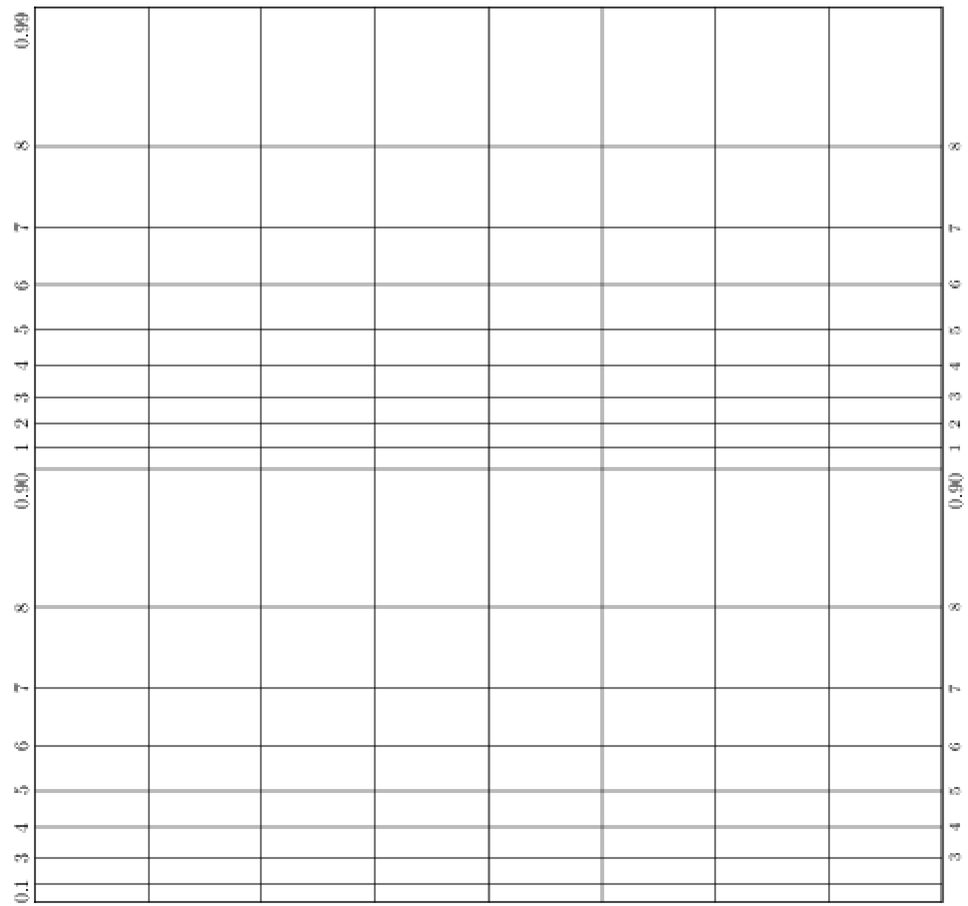
TESTING OF GOODNESS OF FIT: GRAPHICAL METHOD

Probability Paper: Log-Normal



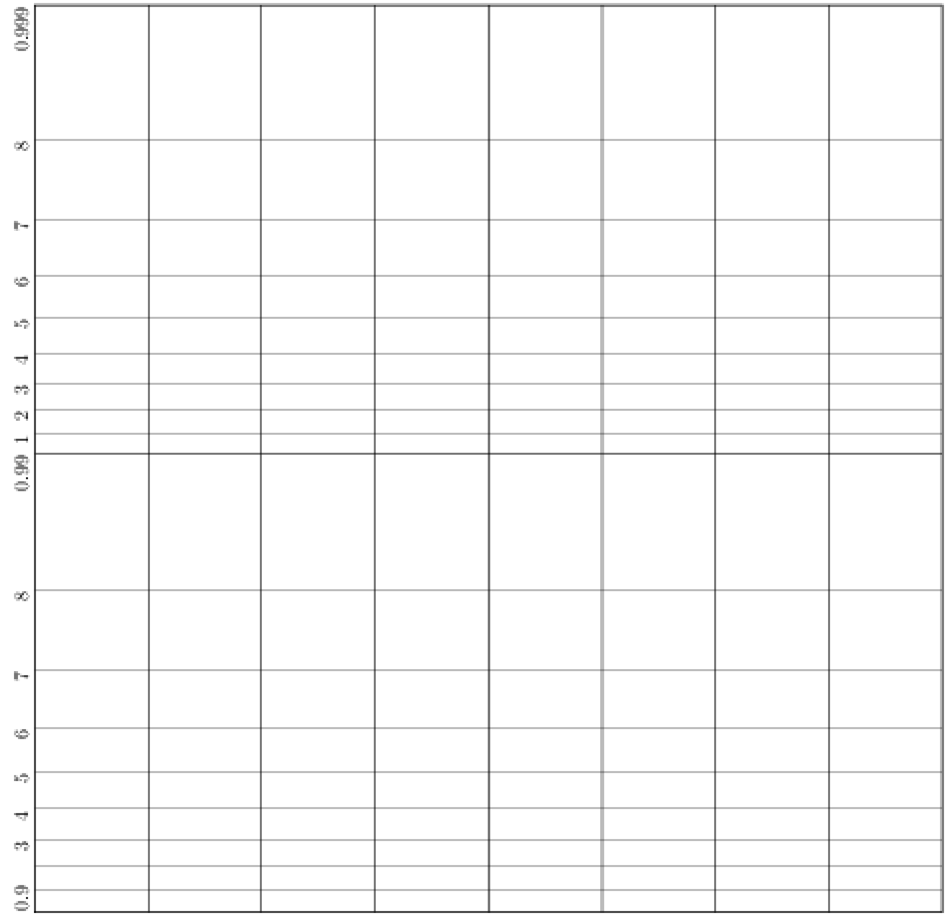
TESTING OF GOODNESS OF FIT: GRAPHICAL METHOD

Probability Paper: Exponential



TESTING OF GOODNESS OF FIT: GRAPHICAL METHOD

Probability Paper: Gumbel



HYDROLOGIC STATISTICS: EXAMPLE5-GRAPHICAL METHOD

The maximum annual flood of the Mae Kong river, Loas since 2466-2508 is shown in the table. Perform a probability plotting analysis with the Gumbel distribution.

Year	Max Annual Flood	Descending	n	Plotting Position
2466	19,300	22,900	1	2.273
2467	21,200	21,200	2	4.545
2468	14,000	20,500	3	6.817
2469	17,700	20,200	4	9.091
2470	17,500	19,400	5	11.364
2471	15,500	19,300	6	13.643
2472	20,500	19,100	7	15.924
2473	18,100	18,900	8	18.182
2474	15,800	18,300	9	20.450
2475	14,900	18,300	10	22.727
2476	16,300	18,200	11	25.000

HYDROLOGIC STATISTICS: EXAMPLE 5-GRAPHICAL METHOD

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Year	Max Annual Flood	Descending	n	Plotting Position
2477	14,900	18,100	12	27.248
2478	17,600	18,000	13	29.586
2479	17,000	18,000	14	31.847
2480	17,300	17,900	15	34.130
2481	18,300	17,700	16	36.364
2482	19,100	17,700	17	38.610
2483	17,900	17,600	18	40.984
2484	19,400	17,500	19	43.103
2485	22,900	17,300	20	45.455
2486	16,200	17,300	21	47.619
2487	14,300	17,200	22	50.00
2488	20,200	17,000	23	52.356
2489	17,700	16,300	24	54.645

HYDROLOGIC STATISTICS:
EXAMPLE 5-GRAPHICAL METHOD

Year	Max Annual Flood	Descending	n	Plotting Position
2490	18,900	16,300	25	56.818
2491	15,600	16,300	26	59.880
2492	14,800	16,200	27	61.350
2493	15,200	15,800	28	63.694
2494	16,300	15,800	29	65.789
2495	17,300	15,700	30	68.493
2496	14,100	15,600	31	70.423
2497	15,700	15,500	32	72.460
2498	18,000	15,400	33	75.190
2499	16,300	15,200	34	77.520
2500	11,300	14,900	35	79.370
2501	11,500	14,900	36	81.970
2502	18,000	14,800	37	84.030

HYDROLOGIC STATISTICS: EXAMPLE 5-GRAPHICAL METHOD

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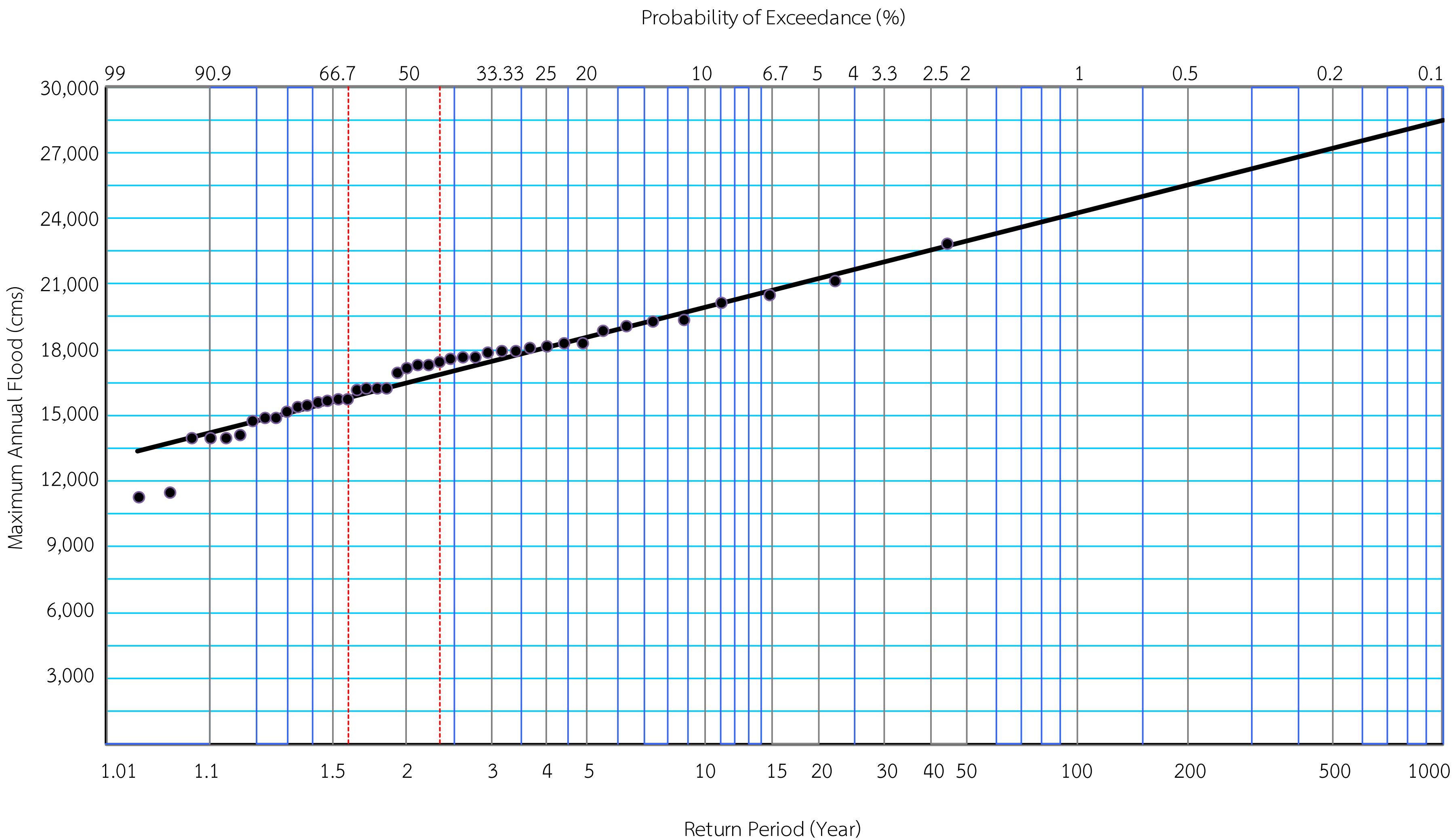
Year	Max Annual Flood	Descending	n	Plotting Position
2503	18,200	14,100	38	86.210
2504	18,300	14,000	39	88.500
2505	15,400	14,000	40	90.910
2506	15,800	14,000	41	93.460
2507	17,200	11,500	42	95.240
2508	14,000	11,300	43	98.040

HYDROLOGIC STATISTICS: EXAMPLE 5-GRAPHICAL METHOD



TESTING OF GOODNESS OF FIT: CHI-SQUARE METHOD

Chi-Square Method

The goodness of fit of a probability distribution can be tested by comparing the **theoretical and sample values of the relative frequency or the cumulative frequency function.**

The chi-square test statistic is given by

$$X_c^2 = \sum_{i=1}^{m} \frac{n[f_s(x_i) - p(x_i)]^2}{p(x_i)}$$

m = the number of intervals

$nf_s(x_i) = n_i$ the observed number of occurrences in interval i

$np(x_i)$ = the corresponding expected number of occurrences in interval

TESTING OF GOODNESS OF FIT: CHI-SQUARE METHOD

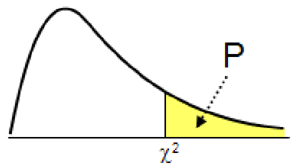
- To describe the χ^2 test, the χ^2 probability distribution must be defined.
- A χ^2 distribution with ν **degrees of freedom** is the distribution for the sum of squares of ν independent standard normal random variables z_i. This sum is the random variable.

$$\chi_\nu^2 = \sum_{i=1}^{\nu} z_i^2$$

ν = m-p-1= degree of freedom
m = number of intervals
p = number of parameters
α = significant level
1-α = confident level

- The null hypothesis for a test is that the proposed probability distribution fits the data adequately. This hypothesis is rejected if the value of χ_c^2 in the formula is larger than a limiting value $\chi_{\nu,1-\alpha}^2$.

TESTING OF GOODNESS OF FIT: CHI-SQUARE STATISTICS



DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
1	0.0000393	0.000982	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.550	10.828
2	0.0100	0.0506	3.219	4.605	5.991	7.378	7.824	9.210	10.597	12.429	13.816
3	0.0717	0.216	4.642	6.251	7.815	9.348	9.837	11.345	12.838	14.796	16.266
4	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
5	0.412	0.831	7.289	9.236	11.070	12.833	13.388	15.086	16.750	18.907	20.515
6	0.676	1.237	8.558	10.645	12.592	14.449	15.033	16.812	18.548	20.791	22.458
7	0.989	1.690	9.803	12.017	14.067	16.013	16.622	18.475	20.278	22.601	24.322
8	1.344	2.180	11.030	13.362	15.507	17.535	18.168	20.090	21.955	24.352	26.124
9	1.735	2.700	12.242	14.684	16.919	19.023	19.679	21.666	23.589	26.056	27.877
10	2.156	3.247	13.442	15.987	18.307	20.483	21.161	23.209	25.188	27.722	29.588
11	2.603	3.816	14.631	17.275	19.675	21.920	22.618	24.725	26.757	29.354	31.264
12	3.074	4.404	15.812	18.549	21.026	23.337	24.054	26.217	28.300	30.957	32.909
13	3.565	5.009	16.985	19.812	22.362	24.736	25.472	27.688	29.819	32.535	34.528
14	4.075	5.629	18.151	21.064	23.685	26.119	26.873	29.141	31.319	34.091	36.123
15	4.601	6.262	19.311	22.307	24.996	27.488	28.259	30.578	32.801	35.628	37.697
16	5.142	6.908	20.465	23.542	26.296	28.845	29.633	32.000	34.267	37.146	39.252
17	5.697	7.564	21.615	24.769	27.587	30.191	30.995	33.409	35.718	38.648	40.790
18	6.265	8.231	22.760	25.989	28.869	31.526	32.346	34.805	37.156	40.136	42.312
19	6.844	8.907	23.900	27.204	30.144	32.852	33.687	36.191	38.582	41.610	43.820
20	7.434	9.591	25.038	28.412	31.410	34.170	35.020	37.566	39.997	43.072	45.315
21	8.034	10.283	26.171	29.615	32.671	35.479	36.343	38.932	41.401	44.522	46.797
22	8.643	10.982	27.301	30.813	33.924	36.781	37.659	40.289	42.796	45.962	48.268
23	9.260	11.689	28.429	32.007	35.172	38.076	38.968	41.638	44.181	47.391	49.728
24	9.886	12.401	29.553	33.196	36.415	39.364	40.270	42.980	45.559	48.812	51.179
25	10.520	13.120	30.675	34.382	37.652	40.646	41.566	44.314	46.928	50.223	52.620
26	11.160	13.844	31.795	35.563	38.885	41.923	42.856	45.642	48.290	51.627	54.052
27	11.808	14.573	32.912	36.741	40.113	43.195	44.140	46.963	49.645	53.023	55.476
28	12.461	15.308	34.027	37.916	41.337	44.461	45.419	48.278	50.993	54.411	56.892
29	13.121	16.047	35.139	39.087	42.557	45.722	46.693	49.588	52.336	55.792	58.301
30	13.787	16.791	36.250	40.256	43.773	46.979	47.962	50.892	53.672	57.167	59.703

HYDROLOGIC STATISTICS: EXAMPLE 6-CHI-SQUARE METHOD

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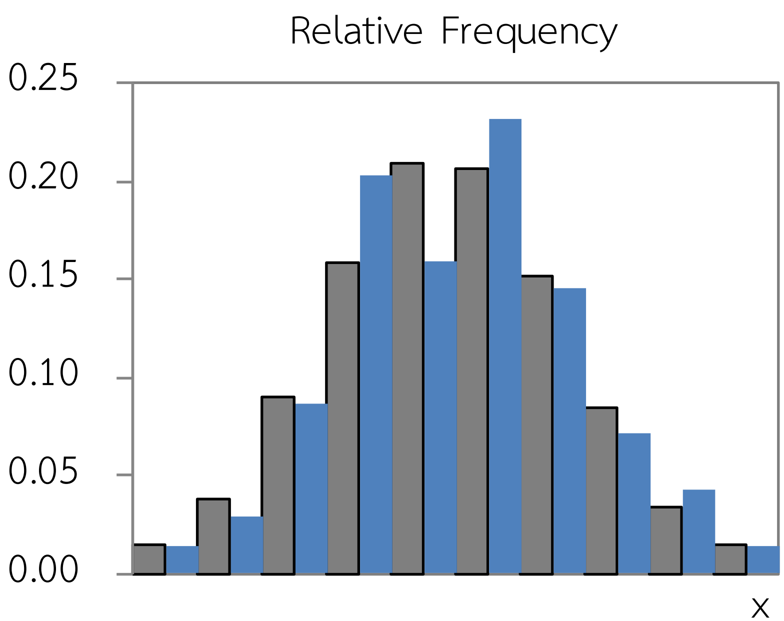
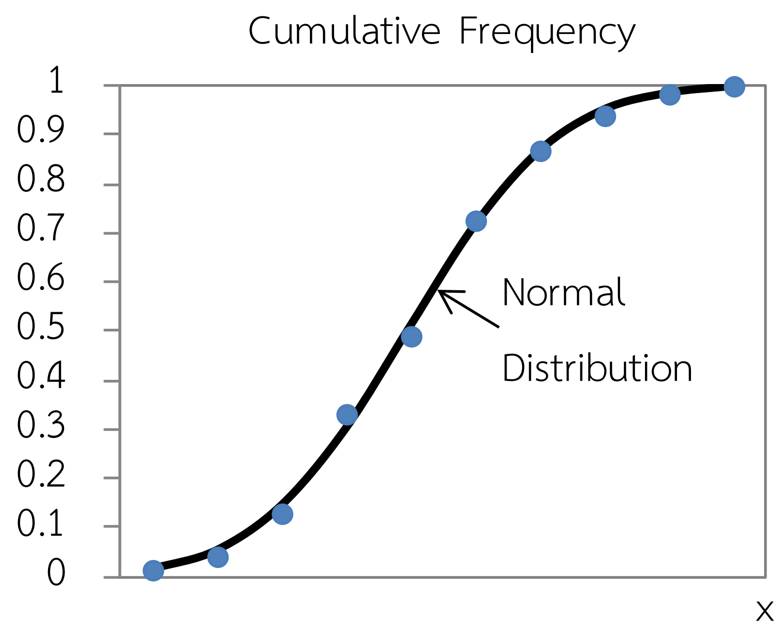
Test goodness of fit of annual precipitation in College Station, Texas, in Example1 with the Normal distribution.

Interval	Range (in)	n_i	$f_s(x_i)$	$F_s(x_i)$	z_i	$F(x_i)$	$f(x_i)$	χ_c^2
1	< 20	1	0.014	0.014	-2.157	0.015	0.015	0.004
2	20-25	2	0.029	0.043	-1.611	0.053	0.038	0.147
3	25-30	6	0.087	0.130	-1.065	0.144	0.090	0.008
4	30-35	14	0.203	0.333	-0.520	0.301	0.158	0.891
5	35-40	11	0.159	0.493	0.026	0.510	0.209	0.805
6	40-45	16	0.232	0.725	0.571	0.716	0.206	0.222
7	45-50	10	0.145	0.870	1.117	0.868	0.151	0.019
8	50-55	5	0.072	0.942	1.662	0.952	0.084	0.114
9	55-60	3	0.043	0.986	2.208	0.986	0.034	0.163
10	> 60	1	0.014	1.000	2.753	1.000	0.014	0.004
Total		69	1.000				1.000	2.377
Avg.	39.77							
Stdev.	9.17							

HYDROLOGIC STATISTICS:
EXAMPLE 6-CHI-SQUARE METHOD





HYDROLOGIC STATISTICS: PROBABILITY DISTRIBUTIONS

Distribution	Probability density function	Range	Equations for parameters in terms of the sample moments
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$-\infty \le x \le \infty$	$\mu = \bar{x},\ \sigma = s_x$
Lognormal	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right)$ where $y = \log x$	$x > 0$	$\mu_y = \bar{y},\ \sigma_y = s_y$
Exponential	$f(x) = \lambda e^{-\lambda x}$	$x \ge 0$	$\lambda = \frac{1}{\bar{x}}$
Gamma	$f(x) = \frac{\lambda^\beta x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$ where Γ = gamma function	$x \ge 0$	$\lambda = \frac{\bar{x}}{s_x^2}$; $\beta = \frac{\bar{x}^2}{s_x^2} = \frac{1}{CV^2}$

Source: Chow et al. (1988)

HYDROLOGIC STATISTICS: PROBABILITY DISTRIBUTIONS

Mahidol University
Wisdom of the Land

Distribution	Probability density function	Range	Equations for parameters in terms of the sample moments
Pearson Type III (three parameter gamma)	$f(x) = \frac{\lambda^{\beta}(x-\epsilon)^{\beta-1}e^{-\lambda(x-\epsilon)}}{\Gamma(\beta)}$	$x \geq \epsilon$	$\lambda = \frac{s_x}{\sqrt{\beta}}, \ \beta = \left(\frac{2}{C_s}\right)^2$ $\epsilon = \bar{x} - s_x\sqrt{\beta}$
Log Pearson Type III	$f(x) = \frac{\lambda^{\beta}(y-\epsilon)^{\beta-1}e^{-\lambda(y-\epsilon)}}{x\Gamma(\beta)}$ where $y = \log x$	$\log x \geq \epsilon$	$\lambda = \frac{s_y}{\sqrt{\beta}},$ $\beta = \left[\frac{2}{C_s(y)}\right]^2$ $\epsilon = \bar{y} - s_y\sqrt{\beta}$ (assuming $C_s(y)$ is positive)
Extreme Value Type I	$f(x) = \frac{1}{\alpha}\exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$	$-\infty < x < \infty$	$\alpha = \frac{\sqrt{6}s_x}{\pi}$ $u = \bar{x} - 0.5772\alpha$

Source: Chow et al. (1988)

TESTING OF GOODNESS OF FIT: KOLMOGOROV-SMIRNOV METHOD

Kolmogorov-Smirnov Method (KS)
The K-S test statistic measures the largest distance between the $F(x_i)$ and the theoretical function $F'(x_i)$, measured in a vertical direction (Kolmogorov as cited in Stephens 1992).

The test statistic is given by:

$$\Delta = \left| F'(x_i) - F(x_i) \right|$$

Where (for a two-tailed test):
$F(x_i)$ = the cdf of the hypothesized distribution
$F'(x_i)$ = the empirical distribution function of observed data

TESTING OF GOODNESS OF FIT: KOLMOGOROV-SMIRNOV STATISTICS

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N	α			
	0.20	**0.10**	**0.05**	**0.01**
5	0.45	0.51	0.56	0.67
10	0.32	0.37	0.41	0.49
15	0.27	0.30	0.34	0.40
20	0.23	0.26	0.29	0.36
25	0.21	0.24	0.27	0.32
30	0.19	0.22	0.24	0.29
35	0.18	0.20	0.23	0.27
40	0.17	0.19	0.21	0.25
45	0.16	0.18	0.20	0.24
50	0.15	0.17	0.19	0.23
N>50	$\frac{1.07}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

The maximum annual flood of the Mae Kong river, Loas since 2466-2508 is shown in the table. Perform a probability fitting analysis with the Gumbel distribution by using Kolmogorov-Smirnov method.

Solution

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{725,500}{43} = 16,827.093$$

$$S_x = \sqrt{\frac{\sum_{i=1}^{N} X_i^2 - N\bar{X}^2}{N-1}} = \frac{\sqrt{12,471.45x10^6 - 43x(16,872.093)^2}}{42}$$

$$= 2,343.91$$

$$x_o = x - 0.45S_x$$

$$= 16,872.093 - 0.45x2,343.91 = 15,817.33$$

$$\alpha = 0.7797S_x = 0.7797(2343.91) = 1,827.55$$

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

CDF:

$$F(X) = \exp[-\exp[-(\frac{X - 15,817.33}{1,827.55})]]$$

Plotting Position:

$$F'(X) = \frac{m}{n+1}$$

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

No.	Max Annual Flood	F'(X), %	F(X), %	Δ=abs[F'(X)-F(X)]
1	11,300	2.273	0.0007	2.27
2	11,500	4.545	0.002	4.54
3	14,000	6.817	6.7	0.12
4	14,000	9.091	6.7	2.36
5	14,000	11.364	6.7	4.66
6	14,100	13.643	7.7	5.94
7	14,800	15.924	17.5	1.58
8	14,900	18.182	19.2	1.02
9	14,900	20.450	19.2	1.25
10	15,200	22.727	24.6	1.87
11	15,400	25.000	28.5	3.5

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

No.	Max Annual Flood	F'(X), %	F(X), %	Δ=abs[F'(X)-F(X)]
12	15,500	27.248	30.4	3.15
13	15,600	29.586	32.4	2.81
14	15,700	31.847	34.4	2.55
15	15,800	34.130	36.4	2.27
16	15,800	36.364	36.4	0.04
17	16,200	38.610	44.4	5.97
18	16,300	40.984	46.4	5.42
19	16,300	43.103	46.4	3.30
20	16,300	45.455	46.4	0.95
21	17,000	47.619	59.2	11.58
22	17,200	50.000	62.5	12.5*

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

No.	Max Annual Flood	F'(X), %	F(X), %	Δ=abs[F'(X)-F(X)]
23	17,300	52.356	64.1	11.74
24	17,300	54.645	64.1	9.46
25	17,500	56.818	67.2	10.38
26	17,600	59.880	68.6	8.72
27	17,700	61.35	70.0	8.65
28	17,700	63.694	70.0	6.31
29	17,900	65.789	72.6	6.81
30	18,000	68.493	73.9	5.41
31	18,000	70.423	73.9	3.48
32	18,100	72.460	75.1	2.64
33	18,200	75.190	76.2	1.01

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

No.	Max Annual Flood	F'(X), %	F(X), %	Δ=abs[F'(X)-F(X)]
34	18,300	77.520	77.3	0.22
35	18,300	79.370	77.3	2.07
36	18,900	81.970	83.1	1.13
37	19,100	84.030	84.7	0.67
38	19,300	86.210	86.2	0.1
39	19,400	88.500	86.9	1.6
40	20,200	90.910	91.3	0.4
41	20,500	93.460	92.6	0.86
42	21,200	95.240	94.9	0.34
43	22,900	98.040	97.9	0.14

HYDROLOGIC STATISTICS: EXAMPLE 7-KS METHOD

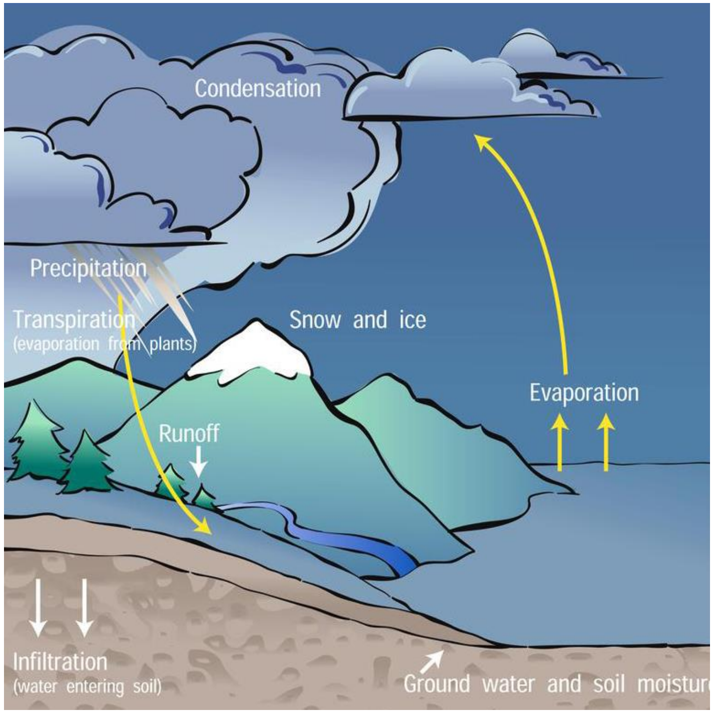
Δ_{max} = abs[F'(X)-F(x)]= 12.5%

$\Delta_{43,5\%}$ = 0.206 = 20.6% (in the table)

$\Delta_{max} \leq \Delta_{43,5\%}$ ⇨ The Gumbel distribution is fitted to maximum annual flood

REFERENCES

Chow, V.T., Maidment, D.R., & Mays, L.W. (1988). *Applied hydrology*. New York: McGraw-Hill Book Company.



LECTURE NOTES
EGCE 323 HYDROLOGY

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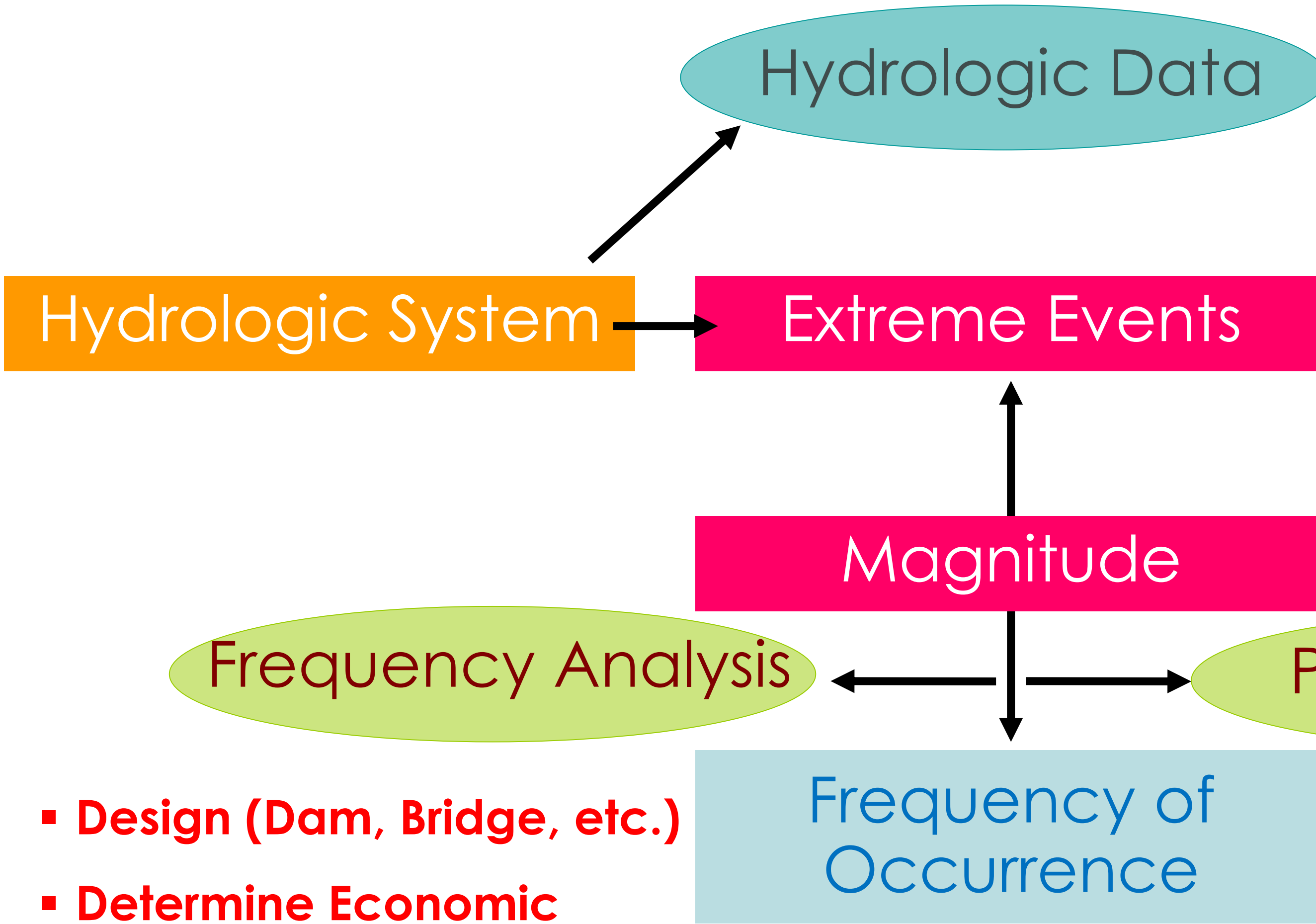
Revised in 2018

Flood Frequency Analysis

- Frequency Analysis
- Extreme Value Distribution
- Frequency Analysis Using Frequency Factors

FREQUENCY ANALYSIS



- Stochastic
- Space-Independent
- Time-Independent

- Severe Storm
- Flood
- Drought

- Design (Dam, Bridge, etc.)
- Determine Economic Value

Frequency Analysis

- Hydrologic systems are sometimes impacted by extreme events such as severe storms, floods, and droughts.
- The magnitude of an extreme events occurring less frequently than more moderate events.
- The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions.
- The results of flood flow frequency analysis can be used for many engineering purposes;
  - (1) for the design of dams, bridges, culvert, and flood control structures.
  - (2) to determine the economic value of flood control projects.
  - (3) to delineate flood plains.
  - (4) to determine the effect of encroachments on the flood plain.

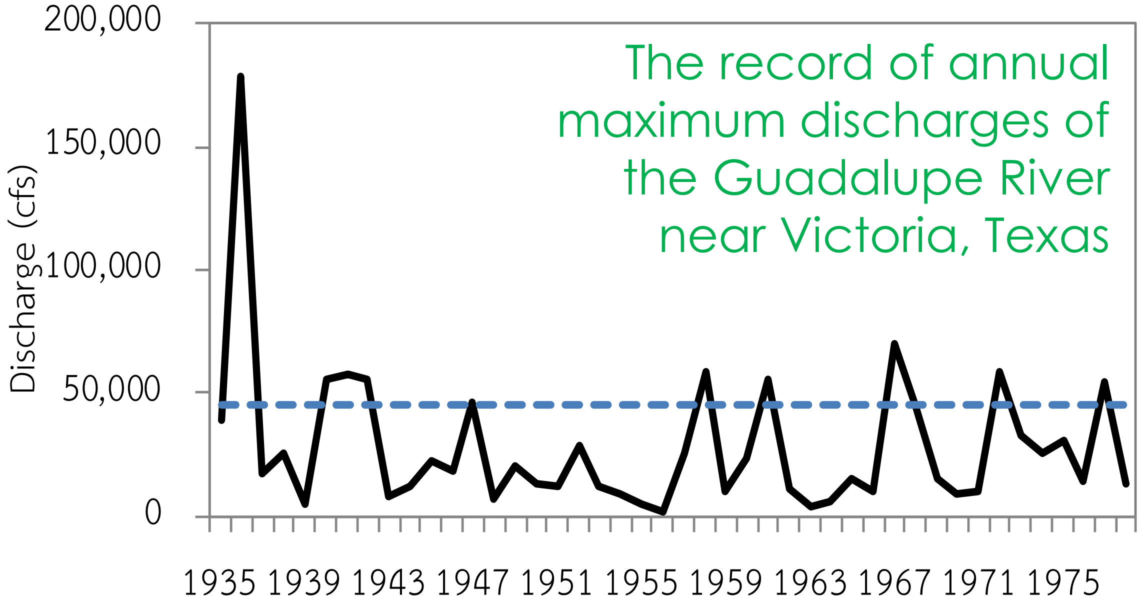
FREQUENCY ANALYSIS: RETURN PERIOD

Return Period

- Suppose that an extreme event is defined to have occurred if a random variable X is greater than or equal to some level x_{Tr}.

$$X \geq x_{Tr}$$

- The **recurrence interval**, τ is the time between occurrences of $X \geq x_{Tr}$



FREQUENCY ANALYSIS: RETURN PERIOD

Year	1930	1940	1950	1960	1970
0		55,900	13,300	23,700	9,190
1		58,000	12,300	55,800	9,740
2		56,000	28,400	10,800	58,500
3		7,710	11,600	4,100	33,100
4		12,300	8,560	5,720	25,200
5	38,500	22,000	4,950	15,000	30,200
6	179,000	17,900	1,730	9,790	14,100
7	17,200	46,000	25,300	70,000	54,500
8	25,400	6,970	58,300	44,300	12,700
9	4,940	20,600	10,100	15,200	

FREQUENCY ANALYSIS: RETURN PERIOD

If xTr = 50,000 cfs
It can be seen that the maximum discharge exceeded this level 9 times during the period of record, with recurrence intervals ranging from 1-16 years.

Exceedence Year	1936	1940	1941	1942	1958	1961	1967	1972	1977	Avg.
Recurrence Interval (yr)	4	1	1	16	3	6	5	5		5.1

The return period Tr of the event $X \geq xTr$ is the expected value of τ, $E(\tau)$.
Its average value measured over a very large number of the occurrences.

Therefore, the return period of a 50,000 cfs annual maximum discharge on the Guadalupe River is approximately $\tau = 41/8 = 5.1$ years

FREQUENCY ANALYSIS: RETURN PERIOD

Thus , "**the return period of an event of a given magnitude may be defined as the average recurrence interval between events equalling or exceeding a specified magnitude**".

The probability of occurrence of the event $X \geq x_{Tr}$ in any observation is

$$p = P(X \geq x_{Tr})$$

$$E(\tau) = \frac{p}{[1-(1-p)]^2} = \frac{1}{p}$$

Hence, $E(\tau) = Tr = 1/p$

FREQUENCY ANALYSIS: RETURN PERIOD

The probability of occurrence of an event in any observation is the inverse of its return period.

$$p = P(X \geq x_{Tr}) = \frac{1}{Tr}$$

For example, the probability that the maximum discharge in the Guadalupe River will equal or exceed 50,000 cfs in any year is approximately

$$p = P(X \geq x_{Tr}) = \frac{1}{5.1} = 0.195$$

FREQUENCY ANALYSIS: RETURN PERIOD

What is the probability that a Tr-year return period event will occur at least once in N years?

$P(X < x_{Tr}$ each year for N years$) = (1\text{-}p)^N$

$P(X \geq x_{Tr}$ at least once in N years$) = 1\text{-}(1\text{-}p)^N$ or

$P(X \geq xTr$ at least once in N years$) = 1\text{-}[1\text{-}(1/Tr)]^N$

FREQUENCY ANALYSIS: EXAMPLE 1

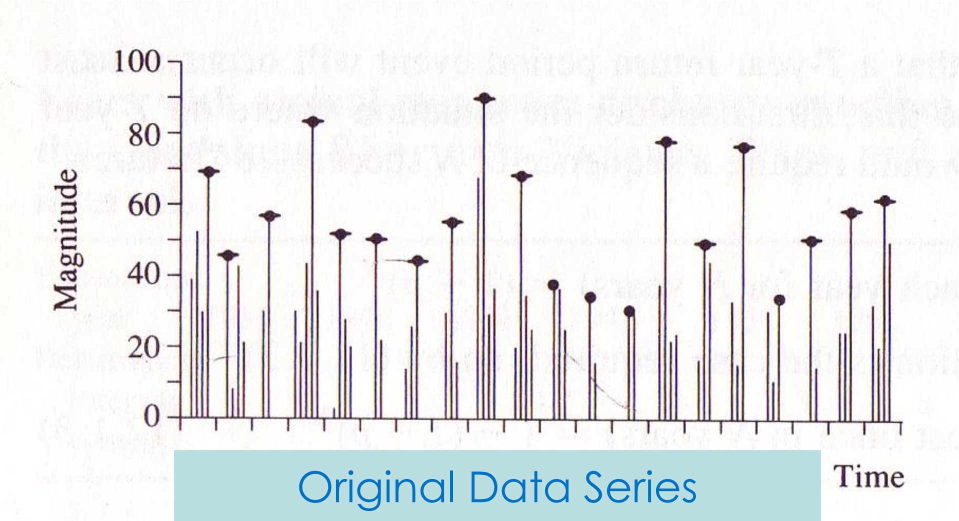
Estimate the probability that the annual maximum discharge Q on the Guadalupe River will exceed 50,000 cfs at least once during the next three years.

Solution

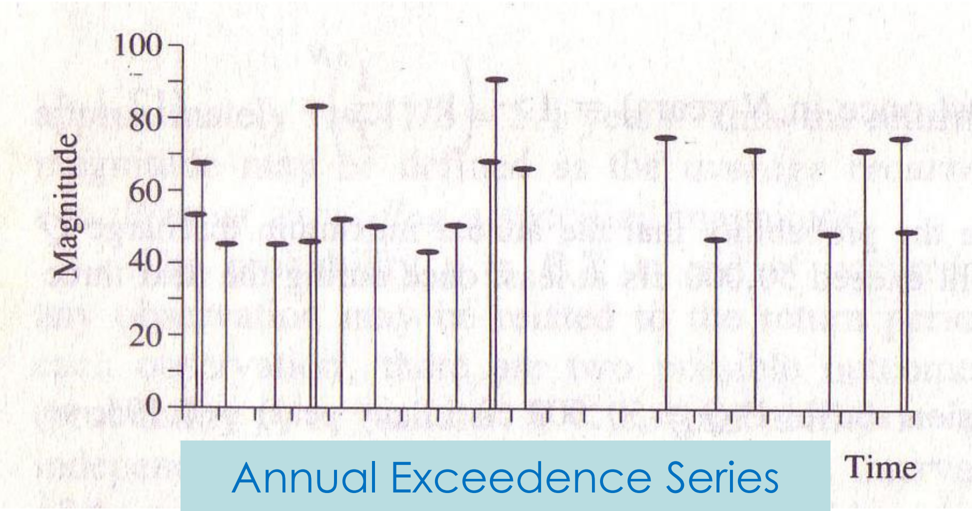
From the discussion above, $P(Q \geq 50{,}000 \text{ cfs in any year}) \approx 0.0195$

So, $P(Q \geq 50{,}000 \text{ cfs at least once during the next 3 years}) = 1-(1-0.195)^3$

HYDROLOGIC DATA SERIES



Complete Duration Series/Original Data Series consists of all the data (N=20 years).

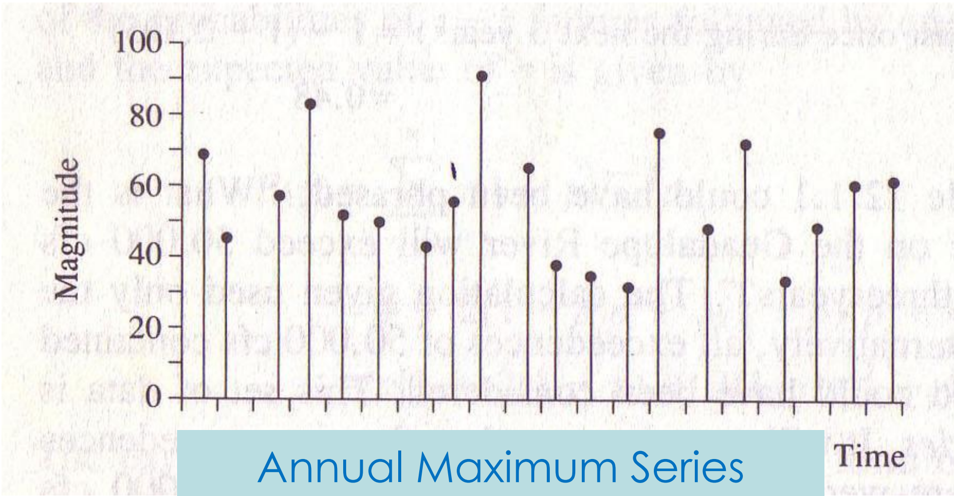


Source: Chow et al. (1988)

Partial Duration Series is a series of data which are selected so their magnitude is greater than a predefined base value.

If the base value is selected so that the number of values in the series is equal to the number of years of the record, the series is called an annual exceedence series.

HYDROLOGIC DATA SERIES



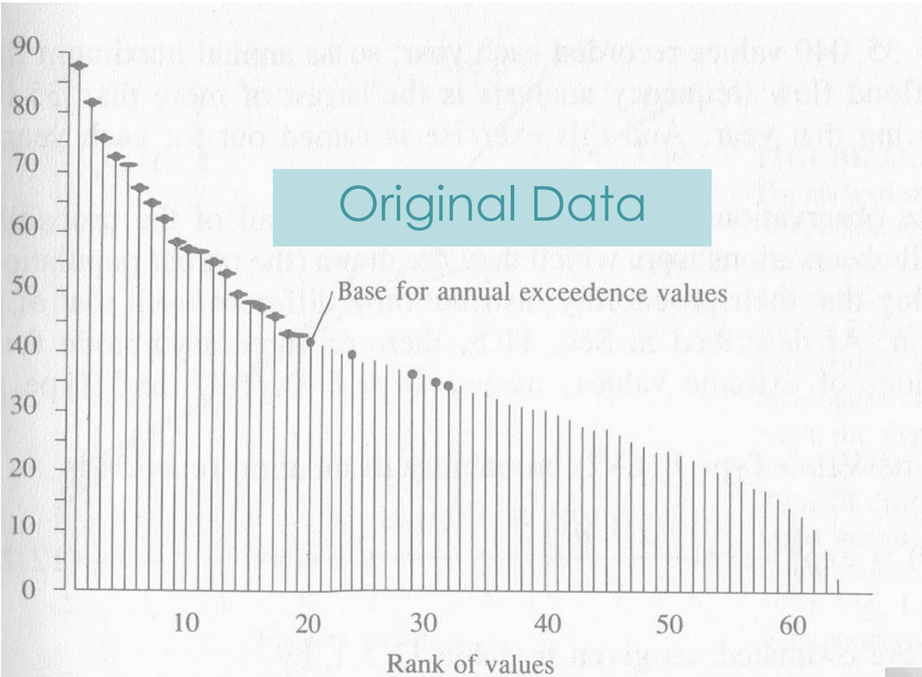
- Using largest annual values, it is an **annual maximum serie**.
- Selecting the smallest annual values produces an **annual minimum series**.

Partial Duration Series

- An extreme value series includes the largest and smallest values occurring in each of the equally-long time intervals of the record.
- The time interval length is usually taken as one year, and a series so selected is called **annual series**.

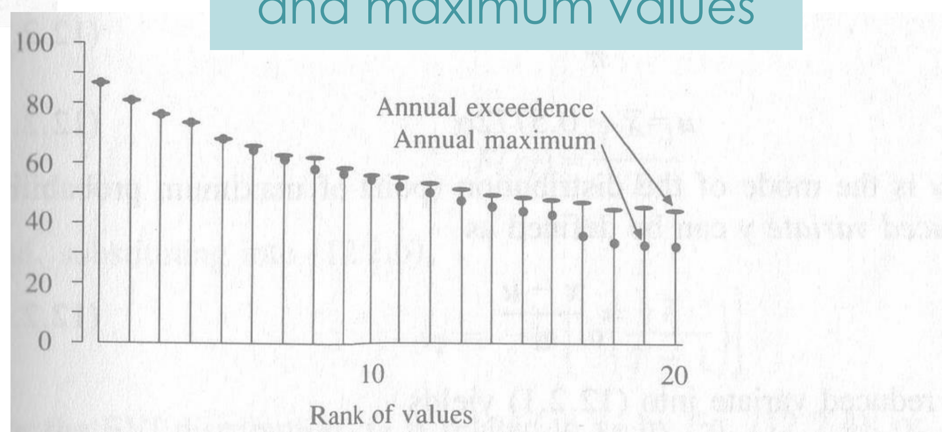
HYDROLOGIC DATA SERIES

Original Data



The annual maximum values and the annual exceedence values of the hypothetical data are arranged graphically in figure in order of magnitude.

Annual Exceedence and maximum values



EXTREME VALUE DISTRIBUTIONS

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The study of extreme hydrologic events involves the selection of a sequence of the largest or smallest observations from sets of data. For example,

Peak Flow

Use just the largest flow recorded each year at a gaging station out of the many thousands of values recorded.

Water Level

Water level is usually recorded every 15 minutes, so

there are 4x24 = 96 values recorded each day

365x96 = 35,040 values recorded each year

EXTREME VALUE DISTRIBUTIONS: EXTREME VALUE TYPE1

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CDF: Extreme Value Type 1

$$F(x) = \exp[-\exp(-\frac{x-\mu}{\alpha})]$$

$$-\infty \leq x \leq \infty$$

Parameters:

$$\alpha = \frac{\sqrt{6}s}{\pi}$$

$$\mu = \bar{x} - 0.5772\alpha$$

Reduced Variate, y:

$$y = \frac{x-\mu}{\alpha}$$

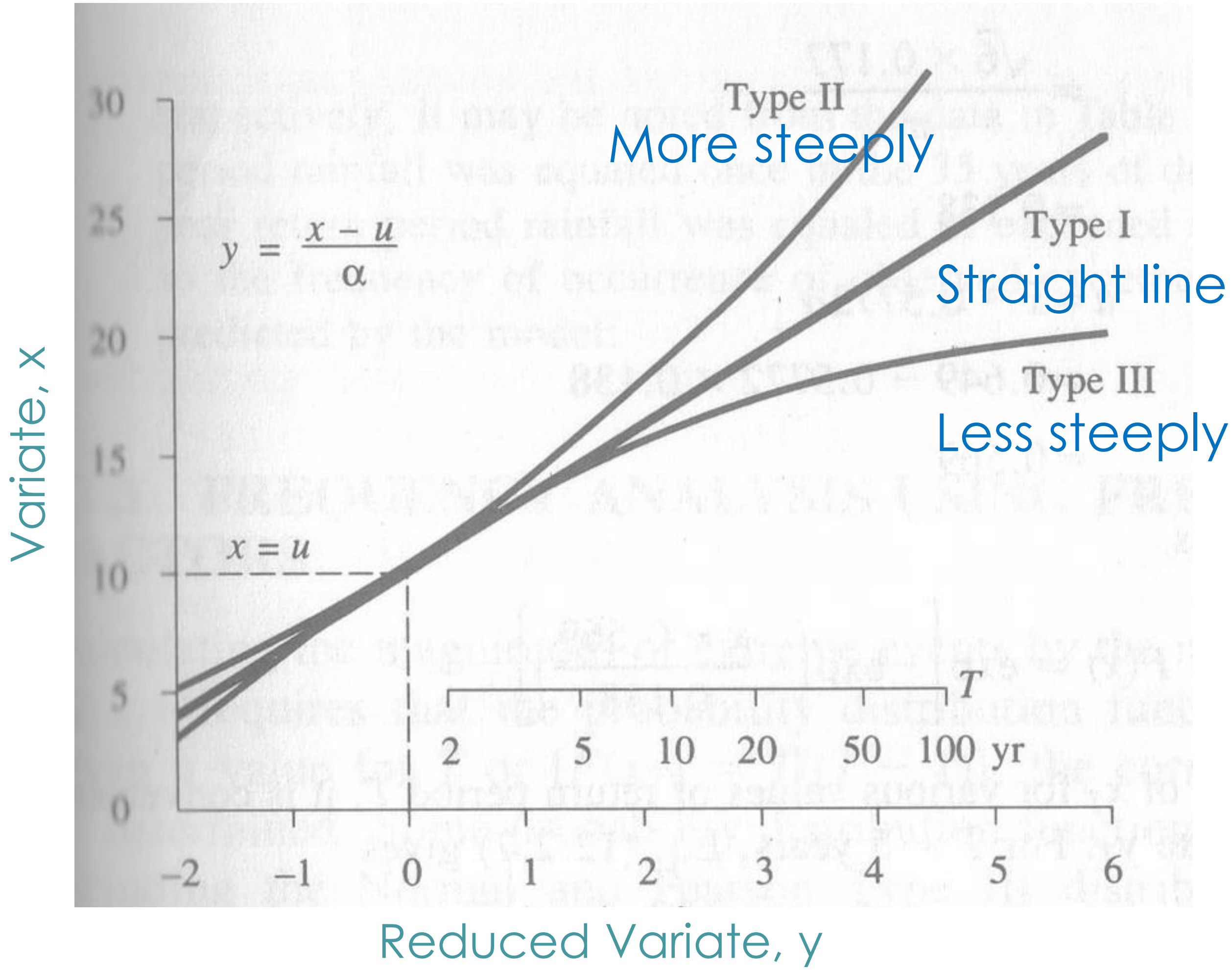
CDF:

$$F(x) = \exp[-\exp(-y)]$$

Solving for y:

F(X) → Define y for Type II, Type II Distributions

EXTREME VALUE DISTRIBUTIONS: EXTREME VALUE TYPE1



EXTREME VALUE DISTRIBUTIONS: EXTREME VALUE TYPE1

Return Period:

$$\frac{1}{Tr} = P(X \geq X_{Tr})$$
$$= 1 - P(X < X_{Tr})$$
$$= 1 - F(X_{Tr})$$

→

$$F(X_{Tr}) = \frac{Tr - 1}{Tr}$$

EV(I) Distribution, y_{Tr}:

$$y_{Tr} = -\ln[\ln(\frac{Tr}{Tr - 1})]$$

EV(I) Distribution, x_{Tr} :

$$x_{Tr} = \mu + \alpha y_{Tr}$$

EXTREME VALUE DISTRIBUTIONS: EXAMPLE 2

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Annual maximum values of 10-minute duration rainfall at Chicago, illinois from 1913 to 1947 are presented in the table. Develop a model for storm rainfall frequency analysis using the Extreme Value Type I distribution and calculate the 5, 10, and 50 year return period maximum values of 10 minute rainfall at Chicago.

Year	1910	1920	1930	1940
0		0.53	0.33	0.34
1		0.76	0.96	0.70
2		0.57	0.94	0.57
3	0.49	0.80	0.80	0.92
4	0.66	0.66	0.62	0.66
5	0.36	0.68	0.71	0.65
6	0.58	0.68	1.11	0.63
7	0.41	0.61	0.64	0.60
8	0.47	0.88	0.52	
9	0.74	0.49	0.64	

Mean = 0.649 in
SD= 0.177 in

EXTREME VALUE DISTRIBUTIONS: EXAMPLE 2

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Solution

$$\alpha = \frac{\sqrt{6}s}{\pi} = \frac{\sqrt{6}x0.177}{\pi} = 0.138$$

$$\mu = \bar{x} - 0.5772\alpha = 0.649 - 0.5772x0.138 = 0.569$$

Probability Model:

$$F(x) = \exp[-\exp(-\frac{x - 0.569}{0.138})]$$

To determine the values of x_{Tr} for Tr = 5 years:

$$y_{Tr} = -\ln[\ln(\frac{Tr}{Tr - 1})] = -\ln[\ln(\frac{5}{5 - 1})] = 1.50$$

$$x_{Tr} = \mu + \alpha y_{Tr} = 0.569 + 0.138x1.50 = 0.78 \text{ in}$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR

The magnitude x_{tr} of a hydrologic event can be represented as the mean plus the departure Δx_{tr} of the variate from the mean.

Mean

$$X_{Tr} = \mu + \Delta X_{Tr}$$

Departure

$$X_{Tr} = \mu + K_{Tr}\sigma$$

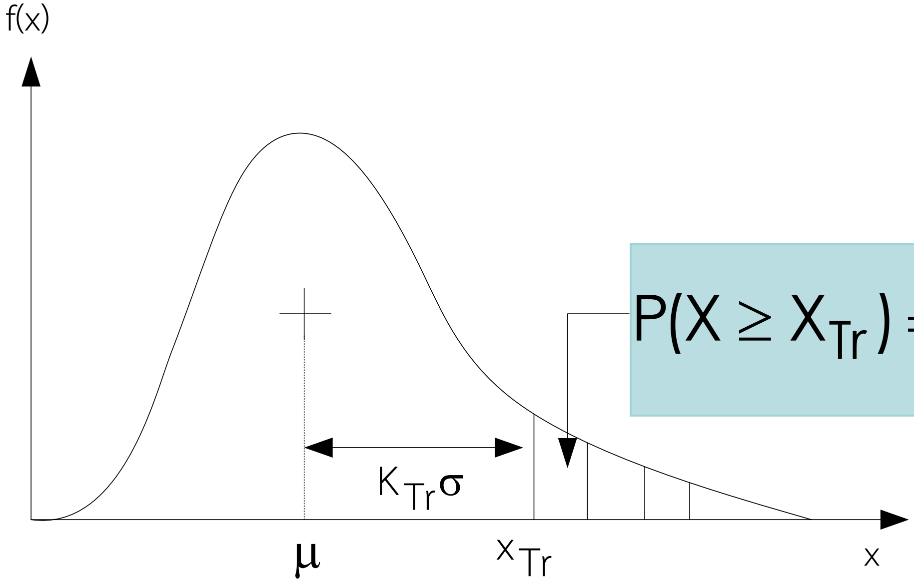
K_{Tr} = Frequency Factor

[Population]

$$x_{Tr} = \bar{x} + K_{Tr}S_x$$

[Sample]

FREQUENCY ANALYSIS USING FREQUENCY FACTOR



$$P(X \geq X_{Tr}) = \frac{1}{Tr} = \int_{X_{Tr}}^{\infty} f(x)dx$$

The theoretical K-Tr relationships for several probability distributions commonly used in hydrologic frequency analysis are now described.

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: NORMAL DISTRIBUTION

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Frequency Factor:

$$K_{Tr} = \frac{x_{Tr} - \mu}{\sigma} = z$$

Value z:

$$w = [\ln(\frac{1}{p^2})]^{1/2} \quad (0 < p \leq 0.5)$$

$$z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}$$

When p > 0.5, 1-p is substituted for p in equation * and the value of z computed by equation ** is given a negative sign.

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: NORMAL DISTRIBUTION/EXAMPLE3

Calculate the frequency factor for the normal distribution for an event with a return period of 50 years.

Solution

For Tr = 50 years, p = 1/50 = 0.02

$$w = [\ln(\frac{1}{p^2})]^{1/2} = [\ln(\frac{1}{0.02^2})]^{1/2} = 2.7971$$

$$K_{Tr} = z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}$$
$$= 2.054$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: EXTREME VALUE (1) DISTRIBUTION

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Frequency Factor:

$$K_{Tr} = -\frac{\sqrt{6}}{\pi}\{0.5772 + \ln[\ln(\frac{Tr}{Tr-1})]\}$$

Return Period:

$$Tr = \frac{1}{1 - \exp\{-\exp[-(\gamma + \frac{\pi K_{Tr}}{\sqrt{6}})]\}}$$

$$\gamma = 0.5772$$

$$x_{Tr} = \mu$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: EXTREME VALUE (1)/EXAMPLE 4

Determine the 5 year return period rainfall for Chicago using the frequency factor of Extreme Value (I) Distribution and the annual maximum rainfall data given in the table.

Solution

For Tr = 5 years

$$K_{Tr} = -\frac{\sqrt{6}}{\pi}\{0.5772 + \ln[\ln(\frac{Tr}{Tr-1})]\} = -\frac{\sqrt{6}}{\pi}\{0.5772 + \ln[\ln(\frac{5}{5-1})]\}$$
$$= 0.719$$

$$x_{Tr} = \bar{x} + K_{Tr}S_x = 0.0649 + 0.719X0.177$$
$$= 0.78\,in$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: LOG-PEARSON (III) DISTRIBUTION

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Frequency Factor:

$$K_{Tr} = z + (z^2 - 1)k + \frac{1}{3}(z^3 - 6z)k^2 - (z^2 - 1)k^3 + zk^4 + \frac{1}{3}k^5$$

where

$$k = \frac{C_S}{6}$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR

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Positive Skew

Skew coefficient C_s or C_w	2	5	10	25	50	100	200
	Exceedence probability						
	0.50	0.20	0.10	0.04	0.02	0.01	0.005
3.0	−0.396	0.420	1.180	2.278	3.152	4.051	4.970
2.9	−0.390	0.440	1.195	2.277	3.134	4.013	4.909
2.8	−0.384	0.460	1.210	2.275	3.114	3.973	4.847
2.7	−0.376	0.479	1.224	2.272	3.093	3.932	4.783
2.6	−0.368	0.499	1.238	2.267	3.071	3.889	4.718
2.5	−0.360	0.518	1.250	2.262	3.048	3.845	4.652
2.4	−0.351	0.537	1.262	2.256	3.023	3.800	4.584
2.3	−0.341	0.555	1.274	2.248	2.997	3.753	4.515
2.2	−0.330	0.574	1.284	2.240	2.970	3.705	4.444
2.1	−0.319	0.592	1.294	2.230	2.942	3.656	4.372
2.0	−0.307	0.609	1.302	2.219	2.912	3.605	4.298
1.9	−0.294	0.627	1.310	2.207	2.881	3.553	4.223
1.8	−0.282	0.643	1.318	2.193	2.848	3.499	4.147
1.7	−0.268	0.660	1.324	2.179	2.815	3.444	4.069
1.6	−0.254	0.675	1.329	2.163	2.780	3.388	3.990
1.5	−0.240	0.690	1.333	2.146	2.743	3.330	3.910
1.4	−0.225	0.705	1.337	2.128	2.706	3.271	3.828
1.3	−0.210	0.719	1.339	2.108	2.666	3.211	3.745
1.2	−0.195	0.732	1.340	2.087	2.626	3.149	3.661
1.1	−0.180	0.745	1.341	2.066	2.585	3.087	3.575
1.0	−0.164	0.758	1.340	2.043	2.542	3.022	3.489
0.9	−0.148	0.769	1.339	2.018	2.498	2.957	3.401
0.8	−0.132	0.780	1.336	1.993	2.453	2.891	3.312
0.7	−0.116	0.790	1.333	1.967	2.407	2.824	3.223
0.6	−0.099	0.800	1.328	1.939	2.359	2.755	3.132
0.5	−0.083	0.808	1.323	1.910	2.311	2.686	3.041
0.4	−0.066	0.816	1.317	1.880	2.261	2.615	2.949
0.3	−0.050	0.824	1.309	1.849	2.211	2.544	2.856
0.2	−0.033	0.830	1.301	1.818	2.159	2.472	2.763
0.1	−0.017	0.836	1.292	1.785	2.107	2.400	2.670
0.0	0	0.842	1.282	1.751	2.054	2.326	2.576

Negative Skew

ew efficient or C_w	2	5	10	25	50	100	200
	Exceedence probability						
	0.50	0.20	0.10	0.04	0.02	0.01	0.005
.1	0.017	0.846	1.270	1.716	2.000	2.252	2.482
.2	0.033	0.850	1.258	1.680	1.945	2.178	2.388
.3	0.050	0.853	1.245	1.643	1.890	2.104	2.294
.4	0.066	0.855	1.231	1.606	1.834	2.029	2.201
.5	0.083	0.856	1.216	1.567	1.777	1.955	2.108
.6	0.099	0.857	1.200	1.528	1.720	1.880	2.016
.7	0.116	0.857	1.183	1.488	1.663	1.806	1.926
.8	0.132	0.856	1.166	1.448	1.606	1.733	1.837
.9	0.148	0.854	1.147	1.407	1.549	1.660	1.749
.0	0.164	0.852	1.128	1.366	1.492	1.588	1.664
.1	0.180	0.848	1.107	1.324	1.435	1.518	1.581
.2	0.195	0.844	1.086	1.282	1.379	1.449	1.501
.3	0.210	0.838	1.064	1.240	1.324	1.383	1.424
.4	0.225	0.832	1.041	1.198	1.270	1.318	1.351
.5	0.240	0.825	1.018	1.157	1.217	1.256	1.282
.6	0.254	0.817	0.994	1.116	1.166	1.197	1.216
.7	0.268	0.808	0.970	1.075	1.116	1.140	1.155
.8	0.282	0.799	0.945	1.035	1.069	1.087	1.097
.9	0.294	0.788	0.920	0.996	1.023	1.037	1.044
.0	0.307	0.777	0.895	0.959	0.980	0.990	0.995
.1	0.319	0.765	0.869	0.923	0.939	0.946	0.949
.2	0.330	0.752	0.844	0.888	0.900	0.905	0.907
.3	0.341	0.739	0.819	0.855	0.864	0.867	0.869
.4	0.351	0.725	0.795	0.823	0.830	0.832	0.833
.5	0.360	0.711	0.771	0.793	0.798	0.799	0.800
.6	0.368	0.696	0.747	0.764	0.768	0.769	0.769
.7	0.376	0.681	0.724	0.738	0.740	0.740	0.741
.8	0.384	0.666	0.702	0.712	0.714	0.714	0.714
.9	0.390	0.651	0.681	0.683	0.689	0.690	0.690
.0	0.396	0.636	0.666	0.666	0.666	0.667	0.667

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: EXAMPLE 5

Calculate the 5 and 50 year return period annual maximum discharge of the Guadalupe River near Victoria, Texas, using the Log-Normal and Log-Pearson Type III Distributions. The data from 1935 to 1978 are given in the table.

Solution

The logarithms of the discharge values are taken and their statistics are calculated:

$$\bar{y} = 4.2743$$

$$s_y = 0.4027$$

$$C_s = -0.0696$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: EXAMPLE 5

Log-Normal Distribution:

$$y_{Tr} = \bar{y} + K_{Tr} S_y$$

$$K_{50} = 2.054$$

$$y_{50} = 4.2743 + 2.054x0.4027 = 5.101$$

$$x_{50} = (10)^{5.101} = 126,300\,cfs$$

Log-Pearson Type III Distribution:

$$k_{50} = 2.054 + \frac{(2.00 - 2.054)}{(-0.1 - 0)} = 2.016$$

$$y_{Tr} = \bar{y} + K_{Tr} S_y$$

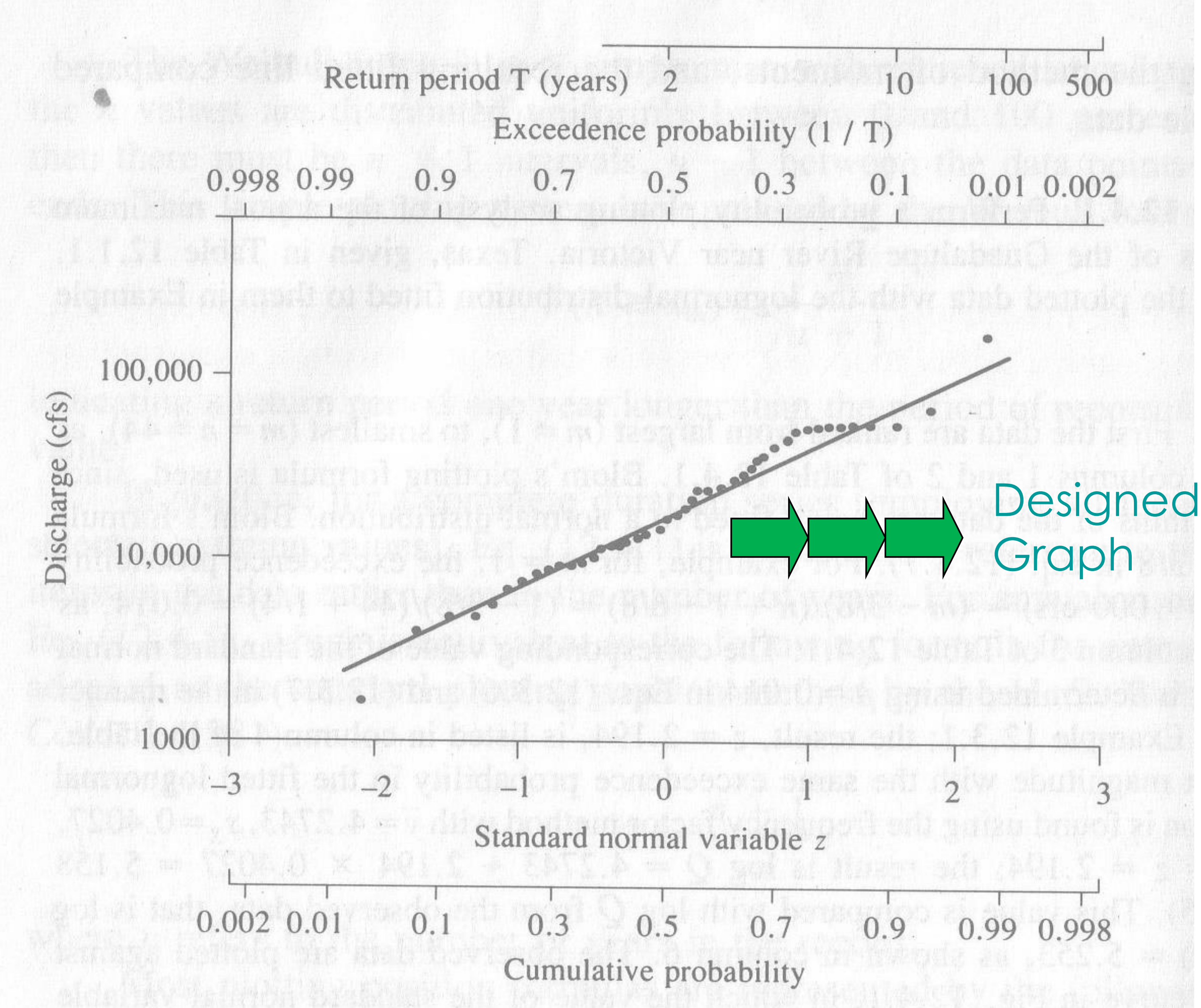
$$y_{50} = 4.2743 + 2.016x0.4027 = 5.0863$$

$$x_{50} = (10)^{5.0863} = 121,990\,cfs$$

FREQUENCY ANALYSIS USING FREQUENCY FACTOR: EXAMPLE 5

PDF	Return Period	
	5 years	50 years
Log-Normal (Cs=0)	41,060	126,300
Log-Pearson Type III (Cs=-0.07)	41,700	121,900

PROBABILITY PLOTTING



Designed Probability Graph

PROBABILITY PLOTTING

Simple Formula:

$$P(X \geq x_m) = \frac{m}{n}$$

California's Formula:

$$P(X \geq x_m) = \frac{m-1}{n}$$

Hazen's Formula (1930):

$$P(X \geq x_m) = \frac{m-0.5}{n}$$

Chegodayev's Formula:

$$P(X \geq x_m) = \frac{m-0.3}{n+0.4}$$

Weibull's Formula:

$$P(X \geq x_m) = \frac{m}{n+1}$$

When
- m = the rank of a value in a list ordered by descending magnitude
- n = the total number of values to be plotted
- x_m = the exceedence probability of the mth largest value

PROBABILITY PLOTTING

Form of most plotting position formulas:

$$P(X \geq x_m) = \frac{m-b}{n+1-2b}$$

When m = the rank of a value in a list ordered by descending magnitude

n = the total number of values to be plotted

x_m = the exceedence probability of the m^{th} largest value

b = a parameter

REFERENCES

Chow, V.T., Maidment, D.R., & Mays, L.W. (1988). *Applied hydrology*. New York: McGraw-Hill Book Company.

Assignment_Hydrologic Cycle
EGCE 323 Hydrology
Department of Civil and Environmental Engineering,
Faculty of Engineering, Mahidol University

The following figure shows the whole process of the hydrologic cycle. Please fill up the hydrological terms to describe the hydrologic cycle.

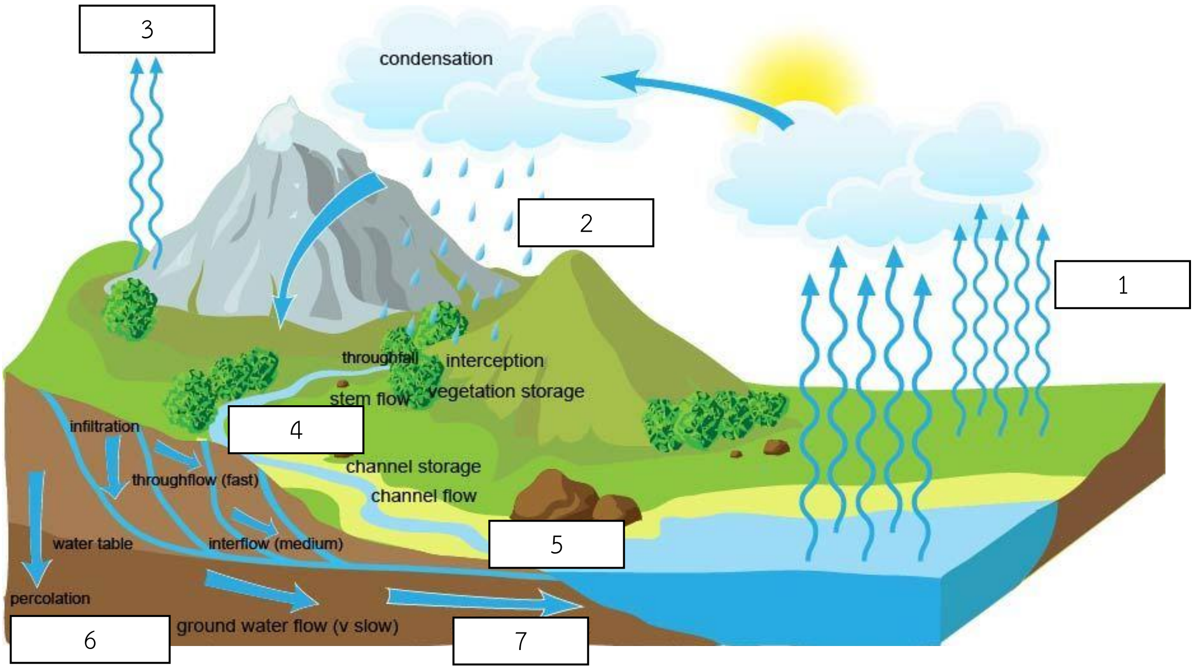


Fig.1 Hydrologic Cycle
Source : Xlskoor (2016)

Assignment_Water Budget
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

In a given year, a 15,000 mi^2 watershed receives 20 inches of precipitation. The average rate of flow in the river draining area was found to be 6,600 cfs. Estimate ET. Assume that the groundwater divide coincides with the watershed boundary, so G = 0. In one year, it can be assumed that the soil/groundwater conditions are unchanged, so $\Delta S = 0$ (1 mi = 5,280 ft).

Assignment_Residence Time
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

1. Assuming that all surface runoff to the oceans comes from rivers, calculate the average residence time of water in rivers.

2. Assuming that all groundwater runoff to the oceans comes from fresh groundwater, calculate the average residence time of this water.

	Assignment_Precipitable Water **EGCE 323 Hydrology** **Department of Civil and Environmental Engineering** **Faculty of Engineering, Mahidol University**

Calculate the precipitable water in a saturated air column 35 km high above 1 m^2 of ground surface. The surface pressure is 101 kPa, the surface air temperature is $33^{o}C$, the lapse rate is $6.0^{o}C/km$, and the increment in elevation is 5 km.

Assignment_Areal Rainfall
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

Four rain gage stations located within a rectangular river basin with four corners at (0,0), (0,13), (14,13), and (14,0) have the following coordinates and recorded rainfalls in the table below.

Rain Gage Station	Rain Gage Location
1	(2,9)
2	(7,11)
3	(12,10)
4	(6,2)

Year	Station Name				Year	Station Name			
	1	2	3	4		1	2	3	4
1	1,486	2,472	1,113	928	16	819	1,922	827	805
2	1,476	2,469	1,483	1,483	17	865	2,379	835	741
3	1,404	2,001	953	1,345	18	1,169	2,610	710	842
4	794	1,918	521	786	19	939	2,177	776	980
5	963	2,131	813	785	20	984	1,929	875	859
6	964	1,820	1,674	1,076	21	970	3,088	944	1,003
7	1,057	1,852	925	1,268	22	1,276	2,443	961	1,233
8	1,217	2,783	794	1,145	23	1,810	2,689	647	872
9	1,737	3,034	775	885	24	1,111	2,938	1,170	601
10	1,097	1,493	1,355	1,390	25	1,350	1,571	1,175	1,198
11	1,166	2,020	576	705	26	1,580	2,569	1,312	1,585
12	1,458	2,505	1,127	611	27	1,214	2,127	323	938
13	1,133	2,043	819	611	28	1,229	2,317	1,195	1,420
14	1,273	2,115	828	349	29	927	2,199	566	1,088
15	1,484	2,237	1,036	378	30	1,465	2,118	773	1,105

Unit : mm/yr

All coordinates are expressed in miles, Compute the average rainfall in the area by

(1) Arithmetic-mean method
(2) Thiessen method
(3) Isohyetal method

Assignment_ET Calculation
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

Calculate the reference evapotranspiration (ETo) using the meteorological data of station 48425 Suphanburi below.

CLIMATOLOGICAL DATA FOR THE PERIOD 1971-2000

Station: Suphanburi		Elevation of station above MSL	7 Meters
Index station	48425	Height of barometer above MSL	9 Meters
Latitude	14 28 N	Height of thermometer above grou	1.20 Meters
Longitude	100 08 E	Height of wind vane above ground	11.65 Meters
		Height of rain gauge	0.80 Meters

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	ANNUAL
Pressure (Hectopascal)													
Mean	1,012.90	1,011.40	1,010.00	1,008.40	1,007.20	1,006.40	1,006.50	1,006.70	1,008.20	1,010.20	1,012.40	1,013.90	1,009.50
Ext. max.	1,025.30	1,022.60	1,022.90	1,018.30	1,014.40	1,012.80	1,013.90	1,013.30	1,018.20	1,019.80	1,021.90	1,024.10	1,025.30
Ext. min.	1,004.60	1,001.90	1,001.00	999.60	999.70	999.00	998.60	998.80	1,000.10	1,001.90	1,003.10	1,004.20	998.60
Mean daily range	5.00	5.30	5.50	5.40	4.90	4.10	4.00	4.10	4.70	4.70	4.60	4.80	4.80
Temperature (Celsius)													
Mean	25.40	27.20	28.90	30.30	29.80	29.10	28.60	28.40	28.10	27.80	26.40	24.60	27.90
Mean max.	32.00	33.90	35.60	36.70	35.70	34.40	34.00	33.60	32.90	31.90	31.00	30.50	33.50
Mean min.	19.90	21.90	23.80	25.30	25.50	25.20	24.80	24.70	24.60	24.40	22.40	19.60	23.50
Ext.max.	36.20	39.10	40.10	41.50	41.70	39.30	40.00	37.70	36.70	36.90	35.90	35.50	41.70
Ext.min	11.30	14.10	14.90	20.70	20.90	20.20	21.10	20.80	20.80	18.00	14.50	10.00	10.00
Relative Humidity (%)													
Mean	70.00	71.00	71.00	70.00	73.00	73.00	75.00	76.00	80.00	80.00	75.00	70.00	74.00
Mean max.	89.00	92.00	92.00	90.00	89.00	88.00	89.00	90.00	93.00	93.00	90.00	87.00	90.00
Mean min.	44.00	44.00	44.00	45.00	51.00	56.00	56.00	57.00	62.00	63.00	56.00	48.00	52.00
Ext. min.	17.00	9.00	15.00	14.00	24.00	25.00	34.00	33.00	38.00	40.00	29.00	22.00	9.00
Dew Point (Celsius)													
Mean	18.90	20.80	22.20	23.40	23.90	23.70	23.50	23.50	24.20	23.70	21.00	18.30	22.30
Evaporation													
Mean-pan	129.50	138.40	181.10	195.90	188.70	169.00	163.60	155.40	135.50	133.00	130.00	133.40	1,853.50
Cloudiness (0-10)													
Mean	3.90	4.00	4.20	5.30	7.00	8.10	8.30	8.70	8.20	7.00	5.00	3.70	6.10
Sunshine Duration (hr.)													
NO OBSERVATION													
Visibility (km.)													
0700 L.S.T.	3.70	3.20	4.90	6.80	9.20	10.60	10.90	10.70	9.90	8.80	7.80	6.50	7.80
Mean	6.50	6.20	6.70	7.90	10.20	11.30	11.60	11.50	11.00	10.60	9.60	8.10	9.30
Wind (Knots)													
Mean wind speed	2.10	2.80	3.50	3.60	3.20	3.60	3.50	3.50	2.20	2.30	3.10	2.90	-
Prevailing wind	N	S	S	S	S	S	S	S	S	N	N	N	-
Max. wind speed	20.00	20.00	48.00	42.00	42.00	35.00	35.00	27.00	32.00	30.00	28.00	33.00	48.00
Rainfall (mm.)													
Mean	6.50	7.30	18.30	59.10	120.60	100.20	106.00	127.20	253.90	209.30	42.20	9.30	1,059.90
Mean rainy day	0.80	0.90	1.80	4.70	11.30	12.60	14.10	15.90	19.30	13.60	4.10	1.00	100.10
Daily maximum	63.90	49.40	95.60	146.00	137.80	66.50	89.40	66.10	120.90	187.80	84.70	73.40	187.80
Number of days with													
Haze	28.60	26.20	28.70	22.70	6.80	1.40	1.80	0.70	0.70	3.70	12.40	22.60	156.30
Fog	8.50	9.60	2.90	0.20	0.00	0.10	0.00	0.00	0.10	0.00	0.40	1.20	23.00
Hail	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Thunderstorm	0.20	0.50	1.40	5.80	10.90	6.10	5.80	5.40	12.70	9.10	1.90	0.30	60.10
Squall	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10

If the crop of interest is sweet corn, compute the potential evapotranspiration on a weekly basis.

Assignment_Flow Duration Curve
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

The observed monthly streamflow of Khaew Noi river is given in the table. Plot the flow-duration curve and estimate the flow that can be expected 80% of the time.

Year	Monthly Streamflow (cms)											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1980	192	229	254	103	166	231	135	83	105	194	156	87
1981	190	228	267	191	237	251	203	245	376	413	202	114
1982	140	192	271	211	242	194	117	347	285	486	342	142
1983	69	74	108	159	184	163	147	224	313	346	218	84
1984	36	33	34	201	140	90	60	62	67	84	43	77
1985	149	124	150	27	32	71	441	342	659	385	187	202
1986	101	119	122	199	271	346	208	337	322	302	139	124
1987	85	169	220	121	133	208	140	206	218	179	206	82
1988	105	87	133	205	218	255	164	230	279	457	117	72
1989	114	183	238	118	189	251	108	135	199	170	101	118
1990	111	85	167	232	247	166	159	240	270	284	212	84
1991	135	165	237	169	122	116	180	487	463	408	324	215
1992	54	117	146	212	192	243	253	268	203	356	227	116
1993	103	167	211	182	150	104	99	249	213	200	130	88
1994	188	183	219	192	173	125	220	655	695	381	415	322
1995	126	157	213	213	223	280	181	177	525	408	205	125
1996	162	222	259	236	222	309	435	476	758	894	479	229
1997	208	232	297	207	217	206	236	1077	630	481	339	229
1998	95	99	116	282	301	233	178	201	184	283	137	77
1999	136	193	239	123	137	104	122	310	237	375	316	141
2000	123	166	232	241	278	276	228	236	357	257	200	167
2001	165	205	268	264	214	176	202	342	372	245	155	161
2002	222	238	224	277	298	235	263	528	845	484	276	195
2003	151	204	239	247	265	223	237	261	215	288	162	137

Assignment
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

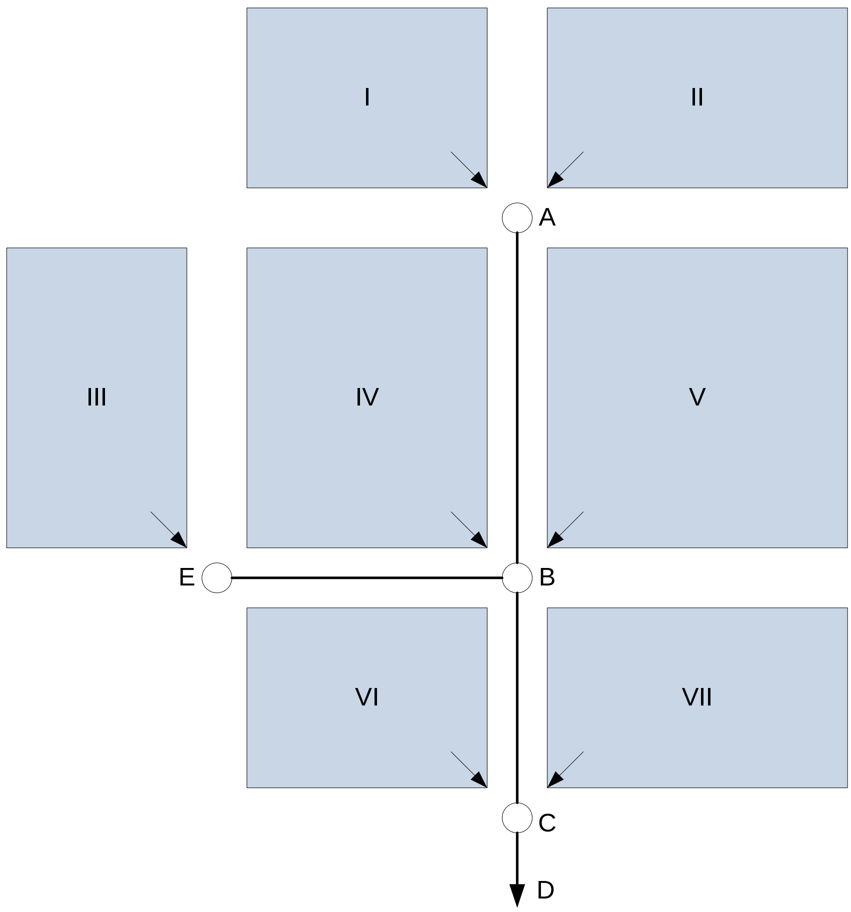
A hydrological drainage basin comprising 7 subcatchments is shown in figure. Determine the required capacity of the storm sewer EB draining subarea III for a five-year return period storm. This subcatchment has an area o f4 acres, a runoff coefficient of 0.6 and an inlet time of 10 minutes.

The design precipitation intensity for this location is given by

$$i = 120T^{0.175}/(Td+27)$$

Where i = The intensity in inch per hour
T = Return period
Td = Duration in minutes

The ground elevations at point E and B are 498.43 and 495.55 ft above sea level, respectively, and the length of pipe is 450 ft. Assume Manning's n is 0.015. Calculate the flow time in pipe.



The drainage basin and storm sewer system

Assignment_Unit Hydrograph EGCE 323 Hydrology Department of Civil and Environmental Engineering Faculty of Engineering, Mahidol University

1. The excess rainfall and direct runoff recorded for a storm are as follows:

Time (hr)	1	2	3	4	5	6	7	8	9
Excess rainfall (in)	1.0	2.0		1.0					
Direct runoff (cfs)	10	120	400	560	500	450	250	100	50

Calculate the one-hour unit hydrograph.

2. The 10-minute unit hydrograph for a 0.86 mi2 watershed has 10 minute ordinates in cfs/in of 134, 392, 475, 397, 329, 273, 227, 188, 156, 129, 107, 89, 74, 61, 51, 42, 35, 29, 24, 10, 17, 14, 11,…. Determine the peaking coefficient Cp for Snyder's method. The main channel length is 10,500 ft, and Lc = 6000 ft. Determine the coefficient Ct.

Assignment_Open Channel Flow
EGCE 323 Hydrology
Department of Civil and Environmental Engineering
Faculty of Engineering, Mahidol University

A trapezoidal open channel has bottom width of 4 ft with a slope of 0.001 ft/ft. The side slope of the channel sides (z) is 2 and the channel is constructed out of concrete (n=0.013). Determine the flow rate for steady uniform flow if the normal depth is 2 ft.